Mathematics of the Bond and Interest-Rate Swap Markets

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ABSTRACT. As a financial derivative instrument, interest rate swap has been invested by an increasing number of investors due to a variety of reasons, but this paper emphasizes the most significant ones which are speculative function and the role of risk avoidance. In addition, this report aims to look for relevance, including both similarities and differences between interest rate swap and bond. Important interest rates such as LIBOR rate and swap rate are also introduced, from which swap payments or cash flows in each payment date are calculated. One method to value a swap is based on the present value of two bonds. Although this report itself may have limitations, it provides a foundation for further research on either swap or other financial products.

KEYWORDS: Swap; Bond; LIBOR; Cash flow; Present value; Valuation

1. Introduction

In finance, a swap is a derivative contract involves two cash flows of one asset exchange. Of the two cash flows, one value is fixed, and the other is variable which is based on an index price, interest rate (or currency exchange rate)**.** The size of fixed rate can be predetermined when the contract is signed, while the size of floating rate is usually based on the floating rate in some international financial market, like LIBOR[1].

For valuation, considering financial markets exist mass of variate and transaction cases. Xiao provided the insight of valuing interst rate swap under uncertain market[2]. Mitra reformulated the swap calculations by using a stochastic interest rate[3]. Liang gave a new pricing model for Credit Contingent Interest Rate Swap, the variant of interest rate swap, the model is sensitive to a stochastic interest rates and counterparty deflaut risk[4]. Frino analyzed the real trading session information and indicated that there exist a distinct bidirectional flow of information between the interest rate swap and futures markets[5].

In this paper, the discussion about swap is related with bond, another vital financial inrtustment. Swap and bond both are a series of cash flow in the payment with a certain maturity. However, buyers and sellers of swaps and bonds do not trade directly, they all trade

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via an over-the-counter(OTC) market. Different from bonds that are traded in a public OTC market, swap market are a mystery to everyday individual investors and casual followers of the financial markets, because it is traded in a private way. One significant advantage of swaps is that it does not have payment of principal. Which means, the principal is nominal. As it is known, bondholders have to pay the face value if they buy a bond and will receive both the principal as well as coupon until the maturity. However, face value of a swap is only used to calculate interests based on certain rates. Swap dealers do not need to prepare a large amount of principal so the transaction is cost-saving and less risky. Therefore, we assume two application of swap for bond holders. Secondly, a method to the valuation of swap is designed with the features of bond.

The organization of this reseach paper is as follows. In section 2 we discuss the functions of interest rate swap for bond traders, and the tranction when a financial intermediary involved. In section 3, some typical terminologies are introduced for a better understanding of valuation. In section 4 and 5, swap payment and valuation method are provided detailedly.

2. Functions

After introducing a series of concepts of interest swap, it is worth mentioning that how are interest swaps used and what people can obtain by trading them. In this part, some examples are given combining simple graphical representation to illustrate our points more clearly and understandable.

2.1 Application 1

One practical application is traders can bet on the interest rate and make profit if their bet is right. Figure 1 is a scenario: Assume that there are 2 parties, A and B respectively. Now A predicts that the interest rate will decline later on and make a swap agreement with B. A receives the fixed rate and pays the floating rate.

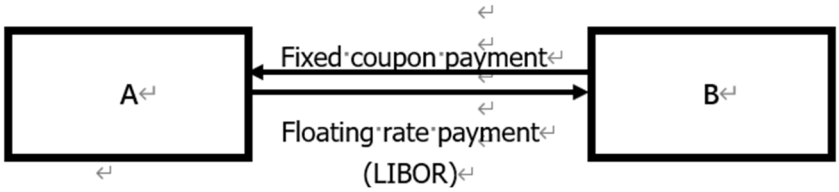

Figure 1. Interest rate swap between A and B

If A bets correctly, floating rate falls as interest rate falls. A receives the fixed rate that has been agreed but pays less floating rate (since interest rate falls), so A earns interest spread in this process. Now there is a problem, B realizes the downward trend of interest rate may cause losses and decides to terminate the agreement. Obviously, A would not easily agree unless B promises to pay a certain amount of penalty or providing other benefits.

If A misjudges the change of interest rate, the consequence is just contrary to that mentioned above. A has negative NPV temporarily because paying a higher floating rate while the fixed rate remains the same.

Thus, it is vital for interest rate swap traders to win the bet, that is, make correct judgment on the trend of interest rate if they expect to profit from the agreements.

