

Application of Monte Carlo Simulation in Calculating the Maximum Sharpe Ratio Based on American Funds

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Abstract— Markowitz's portfolio theory has been widely used and confirmed in practice. Most rational models of portfolio choice suggest that investors hold diversified portfolios to reduce or eliminate the non-compensated risk, thus getting a higher Sharpe ratio. Based on the data including the daily return rate on the first day of every month from 1968 to 1982 collected from CRSP, using R, this paper verifies the effectiveness of Markowitz's portfolio theory and creatively introduces the method of “Monte Carlo Simulation” into the process of finding the maximum Sharpe ratio of portfolios and evaluating the corresponding weights. This paper finds that portfolios have a higher yield, lower standard deviation and a higher Sharpe ratio than any single fund. Additionally, the maximum Sharpe ratio of the portfolio with T-Bills is higher than the one of portfolio without T-Bills. Therefore, when picking funds in the data set to make an investment, the risk-free assets should be chosen to construct the portfolio. This paper highlights the use of Sharpe ratio and providing a feasible decision-making method for investors. Moreover, this paper analyzes both risk and return factors, which improves the utilization and comprehension of data relevant to yields.

Keywords- Sharpe Ratio; Monte Carlo Simulation; Markowitz's portfolio theory

1 INTRODUCTION

Investment fund is a supply of capital which belongs to numerous investors, used to collectively purchase securities while each investor can retain ownership and control of his part of shares. The size of investment fund is expanding rapidly, leading to investors eager to seek more arbitrage opportunities. Noel et al. [1] found that the growth in mutual fund assets has not been paralleled by a corresponding focus on the process by which mutual fund investors make investment decisions. On the other hand, Chang and Lewellen [2] found that neither the skillful marketing timing nor clever security selection abilities are evident in abundance in observed mutual fund return data, and the general conclusion of the prior literature that mutual fund has been unable collectively to outperform a passive investment strategy still seems valid. However, the real situation is that vast sums of time and money are invested in the stock market because professional investors always claim the ability to pick investment portfolios that will deliver higher returns than those selected by random dart-throwers. Don et al. [3] found that the survival of actively managed mutual funds depends in part on their managers' abilities to select winning investments.

The investment performance of mutual fund portfolios has been the subject of extensive examination in the literature of finance for decades. There is a vital question in investment of funds: How should the investors most effectively allocate their initial capitals? Portfolios are built and compared under collected data of return and risk, in order to analyze to a conclusion of the most effective portfolio. Among the infinite number of asset combinations, investors would need an algorithm to find the most effective portfolio with given information.

Portfolio is a certain kind of combination of assets, which has different weights of different funds. Donald and Garud [4] found that portfolio selection is the problem of allocating capital over a number of available assets in order to maximize the “return” on the investment while minimizing the “risk.” Despite the benefits of diversification in reducing risk which has been appreciated since the inception of financial markets, the first mathematical model for portfolio selection was formulated by Markowitz (1952, 1959). In Markowitz portfolio selection model, the return on a portfolio is measured by the expected value of the random portfolio return, and the associated risk is quantified by the variance of the portfolio return. Long [5] found that a portfolio formed from a given list of assets is defined as a numeraire portfolio for the list if (a) it is self-financing, (b) its value is always positive, and (c) zero is always the best conditional forecast of the numeraire-dominated rate of return of every asset on the list. This paper makes comparisons between portfolios built with both risk-free asset and without risk-free asset.

Evans and Archer [6] found that a relatively stable and predictable relationship does indeed exist between the number of securities included in a portfolio and the level of portfolio dispersion. Their result also concerned the economic justifications of increasing portfolio sizes beyond 10 or so securities. Based on this theory, we select 10 funds including a risk-free T-bill into the calculation. In real business circumstances, there must be a lot more than just 10 assets for investors to build a portfolio and this paper can give a guiding effect on portfolio establishing.

Ederington [7] found, as did Levy and Markowitz, that for some utility functions the mean-variance approximation is so good that there is virtually no room for improvement.

