The Experimental Verification of Portfolio Management Theory

Based on Normal Distribution and i.i.d. Test

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Abstract—American economist Markowitz put forward the portfolio theory for first time in 1952. Even though it has developed for decades，constant main idea is still diversification. Based on 9 risky assets and one typical risk-free asset, we assume all are subjeccted to normal distribution and i.i.d. after test and use random functions (while generating portfolios' weights) by using R to construct numerous portfolios to check if we can truly reduce unsystematic risk while investing in different assets by diversification. The empirical results indicate that diversifying is an excellent way to effectively improve our investment efficiency and better our investment choices. This paper provides some more detailed to prove the accuracy of portfolio management theory.

Keywords-portfolio theory, data test, diversification, unsystematic risk

1 INTRODUCTION

Portfolio theory refers to an investment portfolio composed of several securities, whose returns are the weighted average of the returns of these securities but whose risks are not the weighted average of the risks of these securities. A portfolio can reduce non-systematic risks. After American economist Markowitz in May 1952 presented it, it has developed for many years. In 1964, William Sharp presented single factor model, which can estimate the covariance matrix, and Sharp also came up with the CAPM in 1964 to provide a basis for the portfolio analysis and fund performance evaluation. And APT model was presented to supply CAPM model's gap. Up to now, modern portfolio theory is mainly composed of portfolio theory, capital asset pricing model, APT model, efficient market theory, and behavioral finance theory. Their development has greatly changed the traditional investment management practice, which mainly relied on the basic analysis in the past and made modern investment management increasingly develop in the direction of systematization, scientization and combination. And portfolio theory is determined as the best analysis of optimal risk management. Families and companies choose it to help them make decisions.

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Therefore, whether seeking the highest short-term return or the lowest long-term risk, we need to study the portfolio theory, which will help us find the right investment direction in the complex and changeable capital market. This paper studies whether there will exact higher expected return, lower variance and higher Sharpe ratio in multiple asset portfolios than any single fund.

Many literatures had discussed the application of asset portfolios in financial markets. Siegel and Warner [1] proposed that assets that were riskless in real terms depend on the underlying productive technology. The return on these assets may be either endogenously or exogenously determined. Porter [2] examined changes in the optimal proportions of investment capital placed in a safe asset and a risky asset by an expected utility maximizing risk-averse investor. He, C. L. [3] studied the effect of higher moments of risky asset return on portfolio choice, and Gârleanu et al. [4] founded that distribution of risky asset return was non-normality. Gârleanu et al. [4] assessed the magnitude of displacement risk used estimates of inter-cohort consumption differences across households and found support for the model.

Some other documents had built some models to turn up Optimal portfolio. Xie et al. [5] constructed a sentiment-based optimal portfolio model which included the risk-free asset and derived the analytical solution and the efficient frontier equation, and founded that investor sentiment was a key factor, which affected the investment weight of asset and the efficient frontier. The P-value was a random variable derived from the test statistic distribution used to analyze a data set and test a null hypothesis [6]. Koldanov et al. [7] stocked selection by Sharp ratio considered in the framework of multiple statistical hypotheses testing theory. In the paper Kan et al. [8] considered optimal portfolio problems with and without risk-free assets, took into account estimation risk. For the case with a risk-free asset, we derived the exact distribution of out-of-sample returns of various optimal portfolio rules. Yan and Chen [9], based on the theory of portfolio selection with a risk-free asset, a selection model without risk-free asset is introduced. Yankov [10] attracted retail time deposits, over 7,000 FDIC insured U.S. commercial banks publicly posted their yield offers. They documented an economically sizable and highly pro-cyclical cross-sectional dispersion in these yield offers from 1997 - 2011.

This paper generates some random weight combinations derived from normal distributions to verify the portfolio management theory based on Markowitz's portfolio theory. First of all, we choose 9 assets' monthly returns and test if we can use these data under the normal distribution and i.i.d. assumption. We check the distribution by using the Shapiro-Wilk test because the sample size is less than 5000 and test the i.i.d. through the Kolmogorov-Smirnov test. After examining things above, we use the random function to generate some sets of numbers, with 9 data in a set, subjected to normal distribution N (0,3) as our weight combinations. We use those weight combinations to construct some portfolios to test whether we can achieve better indicators in the portfolios we constructed than those in every single asset. The indicators is composed of mean, standard deviation, and sharp ratio. Finally, in the results we tested, all the assets are fairly subjected to the normal distribution and comply with the i.i.d assumption. Moreover, we could observe the consequences of finding a portfolio that performs better than any other single asset through diversification. No matter investing in the risk-free asset(tbill) or not, which means that diversification is the best approach to maximize our profit or minimize our unsystematic risk and the most effective way to invest when managing our assets.

