Determining Trinomial Coefficient With Ladder Multiplication and Its Construction in Circle and Cone

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Abstract: alternative ways in determining trinomial coefficient with ladder multiplication and its construction in a cone. Ladder Multiplication perform by trinomial form factoring, then determine a factor as ladder and as ladder user. Every trinomial coefficient that we get will be constructed in a cone. Coefficient that constructed from a cone is another alternative aside from construction from Pascal's pyramid. Every cone's layer that have trinomial's coefficient shows power of that trinomial's coefficient. Such as third layer means that trinomial coefficient on power n = 3.

Keywords: trinomial, multiplication, cone.

1. Introduction

From Rule of sum and rule of multiplication comes theory about permutation and combination that discuss binomial coefficient that written in$\binom{n}{r}$.Formula$\binom{n}{r}$can be seen as, many combination from r is a factor that comes from$n$ factor which$n$ is a positive integer and$r$ integer$0 \leq r \leq n$(Rosen, 2012). But in algebra(Burton, 2011)binomial coefficient is a coefficient member of$x^{n-r}y^r$expansion on $(x+y)^n$ for$n$ and$r$ is counting number. Then(Munadi, 2011)applied binomial formula on powers of two digits integer number.

Pascal's Triangle is binomial coefficients that arranged in triangle form. Besides helping in algebra expansion, application from binomial coefficient used in a lot of field such as statistic and combinatorics. (Chappell and Osler, 1999)write general form of trinomial formula as.

$$(a+b+c)^p = \sum_{m=0}^{p}\sum_{n=0}^{m}\binom{p}{m}\binom{m}{n}a^{p-m}b^n c^{m-n} \quad (1)$$

By applying multinomial coefficient formula(Bona, 2006), then we get trinomial coefficient as.

$$\binom{p}{k_1, k_2, k_3} = \frac{p!}{k_1!k_2!k_3!} \qquad (2)$$

Trinomial Coefficients is Coefficients that comes from expansion of $(a+b+c)^n$. (Kuhlmann, 2013) Did multiplication by reversing position one factor of $(x+y)^n$to determine binomial coefficient, then generalized to 3-Triangle and 4-Triangle. Trinomial Coefficient can be constructed into triangles that represented coefficient $(a+b+c)^n$(Horn, 2003). Next trinomial coefficient$(a+b+c)^n$constructed on a Layered Pascal's Pyramid(Mueller, 1969). Layers from Pascal's Pyramid shaped as triangle which contains coefficient trinomial powers by n.Pascal Pyramid which contains trinomial's coefficient powers by $n = 1, 2, 3, 4, 5$can be seen on Figure 1. Another method to determine trinomial's coefficient (Kuhlmann, 2013)by multiplies coefficient of $(x+y)^n$in Pascal's Binomial

ICASI 2019, July 18-19, Banda Aceh, Indonesia

DOI 10.4108/eai.18-7-2019.2288520

triangle powers by$n$. To determine trinomial's coefficient can also be done by modifying layered multiplication (Jufri, Sri Gemawati, 2015).

From many method to determines trinomial's coefficients which has been written, in next step given an alternative to determines trinomial's coefficient in ladder multiplication and its construction on cone. Writers hope this article could enriching method to determine trinomial's coefficient, with the result that could make it easier to determine trinomial's coefficient and also could be used in learning.

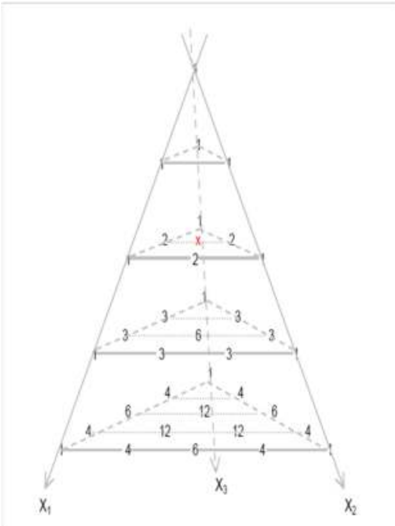

Fig.1. Pascal pyramid containing trinomial coefficients.

