Portfolio Investment Analysis Based on Markowitz Mean-variance Model with a Realistic Fund Dataset

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Abstract—As the financial market is getting increasingly complicated, many investors have confronted the quandary between the investment target and their ability of risk tolerance. To provide investors with insights on portfolio management, this paper is dedicated to boost the return and avoid the risks to the maximum level simultaneously. With 9 separate assets selected, the portfolio which features lower variance, higher expected return, and higher Sharpe Ratio is expected. Throughout the research, normal distribution and independent and identically distributed tests test helped us initially understand the data. With the help of the demonstration of the efficient frontier, we found the best fit portfolios. Corresponding portfolio suggestions have been given, and limitations have also been discussed.

Keywords: Portfolio Management; Markowitz Mean-Variance Model; Stock Market; Sharpe Ratio; Efficient Frontier

1. Introduction

An investment portfolio is a collection of assets and can include investments like stocks, bonds, and different funds. In today's financial market, a well-maintained portfolio is vital to any investor's success. As an investor, you need to know how to determine an asset allocation that best fits the personal investment goals and risk tolerance. Risk tolerance is the ability to accept investment losses in exchange for the possibility of earning higher investment returns. Generally, the more risk you can bear for the same amount of time, the more aggressive the portfolio will be. Conversely, the less risk you can assume, the more conservative the portfolio will be.

Therefore, it is of great significance to improve the way of portfolio construction, as this will promote the profit and control the risk. This paper has carried out related research on the portfolio management of different portfolios.

The stock market is both important for investors who purchase stock and businesses who issue stocks. From the business perspective, the stock market allows a company to be publicly traded and raise capital [1]; from the investors' perspective, they exchange on the stock market to earn money. The more return in the market, the more investors tend to invest [2]. In the long run, the economic growth of a country depends on the stock market performance [3]. The

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unemployment rate and performance of the stock market are also key factors for determining monetary policy [4].

According to the investment theory by Sternberg and Lubart [5], people should buy the stock at alow price and invest high with creativity. Combined with investment theory, the factor model is good to use for determining the stocks that should be invested. Its weights would be obtained based on the portfolio theory. The factor model, also known as the index model, is frequently used in the financial market to explore the risk relationships of security returns [6]. In the model, Sharpe ratio, risk and return are significant determinants for portfolio establishment. Risk can be measured by the standard deviation of the portfolio [7], and return is money made or lost on an investment over some time [8]. Sharpe ratio measures expected excess return per unit of risk [9]. In 1988-1997, the stock market in the U.S. had the best return performance of systematic risk and return volatility [10]. We study the stocks from 1968 to 1982 in the U.S. to investigate how the stock market is like before.

Our research wants to know what kind of portfolios can get us the highest expected return, lowest variance, and highest Sharpe ratio, respectively. We consider two types of portfolios where one contains risky assets, and the other includes both risky and risk-free assets. To reach the goal of choosing the portfolios optimally, we use Markowitz's Mean-Variance Model theory, where one models the rate of returns on assets as random variables. The optimal weight set is the acceptable baseline expected rate of return of the portfolio at the minimum volatility. Here, the variance of the instrumental return is considered as an alternative to its volatility. [11]

Mainly, we use R programming to construct figures such as efficient frontier and use loops to get the portfolios we want:

(1) We use a covariance matrix and solve function in the programming to obtain the higher expected return portfolio.

(2) We purchase high returns and short sell low returns to construct higher return portfolios.

(3) We use loops and random selection to create higher Sharpe Ratio portfolios.

(4) The graphs are drawn by computing loops and matrices, including visualizing the weights with histograms.

In the end, we find the portfolios both with risky assets and risk-free assets that can meet our expectations. To be specific, if we create the portfolio only containing risky assets with the highest Sharpe Ratio, we need to long 0.03 stocks of eqmrkt, 0.09 units of stock Putnminc, and 0.88 units of stock Windsor. Also, if we consider the portfolio with risk-free assets with the highest Sharpe Ratio, we need to long 0.07 units of stock eqmrkt, 0.007 units of stock Putnminc, and 0.8 units of stock Windsor and 0.12 units of stock a risk-free asset.

The remainder of the paper is organized as follows: Section 2 describes the sample and data; Section 3 introduces the Markowitz Mean-Variance Model. For example, we construct portfolios with three assets, including eqmrkt, Putnminc, and Windsor, by analyzing the efficient frontier. Section 4 shows the results of the portfolios we obtain. Section 5 concludes our research. Limitations of the study are indicated in Section 6. It also introduces ideas about the improvement of further investigation.