A vital standard in comparing the risk and the return of assets we add into calculation is Sharpe ratio. Sharpe ratio is a commonly used measure of portfolio performance and has been evaluated for many times. It is based on the mean-variance theory and it is valid only for normally distributed returns or quadratic preferences. In this situation, once we confirmed the data is from the normal distribution, the Sharpe ratio is then a meaningful measure of portfolio performance as long as the risk can be adequately measured by standard deviation which is also satisfied in this paper. Sharpe [8] found that the Sharpe Ratio is designed to measure the expected return per unit of risk for a zero investment strategy. Israelsen [9] found that pragmatically, the SR compares the return of an asset against the return of T-bills. There were also some questions raised, Lo [10] found that by using the appropriate statistical distribution for quantifying the performance of each return history, the Sharpe ratio can provide a more complete understanding of the risks and rewards of a broad array of investment opportunities.

The reason why we choose Sharpe ratio is because it can reflect on the income earned for each unit of risk taken accurately and it is more convenient to add risk-free asset into calculation. This is basically our previous information preparation.

In this article, the empirical rule usually is used to examine whether the data follows the normal distribution and we consider our 9 assets' return data to examine the normal distribution. Then

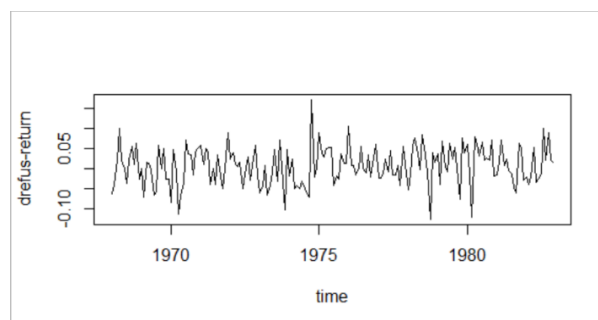
we calculate the autocorrelation coefficient for judgment, which is the similarity between observations as a function of the time lag between them. In this way, we can test the independent identical distribution. After we test the data and confirmed that all the funds are normally distributed, we begin to find portfolios that are more effective than investing in any single fund itself. It means that it has a larger possibility of gaining more return under the same risk taken. We consider three aspects to investigate the different investment portfolio, including the mean return, the standard deviation and the Sharpe ratio. Each standard is tested to find a portfolio that performs better in this standard than any fund we choose. Next we're going to find the largest Sharpe ratio directly using Monte Carlo Simulation. According to the analysis, the portfolio that pays the most per unit of risk is the one with the highest Sharpe ratio after enough simulations. This is how we are going to build the model to find the best portfolio of nine funds and one risk-free tbill.

This article calculates this formula under the situation when the data is generally normally distributed. Based on the collected data (section 2), this article analyzes whether there is a regularity in these data of the chosen funds in a period (section 3). In order to simplify calculations, return-risk is represented by mean-standard deviation. We consider an investor in a single period setting with one riskless fund and multiple risky funds, which follow a normal distribution. And the most effective portfolio, therefore, has the largest Sharpe ratio. Then randomly selects portfolios with a return higher than the maximum of any fund (section 4), risk lower than any fund and Sharpe ratio higher than any fund. With a clear chart comparison, the result of these portfolios can lead us to the solution of finding the largest Sharpe ratio (section 5).

2 DATA SOURCE

We get the real data set from CRSP (The Center for Research in Security Prices). The dataset includes daily return rate on the first day of every month from 1968 to 1982 which has ten assets including drefus, fidel, keystne, tbill and so on.

2.1 Price table



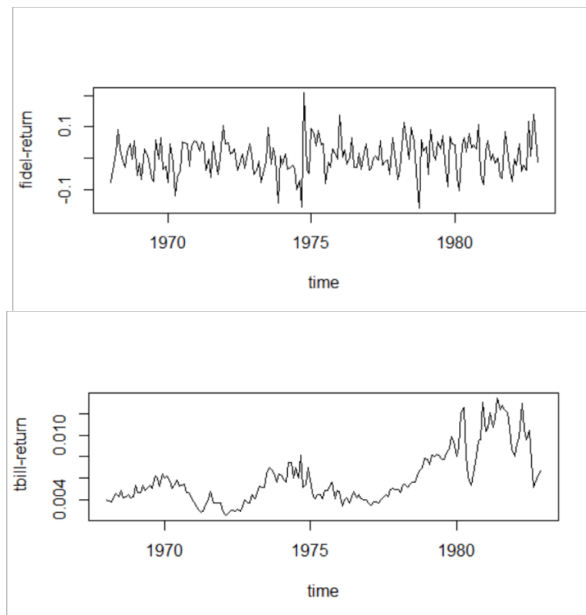


Figure 1 Time series of funds

The three charts (drefus, fidel and tbill) above (Figure 1) are selected from ten asset classes, and you can see that returns do not show a significant trend from 1968 to 1982. In the meantime, drefus and fidel have a similar trend, which both of them got the highest point on October 1st, 1974.

