Investment Portfolio Establishment Based on Markowitz Model and Highest Sharp Ratio

Xinyu Lin[1,*,†], Xin Xu[2,*,†], Yang Xu[3,*,†]
[1*]18798820718@163.com, [2*]201836712178@mail.scut.edu.cn, [3*]yzxuyang@yeah.net

[1]School of Management and Engineering, Capital University of Economics and Business, Beijing, China
[2]School of Economics and Finance, South China University of Technology, Guangzhou, China
[3]School of Finance and Statistics, Hunan University, Changsha, China
[†]These authors contributed equally.

Abstract—Portfolio investment theory is critical to global financial markets. However, the specific actual combination effect is affected by factors such as weight selection and asset selection. Based on the Markowitz model, we use a more reasonable weight selection method to establish an asset portfolio. And we also explore the impact of risk-free assets on risk diversification and magnified returns. We finally use the Sharpe ratio to evaluate the optimal asset portfolio. We found that the use of historical return performance to determine the weight can optimize the portfolio impact. The addition of risk-free assets can diversify risks and increase the rate of return, thereby increasing the Sharpe ratio. Our work is to find a reasonable way to determine the weights and solve the practical problems in using the Markowitz model. This paper provides a better way to use the Markowitz model to find the optimal asset portfolio.

Keywords- Markowitz model; Sharpe ratio; optimal portfolio; risk-free assets

1. INTRODUCTION

Investment portfolio is a collection of stocks, bonds, and financial derivatives held by investors or financial institutions. This concept is divided into two levels. The first level is how to allocate proportionally among different assets; the second level is how to choose bond types and their respective weights in the same asset class. The origin of the concept of securities portfolio can be traced back to the research published in 1959 by the famous economist Markowitz [1]. Markowitz [2] uses both the expected rate of return of risky assets and the risk represented by variance to study asset selection and combination issues. He also uses the linearization theory to analyze investment income, and establishes the basic theory of applying mathematical methods to determine the best portfolio investment [3].

The main problem encountered during the application of the Markowitz model is that the calculation of the covariance matrix is far more complex than expected. To solve this problem, Sharpe's [4] research proposed a single index model. He carefully analyzed "the functional relationship between stock returns and stock market index returns" and proposed a method to simplify effective combinations, although at the expense of certain precision. It also improves the practical value of asset portfolio theory. After that, many researchers did an empirical analysis on this functional relationship and discussed the method of β prediction [3], such as Blume [5], Vasicek [6], Klemkosky, and Martin [7], and so on. Since then, Elton, Gruber, and

ICEMME 2022, November 18-20, Nanjing, People's Republic of China

DOI 10.4108/eai.18-11-2022.2326902

Padberg [8] proposed the EGP model, which uses a single exponential model and an assumed variance to solve the specific application of the Markowitz model. Regarding the related research on portfolio denoising, Daly et al. [9,10] used the traditional RMT denoising method, namely LCP denoising method [11], PG + denoising method[12], and KR denoising method[13]. It is found that the KR denoising method can maximize the accuracy of the risk prediction of the investment portfolio.

In this paper, we first establish the portfolio based on Markowitz Model where mean and variation are employed to serve as a standard to estimate the efficiency of one portfolio. Then we tested the risk diversification effect, and it turned out that adding the risk-free asset helps increase the mean of the portfolio and decrease the variation. Consequently, we contain the risk-free asset in each portfolio when looking for the optimal portfolio with the highest Sharpe ratio. Through making assumptions and building a simplified model, we finally found the optimal portfolio based on our above finding that the risk-free asset is beneficial to decrease the risk of one portfolio while increasing the return of that.

This paper is divided into seven parts: section 1 introduces the background, theme, and method of this research. In section 2, we introduce the data and check if the data is satisfied with the assumption. Section 3 mainly talks about the Markowitz model. And we discuss the advantages of the portfolio compared with the asset in the next section. In section 5, we compare the Sharp ratio of all the portfolios to find the best one which is suitable to invest. In the last two sections, we make a conclusion about this paper and introduce the reference we use.