The remainder of the paper is organized as follows: Section 2 describes the samples and data and check the two basic assumptions; Section 3 performs the building of each portfolio and find out the better ones compared to the single asset; Section 4 presents our conclusions.

2 DATA DESCRIPTION AND TEST

We selected all the samples from Yahoo Finance, including 9 risky assets' returns (drefus, fidel, keystne, Putminc, scudinc, windsor, eqmrkt, valmrkt and mkt) and one risk-free asset's return(tbill) from 1968 to 1982 monthly.

Figures 1-2 show that the rate of returns of every risky asset fluctuated from 1968-01-01 to 1982-01-12, and their volatility showed similarly. The fluctuations of all the samples were presented between -10% to 10%, and the highest points always appeared during the year 1974 or 1975. Consequently, we could conclude that there were usually some sharper fluctuations between 1974 and 1975.

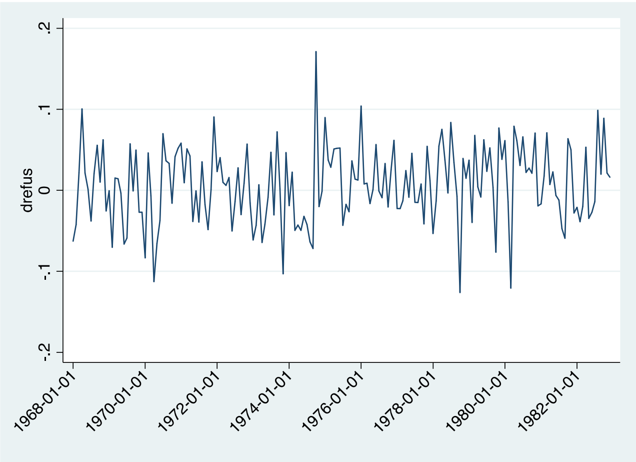

Fig.1. Monthly returns of drefus

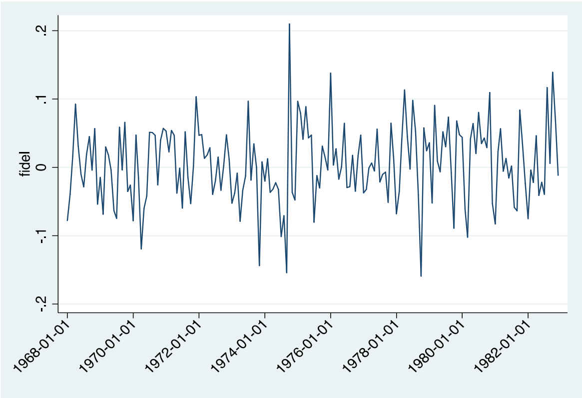

Fig.2. Monthly returns of fidel

From Figure 3 we can see that as the typical risk-free asset, the trend of the tbill was much less fluctuant than other 9 assets. From 1968 to 1972, we could observe that the data experienced a slight increase until 1970-01-01 and then a relatively dramatic downward trend was shown till 1972. We could find the similar movements as before from 1972-07-01 to 1977-07-01. After experiencing a significant increase until 1980-04-01, it showed a sharply fluctuation during the last period.