3 TEST THEORY

First, we describe the funds with the mean and the standard deviation. In addition, in order to construct a better portfolio, we carry out the Normal distribution test and the independent identical distribution test respectively.

3.1 Descriptive Statistic

We calculated the mean and the standard deviation based on the collected data. As we can see from Table 1, the eqmrkt has largest mean, with 0.13 and the smallest mean is scudinc, with 0.05. In the meantime, the keystne and tbill has highest and lowest standard deviation are keystne and tbill.

Table 1 Mean and Standard deviation

| | | | | | |
|------|---------|--------|---------|----------|---------|
| | drefus | fidel | keystne | Putnminc | scudinc |
| mean | 0.08 | 0.06 | 0.07 | 0.07 | 0.05 |
| sd | 0.16 | 0.20 | 0.29 | 0.10 | 0.12 |
| | windsor | eqmrkt | valmrkt | valmrkt | tbill |
| mean | 0.12 | 0.13 | 0.08 | 0.08 | 0.07 |
| sd | 0.17 | 0.24 | 0.17 | 0.17 | 0.01 |

We get the ten figures from R and choose two of Return-density figure to present (Figure 2). So the first picture is scudinc that the return fluctuates -0.13 to 0.14 and from 0 to 0.02 is the highest. The return of Keystne fluctuates by 0.6, which is the higher fluctuation than scudinc and the highest density column is 5 from -0.05 to 0.

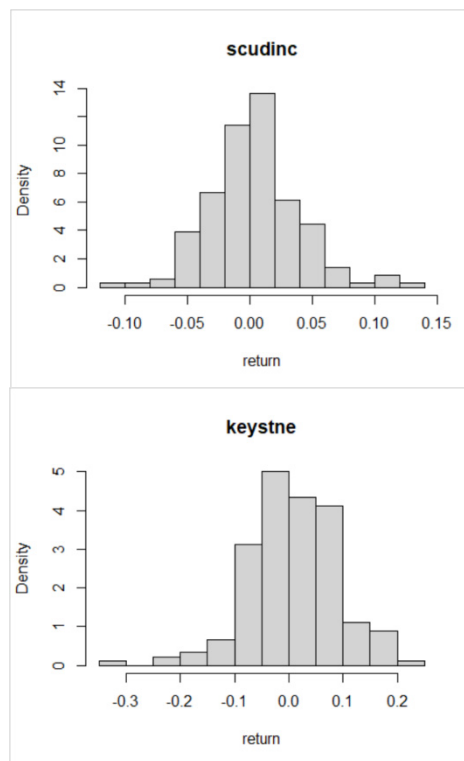


Figure 2 Return-density

3.2 Normal distribution test theory

In the literature, the empirical rule usually is used to examine whether the data follow the normal distribution. The empirical rule means that virtually every piece of data will fall within three standard deviations of the mean in a normal data set. The empirical rule includes three conditions. Firstly, proximately 68% of the data is in the interval $(x - \sigma, x + \sigma) = x \pm \sigma$ called 1sigma. Secondly, 95% of the data is in the interval $(x - 2\sigma, x + 2\sigma) = x \pm 2\sigma$ called 2sigma. Thirdly, 99% of the data is in the interval $(x - 3\sigma, x + 3\sigma) = x \pm 3\sigma$ called 3sigma. According to the outcome, we omit the third situation (99%) because all the assets are in the interval $x \pm 3\sigma$. Then, we consider our 10 assets' return data to examine the normal distribution (Table 2).