2.2 Application 2

Swaps can be used to deal with two common risks for bond owners, interest rate risk and credit risk (default risk). For bondholders, interest rate risk is caused by interest rate fluctuations. The magnitude of bond interest rate risk depends on the sensitivity of bond prices to changes in interest rates. Momentarily, the longer the time to maturity, the greater the interest risk.

Bond value = present value of the coupons + Present value of the face amount

$$= C * [1 - 1 / (1 + r)^t] / r + F / (1 + r)^t \quad (1)$$

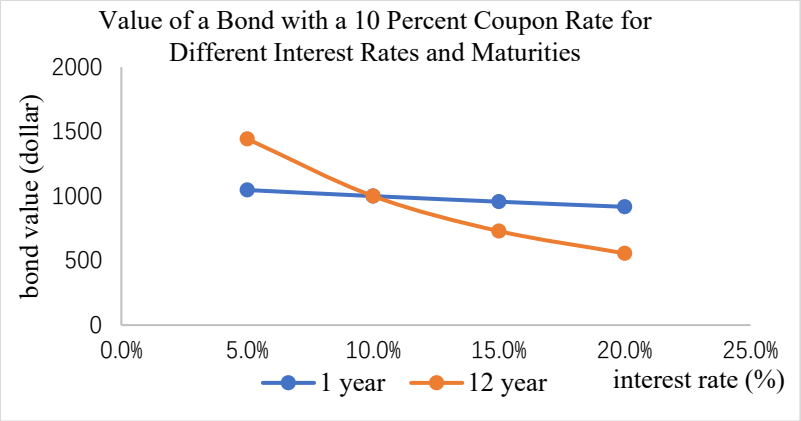

Figure 2. Interest rate risk in application 1

From Figure 2, if the coupon rates of two bonds are the same, the longer meturities are, the significant the bond value changes.

Credit risk refers to the possibility of default by the bond issuer company. Here, we introduce the concept of bond ratings. The bond ratings are an assessment of the creditor worthiness of the corporate issuer. The creditor worthiness is based on how likely the firm is to default and the protection creditors have in the event of a default.

Asset swaps can be used to overlay the fixed interest rates of bond coupons with floating rates. According to Figure 3, the fixed coupon payment that swap buyer pays to swap seller is equal to the fixed rate coupons received from the bond, and in return, the swap buyer receives variable rate payments of LIBOR plus (or minus) an agreed-upon fixed spread. The maturity of this swap is the same as the maturity of the bond.

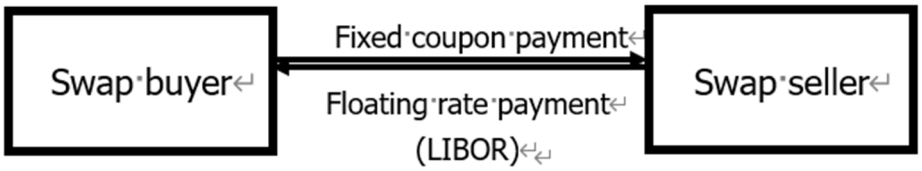

Figure 3. Asset swap in application 2

The mechanism is the same for swap buyers who want to hedge credit risk. Swap buyers are basically buying protection and swap sellers are also selling protection.

2.3 Financial intermediary

Two nonfinancial company get in touch directly.

Figure 4. Interest rate swap without a financial intermediary

As shown in Figure 4, swap buyer

1. Receives 16% per annum on the bonds.
2. Receives LIBOR under the swap contract.

Pays 18% under the swap contract

When Financial intermediary involves, it will earn about 3 or 4 basic points (0.03% or 0.04%) on a pair offsetting transaction, see Figure 5.

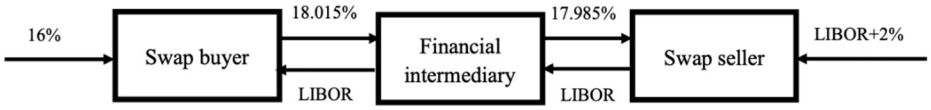

Figure 5. Interest rate swap with a financial intermediary

3. Terminologies

3.1 LIBOR

LIBOR stands for London InterBank Offered Rate. LIBOR is an indicative average interest rate at which a selection of banks (the panel banks) are prepared to lend one another unsecured funds on the London money market. LIBOR comes in 7 maturities (from overnight to 12 months) and in 5 different currencies. As Table 1 shown, LIBOR rates are announced every

working day, though different maturities (Table 1 and Table 2). LIBOR is viewed as the most important benchmark in the world for short-term interest rates. On the professional financial markets LIBOR is used as the base rate for a large number of financial products such as futures, options and swaps [6].