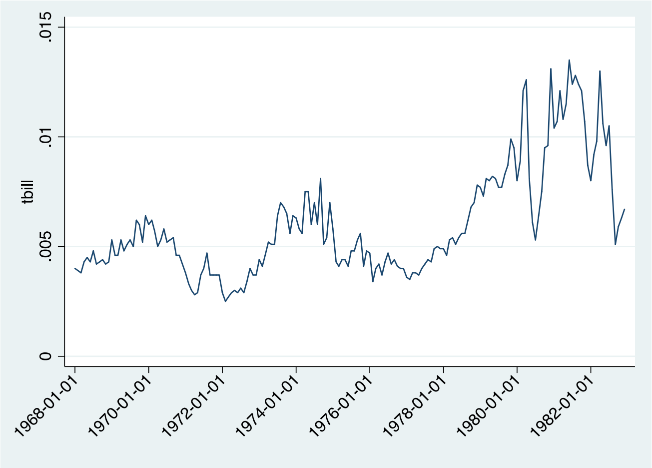

Fig.3. Monthly returns of tbill

We drew all the time series plots, and we consider that there is no need to present them all here because the trends of another 7 risky assets are much similar to these two assets (drefus and fidel).

2.1 Estimate the parameters

We use R to estimate all the parameters here (Table 1), including the mean and standard deviation:

Table 1. Mean and standard deviation of every asset

	mean	sd
drefus	0.006767	0.047237111
fidel	0.004696739	0.056587091
keystne	0.00654255	**0.08423645**
Putnminc	0.005517072	**0.030079074**
scudinc	**0.004432333**	0.035969261
windsor	0.010021906	0.048639473
eqmrkt	**0.010824756**	0.068558043
valmrkt	0.006812983	0.048000146
mkt	0.007019444	0.048572656
tbill	0.005978333	0.002522863

From Table 1, we can get the consequences of assuming that all the assets are subjected to normal distribution. We could notice that when it comes to comparison of mean, eqmrkt is the largest and scudinc is the smallest; when it comes to comparison of standard deviation, keystne has the biggest figure, and Putnminc has the minimum one.

2.2 Examination for normal distributions

We provide a relatively simple approach here: the Shapiro-Wilk test when it comes to normal distribution tests. The sample distribution is statistically compared with the normal distribution to determine whether the data show any deviation from or agreement with the normality. We did the Shapiro-Wilk test for all the assets, and the consequences are below (Table 2).

When using the Shapiro-Wilk test, we develop the null hypothesis: the samples are subjected to normal distribution. If p is smaller than 0.05, we can refuse the null hypothesis, so the tested samples could be recognized as selected from the normal distribution data set.

Table 2. p-value of all assets (Shapiro-Wilk test).

p>0.05	p<0.05
drefus	keystne
fidel	putnminc
valmrkt	scudinc
mkt	windsor
	eqmrkt
	tbill

At the same time, we draw the frequency histograms of all the assets, using the empirical rule, as the supplementary of the Shapiro-Wilk test. We present histograms of three assets below (Fig.4-Fig.6), including drefus, fidel, and tbill.

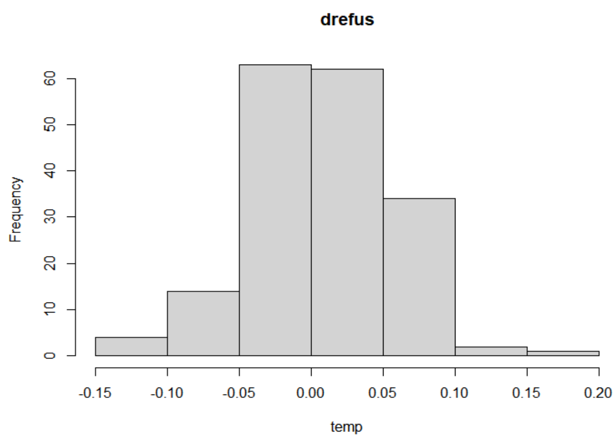

Fig.4. Histogram of drefus

As we could see from Figure 4, we found that most of the data lie in the interval between -0.10 and 0.10, and it could be recognized as approximately symmetric.

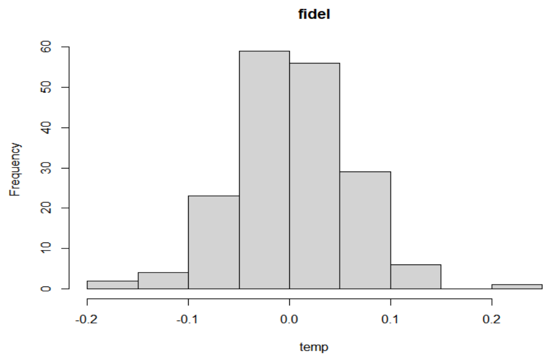

Fig.5. Histogram of fidel

From Figure 5, we could find that most data lies in the interval between -0.2 and 0.2, and there are some extreme values bigger than 0.2. However, the extreme values don't affect its symmetry.