2. DATA

The data of analysis which has various financial assets, is obtained from Yahoo finance. And this research selects four assets, including eqmrkt, valmrkt, mkt, and tbill, in the time period of January 1, 1968 and December 1, 1982. Firstly, we examine if there is none value or deeply extreme value. If they exist, we will use the mean substitution. Finally, there are 180 trading days of research data in total.

According to the descriptive statistics of variables (Table 1), We can find that the mean of eqmrkt, equal to 0.019 is the first largest one amongst the four assets. It also has the largest standard deviation, which means the data is more discrete. The rate of return ranges from -0.192 to 0.333. And the tbill has the smallest standard deviation and the nice mean. It indicates that tbill is more stable and also has a good yield.

TABLE 1. DESCRIPTIVE STATISTICS OF VARIABLES.

Variable	Mean	Std. dev	Min	Max
eqmrkt	0.019	0.069	-0.192	0.333
valmrkt	0.007	0.048	-0.118	0.164
mkt	0.007	0.049	-0.122	0.166
tbill	0.006	0.003	0.003	0.014

This research considers the usual assumptions, including normal distribution and Independent and identically distributed (i.i.d). So we check if the assumptions are reasonable in the data. We can test and verify the data that is satisfied with the assumption that the data is normal distribution by drawing the histograms (Fig.1).

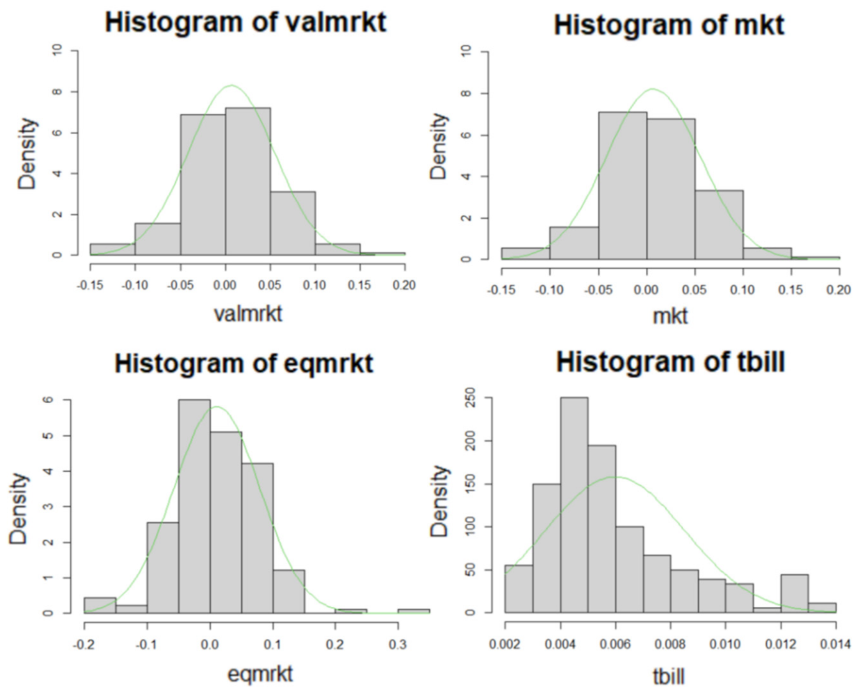

Figure 1. The normal distribution of the four assets

And we check if the i.i.d assumption is reasonable in the data. So we get the autocorrelation of the four assets (fig.2) to check if they are satisfied with the i.i.d assumption. It is obvious to see that valmrkt, mkt and eqmrkt are i.i.d, and the tbill is not i.i.d because tbill is Treasury Bill whose rate of return has a certain regularity and which is a risk-free asset.