Table 1 EUR LIBOR interest rates - maturity 1 day Table 1. EUR LIBOR interest rates under different maturies[7]

Current interest rates	
september 14 2021	-0.59186 %
september 13 2021	-0.59171 %
september 10 2021	-0.59071 %
september 09 2021	-0.59057 %
september 08 2021	-0.59100 %
september 07 2021	-0.59414 %
september 06 2021	-0.59300 %
september 03 2021	-0.59243 %
september 02 2021	-0.59329 %
september 01 2021	-0.59386 %
august 31 2021	-0.59500 %
august 27 2021	-0.59286 %

Table 2 EUR LIBOR interest rates - maturity 12 months

Current interest rates	
september 14 2021	-0.49543 %
september 13 2021	-0.49529 %
september 10 2021	-0.49714 %
september 09 2021	-0.49829 %
september 08 2021	-0.49743 %
september 07 2021	-0.49843 %
september 06 2021	-0.49800 %
september 03 2021	-0.49929 %
september 02 2021	-0.49843 %
september 01 2021	-0.49543 %
august 31 2021	-0.49686 %
august 27 2021	-0.49800 %

3.2. Swap rate

We used interest rate swap as an example. In an interest rate swap, one party pays interest on a variable rate while the other party pays interest on a fixed rate based on a specified principal amount. Notice that, the principal amount under an interest rate swap is never paid by either counterparty.

The fixed interest rate is known as the swap rate. The symbol R is used to represent the swap rate. The swap rate will be determined at the start of the swap and will remain constant for each payment. The swap rate is demanded by the party that receives the fixed rate from the party that pays the fixed rate[8].

Swaps are traded in the over-the-counter market. When a financial intermediary involves in the swap transaction, the fixed rates are sperated as a bid rate an offer rate. The swap rate is the average of the bid and offer rates. In Figure 6, the swap rate is the averge of fixed_rate_1 and fixed_rate_2.

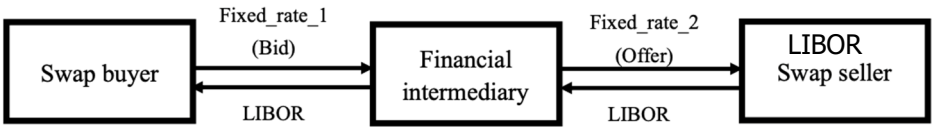

Figure 6: Rates occur in swap payments

$$\text{Swap rate} = \frac{Bid\ Rate + Offer\ Rate}{2} \tag{2}$$

4. Swap payments

An interest rate swap will specify dates during the swap term when the exchange of payments is to occur. These dates are known as settlement dates. The time between settlement dates is known as the settlement period. Settlement periods are typically evenly spaced as shown in Figure 7.

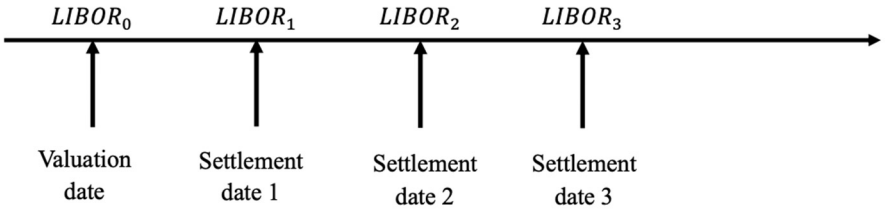

Figure 7: Payments of interest rate swap

In swap transaction, the payments related to the floating rate is called swap payments. In most cases, swap payment is determined by the LIBOR rate. The two parties get a LIBOR rate ($LIBOR_0$) at the valuation date, but don't exchange the payments. At every settlement dates, the two parties will receive a new LIBOR rate.

Saperately,

At settlement date 1, the swap payment is: $P_1 = LIBOR_0$ * nominal principal (3)

At settlement date 2, the swap payment is: $P_2 = LIBOR_1$ * nominal principal (4)

……

Thereby, swap payment formula of one certain settlement period is:

$P_n = LIBOR_{n-1}$ * nominal principal (5)

The nature of swap payment is, starting from the valuation date, each period will receive a libor rate. Each swap payment will be calculated by multiplying the previous period's LIBOR rate and the nominal principal.