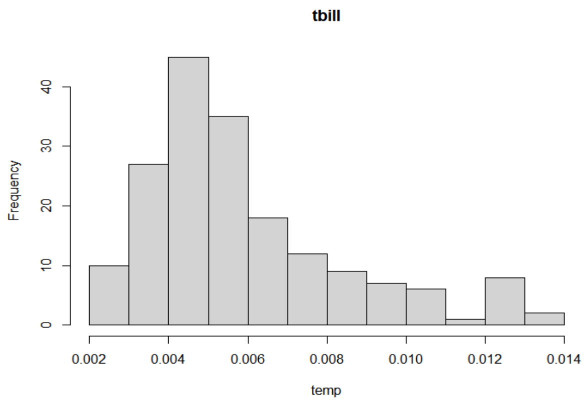

Fig.6. Histogram of tbill

According to Figure 6, we could observe that the most frequent data appear between 0.004 and 0.006, and there is an obvious rightward trend from 0.002 to 0.014 in the graph.

After calculating the quantiles of 68% and 95%, we receive the probability of lying in each interval shown in Table 3 below.

Table 3. The probability of data lying in 68% interval and 95% interval.

	68%	95%
drefus	0.672222222	0.966666667
fidel	0.716666667	0.961111111
keystne	0.733333333	0.933333333
Putnminc	0.738888889	0.955555556
scudinc	0.727777778	0.950000000
windsor	0.744444444	0.944444444
eqmrkt	0.705555556	0.966666667
valmrkt	0.727777778	0.938888889
mkt	0.727777778	0.938888889
tbill	0.772222222	0.938888889

After using the Shapiro-Wilk test and empirical rule check, we can conclude that drefus, fidel, valmrkt, and mkt could be strictly recognized as the normal distribution. Although the p-values of other assets are totally less than 0.05, all are still large than 0.01. Moreover, the sample size is not big enough, so we believe that if more data could be selected in this experiment, we could find that all the p-value will be larger than 0.05. Apart from that, looking at the table of empirical rule, we could see that the degree of deviation from the quantile is not too much.

So, we could roughly consider that all the assets are subjected to normal distribution from what has been discussed above.

2.3 Check if the samples are i.i.d.

- Check if the data of every asset are identical in distribution.

We have concluded that all the samples are subjected to the normal distribution, so we can certainly assume that the data of every asset are identical distribution.

- Check if the data of every asset are independent

As we know, the correlation coefficient of two random variables reflects the degree of linear correlation between them. If two random variables are independent, their correlation coefficient must be 0(vice versa is not necessary). Therefore, the correlation coefficient can be used to test the independence of random variables.

Given a series of random variables: calculate the coefficient of the samples with the distance k before and after:

$$\rho_k = \left(\frac{1}{n-k}\sum_{i=1}^{n-k} r_i\, r_{i+k} - (\bar{r})^2\right)/S^2 \ , k = 1,2,..., \quad (1)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2 \quad (2)$$

Check under the different values of k and propose the null hypothesis: if we can reject the null hypothesis, approaches to standard normal distribution N(0,1). Given the significance level, if we can't reject the null hypothesis, then it could be considered that they have some linear relationship, so they are not independent.

After calculating in R with Kolmogorov-Smirnov test (Table 4), we get the consequences that p-values of all the assets are smaller than 0.05, so we could conclude that all the asset samples are independent.

Table 4. p-value of every assets (Kolmogorov-Smirnov test).

	p-value
drefus	2.20E-16
fidel	2.20E-16
keystne	2.20E-16
Putnminc	2.22E-16
scudinc	2.20E-16
windsor	2.20E-16
eqmrkt	2.20E-16
valmrkt	2.20E-16
mkt	2.20E-16
tbill	6.67E-06

3 CONSTRUCT THE PORTFOLIOS

In this chapter, we decide to construct some portfolios to explore whether some portfolios can have a better-expected return, a smaller standard deviation, and a higher sharp ratio than any other single fund. And this chapter consists of the following details:

(1) Constructing five groups of portfolios and combine them with the nine funds above.