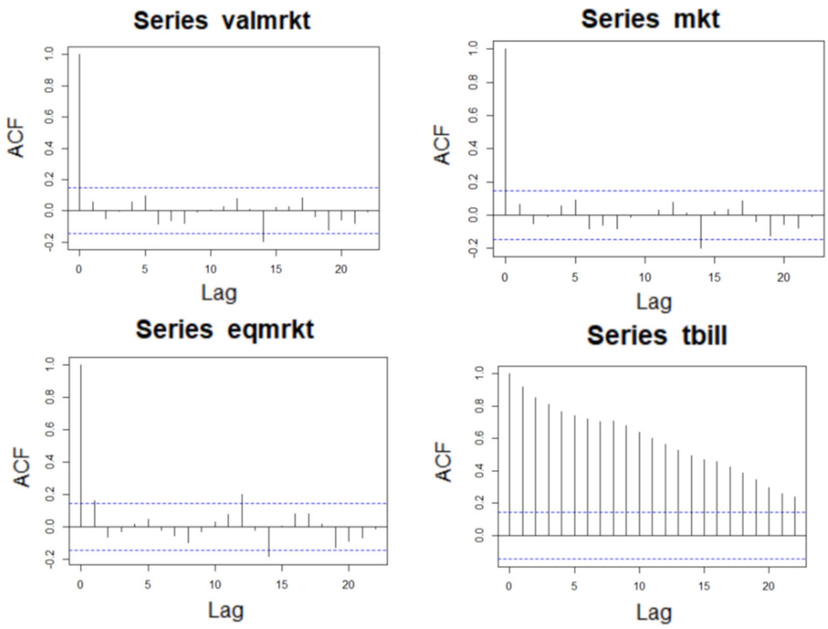

Figure 2. The autocorrelation of the four assets

3. Markowitz Model

The core idea of the Markowitz model is to maximize the expected profit under the premise of a given risk or minimize the risk under the premise of a given expected profit [14]. The three main elements of the model construction are the expected profit of each asset; the variance of all assets; the covariance between the assets in the portfolio [15]. In this paper, in order to find the optimal asset portfolio, the Markowitz model is used to process the data.

3.1 Model

Markowitz's mean value-the variance model has six assumptions [15]. The formula for calculating the portfolio rate of return and standard deviation of the investment portfolio model is as follows.

1) Return on Assets (ROA)

Assuming that there are n kinds of risky assets in the market, their rate of return vector during the period 1 to T is:

$$\mathrm{X} = (\mathrm{X}_1, \mathrm{X}_2, \cdots, \mathrm{X}_\mathrm{n})^\mathrm{T} \tag{1}$$

According to the above formula,$X = (p_{i1}, p_{i2}, \cdots, p_{ij}, \cdots, p_{iT})^T$, $i = 1,2, \cdots, n, j = 1,2, \cdots, T$; p_{ij}represents the rate of return of the i-th risk asset in the j-th period.

2) Portfolio yield

The expected value of the expected return rate of the *n* securities invested by the investor at a certain stage:

$$E(R) = \sum_{i=1}^{n} x_i r_i \tag{2}$$

x_i represents the proportion of investors investing in the i-th security; r_i represents the yield of the i-th bond.

3) Portfolio standard deviation

In the context of economics, the expected variance of a portfolio can also be seen as an expression of portfolio risk, namely:

$$R_{predict}^2 = \sum_{a=1,b=1}^{n} w_a a_{a,b}^2 w_b \tag{3}$$

Then the risk of the risky asset portfolio during the investment period is:

$$R_{realize}^2 = \sum_{a=1,b=1}^{n} w_a \hat{\sigma}_{a,b}^2 w_b \tag{4}$$

According to the hypothesis: the values of $R_{predict}^2$ and $R_{realize}^2$ are both positive. The smaller the difference between the two values, the higher the model's accuracy in predicting portfolio risk. The higher the level of risk optimization, which is more beneficial to investors.

4) Risk Optimization

The Markowitz model also proposes a method to quantify the optimal combination:

$$\min \frac{1}{2} w^T \sum w \tag{5}$$

$$s.t. \, w^T X = b \tag{6}$$

$$w^T d = 1 \tag{7}$$

Among them, under the condition that the rate of return of the risk investment portfolio $w^T X$ is expected to be certain, the weight w of the investment portfolio is determined to minimize the return risk of the risk investment portfolio $\frac{1}{2} w^T \sum w$.