According to the empirical rule, if 1 sigma is approximately 68% and 2 sigma is about 95%, which means that the data is normally distributed. However, the condition needs relaxation so that the data is normally distributed as long as 2 sigma is around 95% cause of the harsh

condition. It can be clearly seen in the picture on the right, the two sigma values of the ten assets are all close to 0.95, so all funds are normally distributed.

Table 2 1sigma and 2sigma

| | dref us | fidel | keystn e | Putnmin c | scudin c | windsor | eqmr kt | valmr kt | mkt | tbill |
|-----------------|------------|-------|-------------|--------------|-------------|---------|------------|-------------|------|-------|
| 1sigma (68%) | 0.67 | 0.72 | 0.73 | 0.74 | 0.73 | 0.74 | 0.71 | 0.73 | 0.73 | 0.77 |
| 2sigma (95%) | 0.97 | 0.96 | 0.93 | 0.96 | 0.95 | 0.94 | 0.97 | 0.94 | 0.94 | 0.94 |

3.3 I.I.D test formula and theory

Autocorrelation is the similarity between observations as a function of the time lag between them. In order to test the independent identical distribution, we calculate the autocorrelation coefficient for judgment. The formula of autocorrelation coefficient is as following.

$$\rho(i, j) = \frac{E[(X_i - \mu_i)(X_j - \mu_j)]}{\sqrt{DX_i DX_j}} \quad (1)$$

If the autocorrelation coefficient is equal to -1, 0 and 1, it means that the data has negative correlation, uncorrelation and positive correlation respectively. Therefore, we can draw a conclusion that the assets are I.I.D when their autocorrelation coefficients are closed to 0.

Table 3 autocorrelation coefficient

| | drefus | fidel | keystne | Putnminc | scudinc | windsor | eqmrkt | valmrkt | mkt |
|------|--------|-------|---------|----------|---------|---------|--------|---------|------|
| j=1 | 0.07 | 0.07 | 0.14 | 0.22 | 0.15 | 0.09 | 0.16 | 0.06 | 0.07 |
| j=6 | -0.04 | -0.09 | 0.01 | 0.03 | -0.02 | -0.07 | -0.02 | 0.10 | 0.10 |
| j=12 | 0.05 | 0.12 | 0.08 | 0.05 | 0.11 | 0.07 | 0.21 | 0.08 | 0.09 |

We calculate the autocorrelation coefficients of those assets when lag is 1 to 12 severally, but we only show the 3 situation when j=1, j=6 and j=12 because of too much data. As shown in Table 3 above, it can be seen obviously that autocorrelation coefficients are all going to be zero. Therefore, every fund is uncorrelation which stands for the fact that all funds are independent identical distribution.

From what has been mentioned, we prove that all the assets studied conform to the normal distribution and independent identical distribution. Next, we will analyze these assets and build a portfolio in the fourth part.

4 PORTFOLIO ESTABLISHMENT

After we test the data and confirmed that all the funds are normally distributed, we begin to find portfolios that are more effective than investing in any single fund itself. It means that it has a larger possibility of gaining more return under the same risk taken. There are basically three standards: mean return, standard deviation and Sharpe ratio, which stands for the return, the risk and the return gained per unit of risk. And we managed to find certain portfolios with a mean larger than any fund, a standard deviation lower than any fund, Sharpe ratio larger than any fund

through simple formula which is written into codes in R. In this article, all the codes are tested in R.

4.1 Lower Risk

Through the code we randomly pick a portfolio with weight that satisfies the condition that its standard deviation is lower than the standard deviation of any fund (Table 4). Then we add the risk-free till into calculation and form a new portfolio includes risk-free asset (Table 5). After we selected each weight for each fund, we can calculate the standard deviation of the portfolio, then compare it with the minimum standard deviation of one fund.

$$D(R_p) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j Cov(R_i, R_j)} \quad (2)$$

In this formula, $D(R_p)$ means the standard deviation of a portfolio, w is the weight of a fund from a portfolio and R is the standard deviation of this fund. Using this formula, we can calculate the standard deviation of each portfolio and then compare them with the maximum standard deviation of a single fund.