(2) Calculating these portfolios' expected return, standard deviation, and sharp ratios.

(3) Comparing these indexes with all the single funds, and find the portfolios that we are looking for.

3.1Choose five groups of portfolios

(1) Choose nine random numbers and make them the weight of a single asset in each portfolio. Every random number can be either positive or negative, which means shorting any single asset is allowed.

(2) The sum of these random numbers always equals 1. We will find five groups of random numbers.

Through the above two steps, we have five portfolios, their respective weights shown in Table 5. And we will compare these portfolios with other individual assets to get the portfolio with higher returns, smaller standard deviation, or higher Sharpe ratios relative to the individual assets. And it is permitted that the portfolios collected are different, which means there may not exist a portfolio that can meet all the requests.

According to the Table 5, we can know all the random numbers in each group, and some of them are negative which means that some funds will be shorted in such a portfolio, others will not. And these random numbers have a wide range of fluctuations, but they all obey the rule: the sum of them equals 1.

Table 5. Groups of numerical.

Groups	derfus	fidel	keystne	Putnminc	scudinc	windsor	eqmrkt	eqmrkt	mkt
W1	0.13	-0.30	0.41	0.82	0.01	0.18	-0.62	0.35	0.02
W2	7.17	-11.8	8.06	-4.47	2.52	4.10	-0.37	-11.7	7.49
W3	-0.34	2.62	0.62	-1.67	2.40	-1.41	0.22	-1.40	-0.05
W4	1.35	0.80	-0.32	-0.38	0.28	-0.67	-0.61	0.65	-0.10
W5	-0.52	-1.33	0.16	1.32	0.73	0.36	0.35	0.14	-0.20

Table 6 (following table) shows all the portfolios' mean return, standard deviation, and sharp ratio, whether risk-free is considered. For example, if considering risk-free, the portfolio w1's mean return is 0.611983. The standard deviation is 1.087740. The sharp ratio is 0.945746, while the portfolio w2's mean return is 2.874420, the standard deviation is 9.330657, and the sharp ratio is 0.204753. If not consider risk-free, the w1's mean return is 0.397688, the standard deviation is 1.089101, and the sharp ratio is 0.158991, while w2's mean return is 3.909630, the standard deviation is 9.382388, and the sharp ratio is 1.369671. And the data of other portfolios are also included.

Table 6. The mean return, standard deviation, and sharp ratio for all the portfolios

Groups	With risk-free			Without risk-free		
	mean return	standard deviation	sharp ratio	mean return	standard deviation	sharp ratio
W1	0.611983	1.087740	0.945746	0.397688	1.089101	0.158991
W2	2.874420	9.330657	0.204753	3.909630	9.382388	1.369671
W3	-0.005008	3.221938	-0.630754	-0.560058	3.257336	0.475516
W4	0.377412	1.078605	0.475549	0.033566	1.089118	0.158993
W5	0.870522	0.593108	1.394649	0.799013	0.584178	0.085280

We calculate all the single funds' mean return, standard deviation, and sharp ratio and take extreme value in Table 7. According to Table 7, we can see the extreme value in all funds. And eqmrkt has the highest mean return without risk-free, and mrk has the highest mean return. Putnminc has the smallest standard deviation both in with and without risk-free, it also has the highest sharp ratio with risk-free considered. If not consider risk-free, windsor will have the highest sharp ratio.

Table 7. The extreme value in all funds.

Target	Extreme value
The highest mean return among them	eqmrkt:0.010021906（without） mkt:0.01299778(with)
The smallest standard deviation among them	Putnminc:0.0300791(without) Putnminc:0.03027467(with)
The highest sharp ratio among them	windsor:0.20604470(without) Putnminc:0.3797038(with)

3.2To find a better-expected return than any single fund by combining these portfolios.