3.2 Summary statistics

According to the following descriptive statistical analysis of the rate of return and the standard deviation (Table 2), the cumulative annual rate of return of the nine different types of funds is all positive. Among them, the fund *eqmrkt* has the highest cumulative return value of 194.846%, while the fund *scudinc* has the lowest cumulative return value of 79.782%, which is more than double the difference. Moreover, the returns of most of these funds fluctuate in the range of 110% to 130%. The returns of the nine funds are positive and large, mainly because the returns need to be studied based on the investment period and must be the same investment period to be comparable. Considering the timeliness of the original data and market influence, to simplify the research, we selected the entire duration of the research fund as the investment period, so the rate of return is the cumulative rate of return and the value is relatively large. Among the standard deviations of the three assets we selected, the maximum standard deviation of Mkt is 0.072, and the minimum is Eqmrkt of 0.055. Looking at the standard deviations of the three separately, the standard deviation is still large, and there is a certain investment risk.

TABLE 2. RETURN RATE AND STANDARD DEVIATION RESULTS OF NINE FUNDS

Fund	Return	Std. dev.
Eqmrkt	194.846%	0.055
Valmrkt	122.634%	0.056
Mkt	126.350%	0.072

In order to better analyze the impact of the portfolio on a standard deviation of investment (Table 3), a distinction is made between risk-free assets and risk-free assets. Before analyzing the portfolio's return rate and standard deviation, it is necessary to set the weight and other factors.

According to the asset portfolio weight table, the weights of the two combinations are determined through the historical performance of fund returns. We have selected Eqmrkt, Valmrkt, Mkt three assets and assigned a combination weight to each asset. Without adding risk-free assets, the weight of each asset is 0.43, 0.27 and 0.3. In the case of adding risk-free assets, the weight of each asset is: 0.3, 0.27, and 0.3, where the weight of risk-free assets is 0.13. Portfolio_nf represents the combination of no risk-free assets, and Portfolio_f represents the combination of risk-free assets. The return rate of the asset portfolio (154.80%) will be lower than the highest return rate of a single fund but generally higher than the remaining funds. This is in line with the fact that adding low-yield assets will dilute the highest income. The principle of profitability, but this is to a greater extent a sacrificial guarantee for a stable yield. After adding risk-free assets, the total return of the investment portfolio will decrease again, but by a smaller margin, falling to 143.46%. This is due to the fact that the yield of the national debt is generally set low, but the yield is more stable, and the losses caused by the greater fluctuation of interest rates are avoided.

TABLE 3. PORTFOLIO RETURN RATE RESULTS OF MARKOWITZ MODELS

Fund	Return	Portfolio_nf	Weight	P_Return	Portfolio_f	Weight	P_Return
Drefus	121.806%	Eqmrkt	0.43		Eqmrkt	0.3	
Fidel	84.541%	Valmrkt	0.27	154.80%	Valmrkt	0.27	143.46%
Keystne	117.766%	Mkt	0.3		Mkt	0.3	
Putnminc	99.307%				Tbill	0.13	
Scudinc	79.782%						
Windsor	180.394%						
Eqmrkt	194.846%						
Valmrkt	122.634%						
Mkt	126.350%						

According to the above portfolio standard deviation analysis table (Table 4), without adding risk-free assets, the weight of each asset is 0.5, 0.2, and 0.3. In the case of adding risk-free assets, the weight of each asset is 0.3, 0.4, and 0.1, where the weight of risk-free assets is 0.2. Portfolio_nf represents the combination of no risk-free assets, and Portfolio_f represents the combination of risk-free assets. The standard deviation of the excluding risk-free investment portfolio is smaller than the standard deviation of any asset, which is 0.049, which is nearly 0.01 lower than the lowest standard deviation of the three assets of 0.055. This shows that selecting funds to form an asset portfolio can greatly reduce the standard deviation of the portfolio. That is, the investment portfolio can reduce the risk of investment. After adding risk-free assets, the standard deviation of the investment portfolio decreased again to 0.047. This shows that choosing risk-free assets to join the investment portfolio can reduce the standard deviation of investment and reduce risk. This is also because the risk of risk-free assets is negligible.

TABLE 4. PORTFOLIO STANDARD DEVIATION RESULTS OF MARKOWITZ MODELS

Fund	Std. dev.	Portfolio_nf	Weight	P_Std.	Portfolio_f	Weight	P_Std.
Eqmrkt	0.055	Eqmrkt	0.5		Eqmrkt	0.3	
Valmrkt	0.056	Valmrkt	0.2	0.049	Valmrkt	0.4	0.047
Mkt	0.072	Mkt	0.3		Mkt	0.1	
					Tbill	0.2	

4. ADVANTAGES OF PORTFOLIO INVESTMENT

This research finds the advantages of the portfolio by drawing the mean-std.dev graph to compare the asset and portfolio. And we can draw the conclusion that most parts of portfolios have a higher mean and smaller standard deviation than assets. We set three weights randomly, which add up to one to allocate the weight of the assets included in three portfolios. And we also compare the difference between the portfolios without investing in the risk-free asset and the portfolios with investing in the risk-free asset.