2. Tronimial Coefficient

a. General Trinomial Form

General trinomial form is written as formula. For example,$n$is positive integer. For all*a, b, c* applied

$(a + b + c)^n = \sum_{r=0}^{n} \sum_{s=0}^{r} \binom{n}{r} \binom{r}{s} a^{n-r} b^{r-s} c^s$ (3)

b. Determine Trinomial Coefficient with Ladder Multiplication

Ladder multiplication method to determine trinomial coefficient done as follows:
Ladder Multiplication method can be done with following steps.

1. Write$(a + b + c)^n = (a + b + c)(a + b + c)^{n-1}$. Set$(a + b + c)$as ladder and$(a + b + c)^{n-1}$as ladder user object
   as shown on Figure 2.

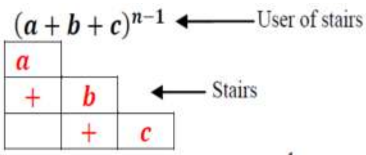

Fig. 2. $(a + b + c)$as ladder and $(a + b + c)^{n-1}$as ladder user.

2. $(a + b + c)^{n-1}$Steps on the ladder from upper stair to ground. Means element that run first is first element, followed by second element, third element and so on.
3. Then, multiply elements $(a + b + c)^{n-1}$with ladder where the element's position is located. For examples first element of$(a + b + c)^{n-1}$is *a*located on ladder*b*, then first element multiplied by *b*which is $a \times b = ab$. Then do normal sum.

Following examples determining trinomial coefficient with ladder model multiplication.

Example 1. Given$(a + b + c)^2$, will determine coefficient$(a + b + c)^2$with ladder multiplication method. Write$(a + b + c)^2 = (a + b + c)(a + b + c)$. Determine first$(a + b + c)$as ladder and second$(a + b + c)$as ladder user. Variabel *a, b, c*in second$(a + b + c)$Sorted first, second, and third element is*a, b,*and*c.* Start down the ladder*a*followed by*b*and*c*. Multiplied*a, b, c*with ladder where*a, b, c*is located, that is$(a \times a)$,$(b \times a)$,$(a \times b)$,...,$(c \times c)$. Illustration can be seen onFigure3.
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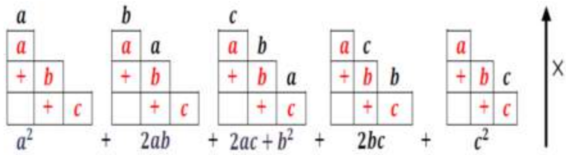

Fig.3. Determine Coefficient$(a + b + c)^2$with ladder model multiplication.

FromFigure 3,we got$(a + b + c)^2 = a^2 + 2ab + 2ac + b^2 + 2bc + c^2$with its coefficients 1, 2, 2, 1, 2, 1.

Example 2. By utilizing result on example 1, will be determine coefficient$(a + b + c)^3$with ladder model multiplication. Determine$(a + b + c)$as ladder and$a^2 + 2ab + 2ac + b^2 + 2bc + c^2$as ladder user. By applying ladder model multiplication step, Expansion illustration of$(a + b + c)^3$can be seen onFigure4.

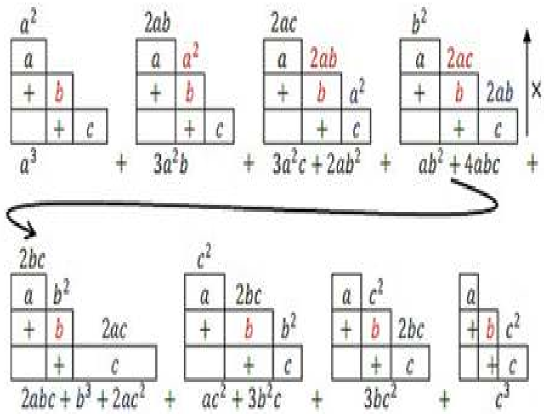

Fig.4. Determine Coefficient$(a + b + c)^3$with ladder model multiplication.