(1) Without risk-free asset

The formula used:

$$\text{m\_port}_1 = \text{w} \times \text{m\_asset}_1 \tag{3}$$

$$\text{outcome} = \frac{\text{m\_port}_1}{\max(\text{m\_asset}_1)} \tag{4}$$

In this formula, w is a sequence of five weighted numbers, it is:

$$\begin{pmatrix} 0.13 & -0.30 & 0.41 & 0.82 & 0.01 & 0.18 & -0.62 & 0.35 & 0.02 \\ 7.17 & -11.8 & 8.06 & -4.47 & 2.52 & 4.10 & -0.37 & -11.7 & 7.49 \\ -0.34 & 2.62 & 0.62 & -1.67 & 2.40 & -1.41 & 0.22 & -1.40 & -0.05 \\ 1.35 & 0.80 & -0.32 & -0.38 & 0.28 & -0.67 & -0.61 & 0.65 & -0.10 \\ -0.52 & -1.33 & 0.16 & 1.32 & 0.73 & 0.36 & 0.35 & 0.14 & -0.20 \end{pmatrix}$$

M_asset1 is a sequence of the mean return of every single fund without risk-free, it is:

$$(0.00677 \quad 0.00470 \quad 0.00654 \quad 0.00552 \quad 0.00443 \quad 0.01002 \quad 0.01082 \quad 0.00681 \quad 0.00702)^{T}$$

We put all the outcomes in Table 8, where we can find that W2's outcome is bigger than 1, which means that portfolio 2 has a better expected return than any single fund, it is:

$$\begin{aligned} &\text{The better expected return portfolio} \\ &\quad = 7.71 \times \text{derfus} - 11.8 \times \text{fidel} + 8.06 \times \text{keystne} - 4.47 \times \text{Putnminc} \\ &\quad + 2.25 \times \text{scudinc} + 4.10 \times \text{windsor} - 0.37 \times \text{eqmrkt} - 11.7 \times \text{valmekr} \\ &\quad + 7.49 \times \text{mkt} \end{aligned}$$

Table 8. The ratio of the average return of each portfolio to the maximum average return of individual assets (without risk-free)

Groups	Outcome
W1	0.39768832
W2	3.90963055
W3	-0.56005770
W4	0.03356624
W5	0.79901284

(2) With risk-free asset

The formula used:

$$\text{m\_port}_2 = \text{w} \times \text{m\_asset}_2 \tag{5}$$

$$\text{otucome} = \frac{\text{m\_port}_2}{\max(\text{m\_asset}_2)} \tag{6}$$

In this formula, w has no change, and the m_asset2 is the mean return of the single fund with risk-free, it is:

$$(0.01275 \quad 0.01068 \quad 0.01252 \quad 0.01150 \quad 0.01041 \quad 0.01600 \quad 0.01680 \quad 0.01279 \quad 0.01300)^{T}$$

We put all the outcomes in Table 9, and we can find that W2's outcome is bigger than 1, which means that portfolio 2 has a better expected return than any single fund, it is:

$$\text{The better expected return portfolio} = 7.71 \times \text{derfus} - 11.8 \times \text{fidel} + 8.06 \times \text{keystne} - 4.47 \times \text{Putnminc} + 2.25 \times \text{scudinc} + 4.10 \times \text{windsor} - 0.37 \times \text{eqmrkt} - 11.7 \times \text{valmekr} + 7.49 \times \text{mkt}$$

Table 9. The ratio of the average return of each portfolio to the maximum average return of individual assets (with risk-free)

Groups	Outcome
W1	0.61198345
W2	2.87441962
W3	-0.00500827
W4	0.37741154
W5	0.87052161

3.3 To find a smaller standard deviation than any single fund by combining these portfolios.

(1) Without risk-free asset

The formula used:

$$\text{sd\_port}_1 = \text{w} \times \text{sd\_asset}_1 \tag{7}$$

$$\text{otucome} = \frac{\text{sd\_port}_1}{\min(\text{sd\_asset}_1)} \tag{8}$$

In this formula, the sd_asset1 is the standard deviation of every single fund without risk-free, it is:

$$(0.04724 \quad 0.05659 \quad 0.08424 \quad 0.03008 \quad 0.03600 \quad 0.04864 \quad 0.06856 \quad 0.04800 \quad 0.04857)^T$$

We put all the outcomes in Table 10, and we can find that W5's outcome is smaller than 1, which means that portfolio 5 has a smaller standard deviation than any single fund, it is:

$$\text{The smaller standard deviation portfolio} = -0.52 \times \text{derfus} - 1.33 \times \text{fidel} + 0.16 \times \text{keystne} + 1.32 \times \text{Putnminc} + 0.73 \times \text{scudinc} + 0.36 \times \text{windsor} + 0.35 \times \text{eqmrkt} + 0.14 \times \text{valmekr} - 0.20 \times \text{mkt}$$