5. Valuation

When valuing a swap, in other words, to know about whether swap traders profit or loss at present, the swap can be regarded as the difference in the present value of two specific bonds[1]. One bond pays fixed coupon rate while the other pays floating coupon rate. That is:

Value of Swap (to swap seller)= PV(Bond_fixed) – PV(Bond_floating) (6)

Value of Swap (to swap buyer)= PV(Bond_floating) – PV(Bond_fixed) (7)

Now this paper further explain how to calculate the present value of these two bonds. Following conditions are given and demonstrated by Figure 8:

Principal: L

Fixed interest rate: r_0

Floating interest rate: $LIBOR_n$

Cash flow: CF_n

Interests are annually compounded

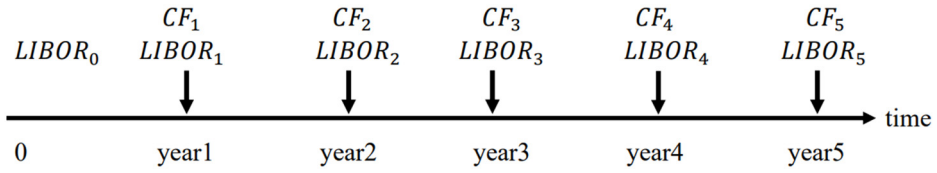

Figure 8: Cash flows and libor rates during each settlemnt time

5.1 Present valule of fixed-rate bond

As calculating any other normal bonds, use this formula:

$$PV = \frac{CF_1}{1+i} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \ldots + \frac{CF_n}{(1+i)^n} \text{ (annually compounded).} \quad (8)$$

What differs in this situation is that cash flow should be discounted by LIBOR rate (the libor rate on valuation date), such as $LIBOR_0$ shown in Figure 8.

$$\text{PV}= \frac{CF_1}{1+LIBOR} + \frac{CF_2}{(1+LIBOR)^2} + \frac{CF_3}{(1+LIBOR)^3} + \ldots + \frac{CF_n}{(1+LIBOR)^n} \text{ (annually compounded).} \quad (9)$$

Cash flow on each payment date is computed based on nominal principal L and fixed rate r_0. Since L and r_0 remain the same, therefore, cash flows are always equal (except the maturity date because the last payment date receives interest plus principal L). That is, $CF_1=CF_2=\ldots=CF_{n-1}= CF_n$- L.

Thus, PV of fixed-rate bond in Figure 8 is:

$$PV_1= \frac{L*r_0}{1+LIBOR_0} + \frac{L*r_0}{(1+LIBOR_0)^2} + \frac{L*r_0}{(1+LIBOR_0)^3} + \frac{L*r_0}{(1+LIBOR_0)^4} + \frac{L*r_0+L}{(1+LIBOR_0)^5} \quad (10)$$

5.2. Present value of floating-rate bond

Solving floating-rate bond can be more complicated. As it has been mentioned in swap payments, usually LIBOR rate of last payment date is used to calculate the cash flow of next payment date. Defined by a formula, that is: $CF_n = L*LIBOR_{n-1}$ (the last payment should add L). For instance, in Figure 8,

$$\begin{aligned} CF_1 &= L*LIBOR_0 \\ CF_2 &= L*LIBOR_1 \\ CF_5 &= L*LIBOR_4 + L \end{aligned} \quad (11)$$

PV of year 5-payment in year 4 is : $\frac{CF_5}{1+LIBOR_4} = \frac{L*LIBOR_4+L}{1+LIBOR_4} = L;$ (12)

Now, in consideration of the PV of the last payment above and the original payment in year 4, cash flow in year 4 actually changes into L+CF4 from just CF4. Hence, PV of year 4-payment

in year 3 is: $\frac{CF_4}{1+LIBOR_3} = \frac{L*LIBOR_3+L}{1+LIBOR_3} = L;$ (13)

Repeat these steps and discount all cash flows, finally it can be easily discovered that the present value is exactly the principal L.

5.3 Results

$PV_1= \frac{L*r_0}{1+LIBOR_0} + \frac{L*r_0}{(1+LIBOR_0)^2} + \frac{L*r_0}{(1+LIBOR_0)^3} + \frac{L*r_0}{(1+LIBOR_0)^4} + \frac{L*r_0+L}{(1+LIBOR_0)^5}$, which is formula (10)
PV_2=L
If $PV_1 > PV_2$, value of the swap is positive to swap seller, swap seller profits
If $PV_1 < PV_2$, then swap buyer makes money.

6. Conclusion

The primary idea of this paper is to analzy interest rate swap under a bond market view. Besides the similarity and differeces of bond and interest rate swap markets, the mathematics of them also have a closely connection. And for bond holders in financial markets, they can take swap into their investment portfolio to hedge risk. Although there exists some realistic problem in bond transction, this paper is pratical for the use of interest rate swap to a bond holder. In addition, a further research view for interest rate swap associated with bond is provided.

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