Table 4 Portfolio without risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt |
|-----------|---------|---------|---------|----------|---------|---------|--------|--------|--------|
| weight | -0.1912 | -0.1712 | 0.5812 | -0.0227 | 0.0232 | 0.1723 | - | 0.2688 | 0.5058 |
| | | | | | | | 0.1663 | | |

Table 5 Portfolio with risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt | tbill |
|-----------|--------|--------|---------|----------|---------|---------|--------|--------|--------|--------|
| weight | - | - | - | 1.2424 | 0.4773 | -1.8801 | 3.5788 | 1.7394 | - | - |
| | 1.3342 | 1.1983 | 0.0014 | | | | | | 0.8384 | 0.7853 |

Table 6 Portfolio without risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt |
|-----------|---------|---------|---------|----------|---------|---------|--------|---------|--------|
| weight | -5.3019 | -0.1776 | -7.1737 | 0.5999 | 1.6577 | 8.4416 | 3.3413 | -7.5187 | 7.1314 |

Table 7 Portfolio with risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt | tbill |
|-----------|--------|--------|---------|----------|---------|---------|--------|--------|--------|--------|
| weight | - | - | 1.3403 | 1.0415 | 0.5937 | 0.5657 | 0.7922 | - | - | 0.2504 |
| | 0.3412 | 1.3593 | | | | | | 1.6366 | 0.2466 | |

Table 8 Portfolio without risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt |
|-----------|--------|--------|---------|----------|---------|---------|--------|--------|--------|
| weight | -0.090 | 0.1033 | 0.2175 | 0.2484 | 0.1336 | 0.2162 | -0.070 | - | 0.4028 |
| | | | | | | | | 0.1619 | |

Table 9 Portfolio with risk-free asset

| fund name | drefus | fidel | keystne | putnminc | scudinc | windsor | eqmrkt | almrkt | mkt | tbill |
|-----------|-------------|--------|-------------|----------|---------|---------|--------|--------|--------|--------|
| weight | - 0.7762 | 0.6044 | - 0.3565 | 0.0627 | 0.1465 | -0.0319 | 0.3029 | 0.0878 | 0.2576 | 0.7027 |

4.2 Larger Return

Through the code we randomly pick a portfolio with weight that satisfies the condition that its mean is larger than the mean of any fund (Table 6). Then we add the risk-free till into calculation and form a new portfolio includes risk-free asset (Table 7).

4.3 Larger Sharpe ratio

$$SR = \frac{E(R_p) - R_f}{\sigma_p} \quad (3)$$

In this formula, SR means Sharpe ratio, $E(R_p)$ means the expected rate of return on the portfolio, R_f means the risk-free rate and σ_p means the standard deviation of portfolio. This is how we can calculate the Sharpe ratio of portfolios.

Through the code we randomly pick a portfolio with weight that satisfies the condition that its Sharpe ratio is lower than the Sharpe ratio of any fund (Table 8). Then we add the risk-free till into calculation and form a new portfolio includes risk-free asset (Table 9).

4.4 Comparison for different situations

This is the simulation of finding a better portfolio with more mean return per unit of risk. While we choose from three sides, mean return, standard deviation for risk and Sharpe ratio to estimate the portfolio we established, we can compare the results to find which of the three factors is the best to reflect on the portfolios. Thus, we put those data into one form and find out the best portfolio out of the six portfolios we built, which contains three portfolios with risk-free asset and three portfolios without risk-free assets (Table 10).

Table 10 Weight of six portfolios

| | | drefus | fidel | keystne | putninc | scudinc | windsor | eqmrkt | almrkt | mkt | tbill |
|----------|-------------------|-----------------|-----------------|-----------------|-------------|------------|-----------------|-----------------|-------------|-------------|-------------|
| Min Sd | without risk-free | - 0.191 2 | - 0.171 2 | 0.581 2 | - 0.0227 | 0.023 2 | 0.172 3 | - 0.16 63 | 0.2688 | 0.5058 | |
| | with risk-free | - 1.334 2 | - 1.198 3 | - 0.001 4 | 1.2424 | 0.477 3 | - 1.880 1 | 3.57 88 | 1.7394 | - 0.8384 | - 0.7853 |
| Max mean | without risk-free | - 5.301 9 | - 0.177 6 | - 7.173 7 | 0.5999 | 1.657 7 | 8.441 6 | 3.34 13 | - 7.5187 | 7.1314 | |
| | with risk-free | - 0.341 2 | - 1.359 3 | 1.340 3 | 1.0415 | 0.593 7 | 0.565 7 | 0.79 22 | - 1.6366 | - 0.2466 | 0.2504 |
| Max R | without risk-free | - 0.090 | 0.103 3 | 0.217 5 | 0.2484 | 0.133 6 | 0.216 2 | - 0.07 0 | - 0.1619 | 0.4028 | |