Table 10. The ratio of the standard deviation of each portfolio to the minimum standard deviation of individual assets (without risk-free)

Groups	Outcome
W1	1.089104
W2	9.382388
W3	3.257336
W4	1.089118
W5	0.584178

(2) With a risk-free asset

The formula used:

$$\text{sd\_port}_2 = \text{w} \times \text{sd\_asset}_2 \tag{9}$$

$$\text{otucome} = \frac{\text{sd\_port}_2}{\min(\text{sd\_asset}_2)} \tag{10}$$

In this formula, the w has no change, and the sd_asset2 is the standard deviation of every single fund with risk-free, it is:

$$(0.04724 \quad 0.05647 \quad 0.08412 \quad 0.03027 \quad 0.03601 \quad 0.04862 \quad 0.06844 \quad 0.04787 \quad 0.04845)^T$$

We put all the outcomes in Table 11, and we can find that W5's outcome is smaller than 1, which means that portfolio 5 has a smaller standard deviation than any single fund, it is:

$$\begin{aligned}\text{The smaller standard deviation portfolio} \\ = -0.52 \times \text{derfus} - 1.33 \times \text{fidel} + 0.16 \times \text{keystne} + 1.32 \times \text{Putnminc} \\ + 0.73 \times \text{scudinc} + 0.36 \times \text{windsor} + 0.35 \times \text{eqmrkt} + 0.14 \times \text{valmekr} \\ - 0.20 \times \text{mkt}\end{aligned}$$

Table 11. The ratio of the standard deviation of each portfolio to the minimum standard deviation of individual assets (with risk-free)

Groups	Outcome
W1	1.0877400
W2	9.3306570
W3	3.2219375
W4	1.0789053
W5	0.5931077

3.4To find a higher sharp ratio than any single fund by combining these portfolios.

(1) Without risk-free asset

The formula used:

$$\text{sp\_port}_1 = \text{w} \times \text{sp\_asset}_1 \tag{11}$$

$$\text{otucome} = \frac{\text{sp\_port}_1}{\max(\text{sp\_asset}_1)} \tag{12}$$

$$\text{sp\_asset}_1 = \frac{\text{m\_asset}_1}{\text{sd\_asset}_1} \tag{13}$$

In this formula, all the factors we have described above, and we put the outcomes in Table 12, and we can find that W2's outcome is bigger than 1, that means that portfolio 5 has a higher sharp ratio than any single fund, it is:

$$\begin{aligned}\text{The higher sharp ratio portfolio} \\ = 7.71 \times \text{derfus} - 11.8 \times \text{fidel} + 8.06 \times \text{keystne} - 4.47 * \text{Putnminc} \\ + 2.25 \times \text{scudinc} + 4.10 \times \text{windsor} - 0.37 \times \text{eqmrkt} - 11.7 \times \text{valmekr} \\ + 7.49 \times \text{mkt}\end{aligned}$$

Table 12. The ratio of the sharp ratio of each portfolio to the maximum sharp ratio of individual assets (without risk-free)

Groups	Outcome
W1	0.1589910
W2	1.3696714
W3	0.4755164
W4	0.1589930
W5	0.0852802

(2) With risk-free asset

The formula used:

$$\mathrm{sp\_port_2} = \mathrm{w} \times \mathrm{sp\_asset_2} \tag{14}$$

$$\mathrm{otucome} = \frac{\mathrm{sp\_port_2}}{\max(\mathrm{sp\_asset_2})} \tag{15}$$

$$\mathrm{sp\_asset_2} = \frac{\mathrm{m\_asset_2}}{\mathrm{sd\_asset_2}} \tag{16}$$

In this formula, all the factors we have described above, and we put the outcomes in Table 13. We can find that the outcome of W1, W2, W4 are bigger than 1, that means that all these portfolios have a higher sharp ratio than any single fund, and let's choose the best one as the portfolio we are looking for,it is:

$$\begin{aligned}\text{The higher sharp ratio portfolio} \\ &= -0.52 \times \mathrm{derfus} - 1.33 \times \mathrm{fidel} + 0.16 \times \mathrm{keystne} + 1.32 \times \mathrm{Putnminc} \\ &+ 0.73 \times \mathrm{scudinc} + 0.36 \times \mathrm{windsor} + 0.35 \times \mathrm{eqmrkt} + 0.14 \times \mathrm{valmekr} \\ &- 0.20 \times \mathrm{mkt}\end{aligned}$$