We create three portfolios by using the assets (valmrkt, mkt, and eqmrkt). And drawing each portfolio and assets respectively in the mean-std.dev graph (Fig.3, Fig.4).w_1,w_2,w_3,$w_4$are the portfolios with investing in the risk-free asset. w_{1_1},w_{2_1} and $w_{3_1}$are the portfolios without investing in the risk-free asset. It indicates that most portfolios have a larger mean and the smaller standard deviation than assets, which means portfolios have a larger return and a smaller risk than assets.

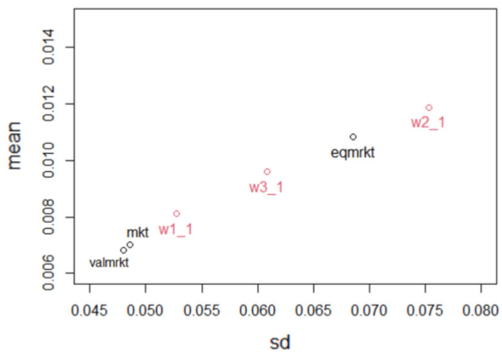

Figure 3. Mean-std.dev graph without investing in the risk-free asset

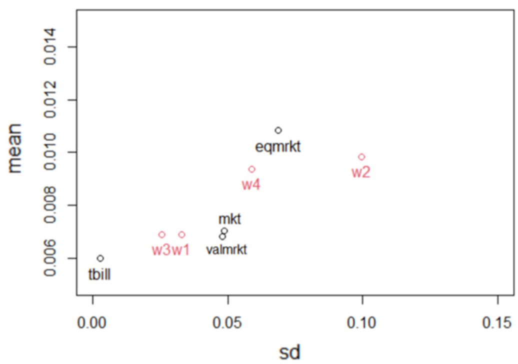

Figure 4. Mean-std.dev graph with investing in the risk-free asset

And we calculate the Sharp ratio of each asset (Table 5) which is included in the data, and each portfolio (Table 6) we have created before. We also can find that most number of portfolios' Sharp ratio is bigger than assets. The Sharp ratio of portfolios with tbill is bigger than the Sharp ratio of portfolios without tbill, which means the portfolios with investing in the risk-free asset are worth investing. Thus, we suggest that investors prefer portfolios with investing in risk-free assets rather than portfolios without investing in risk-free assets or only assets.

TABLE 5. THE SHARP RATIO OF EACH ASSET

Variable	Sharp ratio
drefus	0.143
fidel	0.083
keystne	0.078
putnminc	0.183

scudinc	0.123
windsor	0.206
eqmrkt	0.158
valmrkt	0.142
mkt	0.145
tbill	2.370

TABLE 6. THE SHARP RATIO OF EACH PORTFOLIOS

Variable	Sharp ratio
w_1	0.209
w_2	0.099
w_3	0.268
w_4	0.160
w_{1_1}	0.154
w_{2_1}	0.158
w_{3_1}	0.158

5. THE OPTIMAL PORTFOLIO FOR THE HIGHEST SHARP RATIO

While finding the portfolio that pays most per unit of risk, we found it extremely difficult to figure out the exact portfolio since there are so many factors that we need to consider. Therefore, we made some simplifications for the models according to the following assumption.

To simplify our model, three assumptions are made. Firstly, the weights of our portfolio are separated into two parts. The first part Wa stands for the assets, and the second part Wb stands for tbills. Secondly, we assume that Wa ranges from 0.1 to 1, and the interval is 0.1. Thirdly, Wa is equally divided to each asset in one portfolio. For example, if the portfolio contains three assets--- fidel, eqmkt and keystne, the weight of each asset is Wa/3.