2. Data

2.1 Description

There are 9 funds in total which have been involved in this investigation. All the data were found in Yahoo finance. The sample period starts from January, 1968 and ends in December 1982.

First of all, we map the normal distribution according to each fund's density and monthly return. During this process, we collected, sorted the data and tried to demonstrate the data in bar charts. Then, we extracted the mean and standard deviation of the data and drew the curve, in which way we finally got the normal distribution line. It can be seen from Fig1 that the maximum density of Drefus is around 7.5 when the monthly return is near 0. And the other four (Fidel, Windsor, Valmrkt, and Mkt) almost follow the same pattern. The remaining four funds can be divided into two pairs. The highest density of Keystne and Eqmrkt are relatively low (below 6), and that of Putnminc and Scudinc is much higher (above 10). In general, nearly all figures appear to have no skewness except for two. However, we still assume all the data are in a normal distribution way.

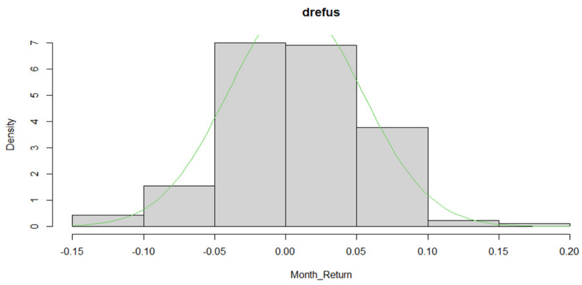

Figure 1: Normal distribution graph of drefus

2.2 Independent and identically distribution test

After that, we use the autocorrelation function (ACF) to draw lag plots to do the independent and identically distributed (i.i.d) test. In statistics, if each random variable has the same probability distribution as the others and all are mutually independent, then the random variables are independent and identically distributed. In statistical modeling, however, the assumption may or may not be realistic. To partially test how realistic the assumption is on the given data set, lag plots aredrawn.

From the graphs, any two numbers between two blue lines are considered to be independent, and their correlations are zeros. For example, in Fig 2, all numbers are in two blue lines except the 0 one, which means almost all the data are independent. By drawing the Lag-ACF plot, we found that all these 9 funds are independent and identically distributed.

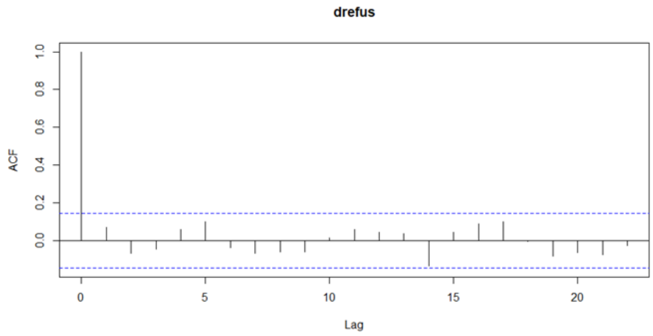

Figure 2: ACF graph of drefus

2.3 Correlation

Last but not least, we calculate the correlation among these 9 different funds. According to table 1, the correlation between Valmrkt and Mkt is the strongest (the coefficient is 0.999), indicaing an intense connection between these two funds. Since it is near to 1, in statistics, they have a perfect positive correlation. It means that if Valmrkt changes, Mkt will change the same amount as follows. Furthermore, it is also worth noticing that the correlation between Drefus and Valmrkt and between Drefus and Mkt is strong (the coefficient is 0.958 and 0.959, respectively). Besides, it can also be found that the correlation between Putnminc and Fidel and that between Putnminc and Keystne are the weakest in comparison, the coefficient of which are 0.572 and 0.557, respectively. To reduce risk, we need to find a portfolio with weak correlation assets.