From Figure4,we obtain$(a+b+c)^3 = a^3 + 3a^2b + 3a^2c + 2ab^2 + ab^2 + 4abc + 2abc + b^3 + 2ac^2 + ac^2 + 3b^2c + 3bc^2 + c^3 = a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$with coefficient of$(a+b+c)^3$is 1, 3, 3, 3, 6, 3, 1, 3, 3, 1.

c. Trinomial Coefficient Constructed on Circle

In addition to the Pascal's Pyramid. Trinomial coefficients can be set into a circle that having same behavior with Pascal's pyramid. Even though have same behavior with Pascal's Pyramid, trinomial's coefficients on circle can be an alternative to present trinomial's coefficient.

Trinomial's coefficient start with powers of $n = 1$. Trinomial' coefficient powers by 1 can be set on circle by dividing circle side into three sides so formed 120-degree circle segment. Can be seen on Figure 5.

Trinomial Coefficient power by 2 can be set by summing nearby number with circle side trinomial powers 1 result obtained 1, 2, 1, 2, 1, 2.See Figure 6.

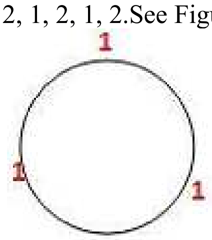

Fig.5. Trinomial coefficient powers by 1 on a circle.

In figure 5 trinomial coefficient is arranged in a way on the side of the circle.

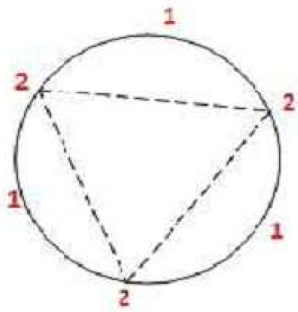

Fig.6. Trinomial coefficient powers by 2 on a circle.

In Figure 6 coefficient 1 is arranged as rule Figure 5. Then insert coefficient 2 between coefficients 1. Related each coefficient 2 with line to form a triangle.

Trinomial coefficients powers by 3, the number of sides is obtained from summary of 2 number nearby to circle side of trinomial powers by 2, then sums 3 nearby number from trinomial powers by 2 circle excepts number 1 which is 2 + 2 + 2 = 6, then write it in circle. SeeFigure 7.

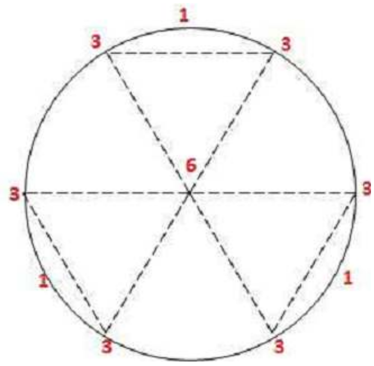

Fig.7. Trinomial coefficient powers by 3 on a circle.

In Figure 7 use the rule in Figure 6. There will be a point on the circle. Write coefficient 6 at the center of the circle

Trinomial coefficients powers by 3, the number of sides is obtained from summary of 2 number nearby to circle side of trinomial powers by 2, then sums 3 nearby number from trinomial powers by 2 circle excepts number 1 which is 3 + 3 + 6 = 12, then write on the circle.For trinomial coefficient powers by 5 apply steps same as powers by 1, 2, 3, and 4.Trinomial coefficients powers by 4 and 5 can be seen on Figure 8 and Figure 9.