| | | | | | | | | | | |
|----------------|---------|--------|---------|--------|--------|---------|--------|--------|--------|--------|
| with risk-free | -0.7762 | 0.6044 | -0.3565 | 0.0627 | 0.1465 | -0.0319 | 0.3029 | 0.0878 | 0.2576 | 0.7027 |
|----------------|---------|--------|---------|--------|--------|---------|--------|--------|--------|--------|

Table 11 Mean, standard deviation and Sharpe ratio of six portfolios

| | port1 | port2 | port3 | port4 | port5 | port6 |
|--------------------|-----------|-----------|-----------|-----------|------------|------------|
| Mean return | 0.0838964 | 0.1856469 | 0.5593642 | 0.1357593 | 0.07674736 | 0.06978986 |
| standard deviation | 0.2213901 | 0.5059279 | 1.378002 | 0.2993688 | 0.1566246 | 0.07246476 |
| Sharpe ratio | 0.3789528 | 0.3669435 | 0.4059242 | 0.4534852 | 0.4900082 | 0.9630869 |

Based on Table 11 we built, we can compare the mean return of these six portfolios. Port 3 and 4 are found to have a larger return than any fund, but their mean returns are not larger than those from Port 2. Although Port 5 and Port 6 have lower mean returns, there is not a big difference value here. Therefore, it is not very effective to find the best portfolio only by finding the largest mean return. Then we can make a comparison into the standard deviations. Port 1 and 2 are built to find lower standard deviations than any fund. But Port 4 has a lower standard deviation than Port 2. In addition, Port 5 and 6 both have lower standard deviations lower than Port 1 and 2. That means Port 5 and 6 perform better in this standard even than the two portfolios we established to satisfy this condition. Therefore, it is also not very effective to find the best portfolio only by finding the lowest standard deviation.

Finally, we can compare the Sharpe ratio of these six portfolios. Port 5 and 6 have Sharpe ratios larger than any other portfolios. What's more, Port 6 has a real large Sharpe ratio which means that this portfolio is without doubt an effective one. Thus, it is reasonable to find that using Sharpe ratio as the standard to find the best portfolio is more reliable and efficient. After lots of comparisons, Port 6 is without doubt the best of these six, for its large Sharpe ratio, low standard deviation and a mean return that is not so low.

5 MAXIMUM SHARPE RATIO

We want to find the best portfolio in the whole dataset, so we need to comprehensively consider the return and the risk of the portfolio, which requires us to choose an index to measure the performance of portfolios. After careful consideration and selection, we regard Sharpe ratio as a good indicator. The numerator of Sharpe ratio is the excess return of a portfolio and the denominator is the volatility or standard deviation of the portfolio, which means that this formula represents the excess return of each unit of risk taken by the portfolio. Therefore, the portfolio that pays the most per unit of risk is the one with highest Sharpe ratio.

In the beginning, we verified the IID and normal distribution assumption, which meets the conditions of Monte Carlo simulation. Therefore, to find the maximum Sharpe ratio in general, we choose the method of Monte Carlo simulation whose core idea is to take a probability model as the basis, follow the process described by the model, and get the result close to the exact value through many simulation experiments.

As Sharpe ratio is the slope of the capital allocation line, the slope of the tangent is the largest Sharpe ratio. However, if we still randomly generate 100000 data, it will be difficult to

distinguish the tangent point from the simulated feasible set and Capital Market Line (CML). Therefore, we're going to find the largest Sharpe ratio directly using Monte Carlo Simulation. According to the analysis above, the portfolio that pays the most per unit of risk is the one with highest Sharpe ratio after enough simulations. Using R, we find the maximum Sharpe ratio and corresponding weights (Table 12 and Table 13).