Table 13. The ratio of the sharp ratio of each portfolio to the maximum sharp ratio of individual assets (with risk-free)

Groups	Outcome
W1	0.9457460
W2	0.2047532
W3	-0.6307541
W4	0.4755486
W5	1.3946489

According to Table 14, we can get the suitable portfolio for each target:

Both in with and without risk-free:

$$\begin{aligned}\text{The better expected return portfolio} \\ &= 7.71 \times \mathrm{derfus} - 11.8 \times \mathrm{fidel} + 8.06 \times \mathrm{keystne} - 4.47 \times \mathrm{Putnminc} \\ &+ 2.25 \times \mathrm{scudinc} + 4.10 \times \mathrm{windsor} - 0.37 \times \mathrm{eqmrkt} - 11.7 \times \mathrm{valmekr} \\ &+ 7.49 \times \mathrm{mkt}\end{aligned}$$

(3) Both in with and without risk-free:

The smaller standard deviation portfolio

$$= -0.52 \times \text{derfus} - 1.33 \times \text{fidel} + 0.16 \times \text{keystne} + 1.32 \times \text{Putnminc} + 0.73 \times \text{scudinc} + 0.36 \times \text{windsor} + 0.35 \times \text{eqmrkt} + 0.14 \times \text{valmekr} - 0.20 \times \text{mkt}$$

(4) Without risk-free:

The higher sharp ratio portfolio

$$= 7.71 \times \text{derfus} - 11.8 \times \text{fidel} + 8.06 \times \text{keystne} - 4.47 \times \text{Putnminc} + 2.25 \times \text{scudinc} + 4.10 \times \text{windsor} - 0.37 \times \text{eqmrkt} - 11.7 \times \text{valmekr} + 7.49 \times \text{mkt}$$

With risk-free:

The higher sharp ratio portfolio

$$= -0.52 \times \text{derfus} - 1.33 \times \text{fidel} + 0.16 \times \text{keystne} + 1.32 \times \text{Putnminc} + 0.73 \times \text{scudinc} + 0.36 \times \text{windsor} + 0.35 \times \text{eqmrkt} + 0.14 \times \text{valmekr} - 0.20 \times \text{mkt}$$

Table 14. The suitable portfolio for each target.

Target	Portfolio
Higher return than single fund	W2(without risk-free)
	W2(with risk-free)
Smaller standard deviation than single fund	W5(without risk-free)
	W5(with risk-free)
Higher sharp ratio than single fund	W2(without risk-free)
	W5(with risk-free)

4 CONCLUSIONS

Portfolio theory development up to now has relatively perfect, including single-factor model. CAPM model is APT. For companies or individuals have important guiding significance, under this premise, this article explores whether the relative single fund portfolio to have higher returns and lower variance and higher Sharpe ratio, the return, variance and Sharpe ratio of the portfolio are compared with the extreme value in all single funds.

We construct the portfolios by generating some random weights as our weight combinations. Then we combine every asset with different weights, trying to find the portfolios with the largest mean, lowest standard deviation, and biggest sharp ratio whenever investing in a risk-free asset or not. Based on the different models we constructed, we could find that there are always some portfolios performing better than any other single asset, no matter the mean, standard deviation, or sharp ratio. These consequences also prove that diversification is the best way to maximize the excess return per risk unit and minimize the investment process's arbitrariness, which portfolio management theory wants to express.

There are still some limitations in our work. There is no way to find the optimal solution for a combination of random combinations, and the calculation process is a bit complicated. Additionally, our article has some deficiencies in innovation, although the article is scientific enough. Moreover, our sample is a bit small in the total process of research, which leads to our result inaccurate. In the process of writing the paper, it is also a process in which we recognize our lack of knowledge and experience. Although We collect materials as much as possible, do our best to write this paper, but the paper still has many shortcomings. Therefore, we will try our best to acquire some achievement to finish complete academic research in the future.

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