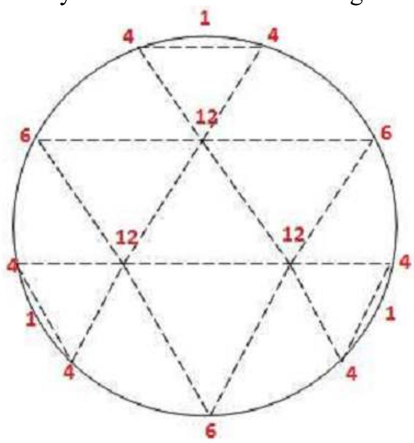

Fig.8. Trinomial Coefficient powers by 4 in a circle.

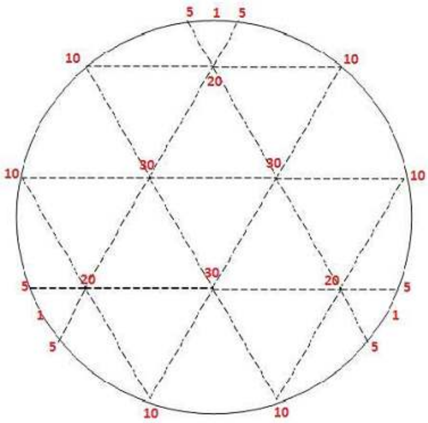

Fig.9. Trinomial Coefficient powers by 5 in a circle.

Figure 8 and figure 8 still used rule figure 6, after arrangement of coefficients on the side of the circle is met, then write other coefficients at the points formed in the circle

d. Trinomial Coefficients Construction on a Cone

Paying attention on behavior that shown by a circle that contains trinomial coefficients, we found that side of circle is binomial coefficient n, in inside part of circle is the sum of three coefficient nearby to trinomial coefficient powers by $n-1$. If trinomial circles compiled in sequence, will be formed a cone contains trinomial coefficients. The cone consists of layers that contains trinomial coefficient powers by$n$.This cone construction is another way to present trinomial coefficient other than Pascal's Pyramid.

Sides of circle which contains binomial coefficient. Numbers in each circle is summary of three numbers that is nearby to previous circle. The number of coefficient in every circle is 3, 6, 10, 15, 21 ...or $(n+1)(n+2)/2$. Summary of numbers on every circle is 3^n. By setting circles that contains trinomial coefficient then construct a cone that contains trinomial coefficient as seen onFigure 10.

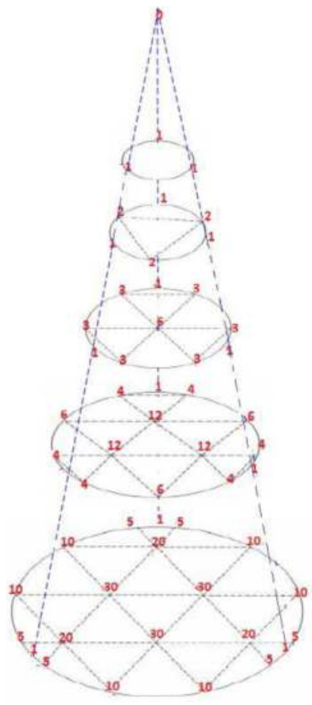

Fig.10. Cone that contains trinomial coefficients

Figure 10 is obtained by arranging a circuit containing a trinomial coefficient rank 1, 2, 3, 4 and 5. This forming a cone comprising layers containing a trinomial coefficient of rank 1, 2, 3, 4, and 5

3. Conclusions

In addition to the combinatorics trinomial coefficient can be found with ladder multiplication. Trinomial coefficient could also construct on a cone. This Alternatives is also another way that can be applied to trinomial coefficient to enriching method and presenting trinomial coefficient learning.

Acknowledgements

An infinite thanks for Ministry of Research, Technology and Higher Education(KEMENRISTEKDIKTI) which has funded this research. And forRector ofUniversitasPasirPengaraian and other colleagues which has help with this research.This Research can be continued to Quadrinomial Coefficient.

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