Table 12 Corresponding Weights

| | drefus | fidel | keystne | Putnminc | scudinc |
|-----------------|---------|--------|---------|----------|---------|
| Without T-Bills | 0.58 | -0.22 | 0.28 | 0.07 | 0.43 |
| With T-Bills | -0.91 | -2.34 | -0.60 | 0.55 | 0.28 |
| | windsor | eqmrkt | valmrkt | mkt | tbill |
| Without T-Bills | -0.23 | 0.52 | -0.16 | -0.27 | |
| With T-Bills | 1.13 | 0.79 | 0.74 | -0.23 | 1.59 |

According to the results in Table 12, in the portfolio without T-Bills, drefus accounts for the largest proportion of the funds bought and mkt accounts for the largest proportion of the funds sold. In the portfolio with T-Bills, tbill accounts for the largest proportion of the funds bought and fidel accounts for the largest proportion of the funds sold.

Table 13 The Maximum Sharpe Ratio and Characteristics

| | Sr | Mean | Std. |
|-----------------|------|------|------|
| Without T-Bills | 1.14 | 0.58 | 0.51 |
| With T-Bills | 1.86 | 0.26 | 0.14 |

According to the results in Table 13, the return of the portfolio without T-Bills is 0.58 and the standard deviation is 0.51, while the return of the portfolio with T-Bills is 0.26 and the standard deviation is 0.14. Moreover, we find that the maximum Sharpe ratio of the portfolio with T-Bills is 1.86, higher than that of the portfolio without T-Bills. This shows that although the average return rate of the portfolio with risk-free assets is lower than that without risk assets, it has a more obvious effect on the risk dispersion and is a better investment option. Therefore, when picking funds in the data set to make an investment, we should choose the risk-free assets to construct the portfolio.

6 CONCLUSION

According to Markowitz's portfolio theory, the variance of the portfolio is not a simple linear combination of the variance of each fund in the portfolio, but largely depends on the correlation between funds, thus providing a theoretical explanation and a practical guidance for the effective diversification and investing options.

This paper verifies the effectiveness of Markowitz's portfolio theory and creatively introduces the method of "Monte Carlo Simulation" into the process of finding the maximum Sharpe ratio of portfolios and evaluating the corresponding weights. Considering investing in risk-free funds and non-risk-free funds, we randomly choose portfolios with minimum standard deviation,

maximum return, and maximum Sharpe ratio. We find that portfolios have a higher yield, lower standard deviation, and higher Sharpe ratio than any single fund, which proves the better performance of portfolios and the effectiveness of Markowitz's portfolio theory.

Finally, we find the maximum Sharpe ratio in general and the most effective portfolio through Monte Carlo simulation. The results show that: the maximum Sharpe ratio of the portfolio with T-Bills is higher than the one of the portfolio without T-Bills. Although the average return rate of portfolio with risk-free assets is lower than that without risk assets, it has a more obvious effect on risk dispersion and is a better investment option. Therefore, when picking funds in the data set to invest, we should choose the risk-free assets to construct the portfolio.

We believe our study makes the following contributions: first of all, this paper creatively transforms the highest return per unit risk into Sharpe ratio, and then uses Sharpe ratio as the standard of asset selection, thus highlighting the use of Sharpe ratio and providing a feasible decision-making method for investors. Since Sharpe ratio is an index that can consider the return and risk at the same time with simple calculation is simple, it has become the most commonly used standardized index to measure the performance of portfolios. Secondly, this paper innovatively introduces the method of "Monte Carlo Simulation" to find the maximum Sharpe Ratio. Through the use of Monte Carlo Simulation in R, this paper can better analyze both risks and return factors, which improves the utilization and comprehension of data relevant to yields.

The shortcomings of this paper mainly include two aspects. First, our method is based on the verified IID and the normal distribution assumption. Nevertheless, in reality, many yield curves have the characteristics of a sharp peak and a thick tail, which gives the yield more probability of extreme cases. Second, the risk-free assets and the other nine funds are treated as the same, which means that the analysis and calculation are directly based on the data, without considering the fact that the standard deviation of T-Bills is 0. Therefore, in the future, we would take into account the actual characteristics of risk-free assets and break through the assumption of normal distribution, making the model conform to the characteristics of real data.

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