When designing the model, the loop is employed to help us find the portfolio which has the highest Sharpe ratio. We start with the portfolio that contains only 1 asset and tbills, so there are 9 groups in total. Then we get to the portfolio that contains 2 assets and tbills, so there are 36 groups in total. By repeating the above steps, we finally get to the portfolio that has 9 assets and tbills, so there will be only one group.

Ultimately, we find that two portfolios with the highest sharpe ratio are shown in the table (Table 7) below.

TABLE 7. TABLE THE OPTIMAL PORTFOLIOS

	Asset1	Asset2	Sharp ratio
Portfolio1	0.2*Windsor	0.8*tbills	0.8946
Portfolio2	0.7*Windsor	0.3*tbills	0.8946

6. CONCLUSION

In this article, the mean and standard deviation are used to measure one particular portfolio's efficiency. In order to find a portfolio that has a higher return and a lower risk, we therefore introduce the Markowitz Model and Sharpe Ratio.

It can be illustrated from our analysis that, compared with portfolios without risk-free asset, those with one risk-free asset tend to have bigger means and lower standard deviation. To be more specific, the standard deviation of the excluding risk-free investment portfolio, 0.049, is smaller than any asset. Moreover, most of the portfolios' Sharp ratios are bigger than those of the single assets. In other words, their Sharpe ratios are much higher. Therefore, including one or more risk-free assets in the portfolio is highly recommended.

However, while finding the optimal portfolio for highest Sharp ratio, we simplified the realistic problem and made some prerequisites. This problem is far more complex in practice, which means our model still requires further optimization considering the various investing situations.

REFERENCES

[1] Li Hua, Li Xingsi. (2005). Research on entropy optimization model in securities investment portfolio. Journal of Dalian University of Technology (01), 153-156. doi:CNKI:SUN:DLLG.0.2005-01-031.

[2] Markowitz Harry M.. (1971). Portfolio Selection:Efficient Diversification of Investments. NEW HAVEN; LONDON: Yale University Press,Yale University Press.

[3] Li Shanmin, Xu Pei. (2000). Research on the Application of Markowitz's Portfolio Theory Model. Economic Science (01), 42-51. doi: 10.19523/j.jjkx.2000.01.006.

[4] Sharpe, W. F. . (1964). Capital asset prices: a theory of market equilibrium under conditions of risk*. Journal of Finance, 19(3), 425-442.

[5] Blume, M. E. . (1971). On the assessment of risk. Journal of Finance, 26(1), 1-10.

[6] O Vasicek. (1977). An equilibrium characterization of the term structure. Journal of Financial Economics.

[7] Klemkosky, R. C. , & Martin, J. D. . (1975). The adjustment of beta forecasts. The Journal of Finance.

[8] Edwin J. Elton and Martin J. Gruber and Manfred W. Padberg. (1977). Simple Criteria for Optimal Portfolio Selection with Upper Bounds. Operations Research, 25(6),

[9] J. Daly and M. Crane and H.J. Ruskin. (2008). Random matrix theory filters in portfolio optimisation: A stability and risk assessment. Physica A: Statistical Mechanics and its Applications, 387(16), pp. 4248-4260.

[10] J. Daly and M. Crane and H.J. Ruskin. (2010). Random matrix theory filters and currency portfolio optimization. Journal of Physics: Conference Series.

[11] Laloux, L. , Cizeau, P. , Potters, M. , & Bouchaud, J. P. . (2000). Random matrix theory and financial correlations. International Journal of Theoretical and Applied Finance (IJTAF), 03.
[12] Plerou Vasiliki et al. (2002). Random matrix approach to cross correlations in financial data.. Physical review. E, Statistical, nonlinear, and soft matter physics, 65(6 Pt 2), pp. 066126.
[13] S. Sharifi et al. (2003). Random matrix theory for portfolio optimization: a stability approach. Physica A: Statistical Mechanics and its Applications, 335(3), pp. 629-643.
[14] Markowitz, H.M.(1952). Portfolio Selection. the Journal of Finance.
[15] Li Junchang, Fan Chongjun, Yang Yunpeng & Yuan Guanghui. (2018). Research on stock portfolio risk optimization based on Monte Carlo wavelet denoising. Computer Application Research (10), 2947-2951+3028.