TABLE 1. CORRELATION BETWEEN 9 FUNDS

	drefus	fidel	keystne	Putnminc	scudinc	windsor	eqmrkt	valmrkt	mkt
drefus	1	0.941613	0.865287	0.663667	0.798314	0.911919	0.843282	0.957652	0.959114
fidel	0.941613	1	0.876073	0.572497	0.760392	0.875926	0.822658	0.953991	0.956729
keystne	0.865287	0.876073	1	0.55711	0.696712	0.809881	0.83076	0.862556	0.86851
Putnminc	0.663667	0.572497	0.55711	1	0.833399	0.638307	0.579748	0.630066	0.632225
scudinc	0.798314	0.760392	0.696712	0.833399	1	0.81709	0.732008	0.812967	0.814016
windsor	0.911919	0.875926	0.908881	0.638307	0.81709	1	0.896418	0.923418	0.927829
eqmrkt	0.843282	0.822658	0.83076	0.579748	0.732008	0.896418	1	0.871975	0.878901
valmrkt	0.957652	0.953991	0.862557	0.630066	0.812967	0.923418	0.871975	1	0.999294
mkt	0.959114	0.956729	0.86851	0.632225	0.814016	0.927829	0.878901	0.999294	1

SOURCE: CALCULATED BY R

3. METHOD

3.1 Choose the 3 assets

The first step we did is check and clean up data. We first look at the dataset and find out that the returns are monthly. So, we try to sum each year's return and divide it by 12 to get a yearly return. However, after that, we find only 15 rows of data since there are only 15 years. We feel that yearly sample size is too small. So, we decided to continue to use monthly returns for further exploration.

The second step we did is to decide which assets we should include in our portfolio. We tried several combinations of different assets to see which one is better. We made the decision based on three perspectives of return, standard deviation, and Sharpe ratio. Finally, we decided to choose the portfolio with a high return asset, low risk asset, and high Sharpe ratio asset.

The first combination we tried is keystne, putnminc, and eqmrkt. Based on the correlation matrix, these three assets are the least correlated with each other. However, equity and mrkt as individual assets are above the efficient frontier (Figure 3). It means investors can only purchase eqmrkt for a higher return and lower risk than the portfolio. Thus, we reject the hypothesis of using this port

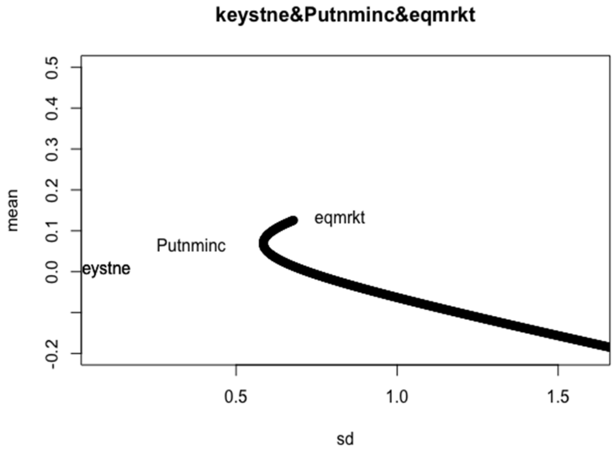

Figure 3: efficient frontier of keystone & Putnminc & Eqmrkt

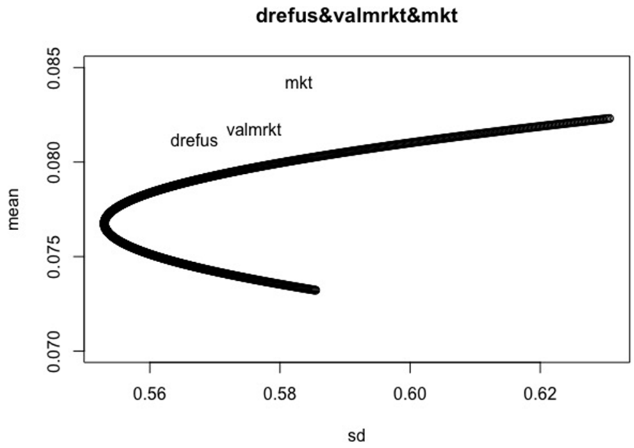

Figure 4: efficient frontier of Drefus & Valmrkt & Mkt

The second tried portfolio combines 3 assets with the minimum standard deviation: Drefus, valmrkt, and mkt. However, all 3 individual assets are above the efficient frontier, which means investors can just purchase any one of them to receive a higher return with the same risk than the portfolio (Figure 4).

The third combination we tried is 3 assets with the highest return, namely windsor, valmrkt, and eqmrkt. Based on the graph, Although the three points are below the efficient frontier, we can have a higher return by purchasing the portfolio (Figure 5). On the other hand, investors also need to take higher risks, and it's not optimal for risk-averse conservative investors. Thus, we also reject the hypothesis of using this combination.

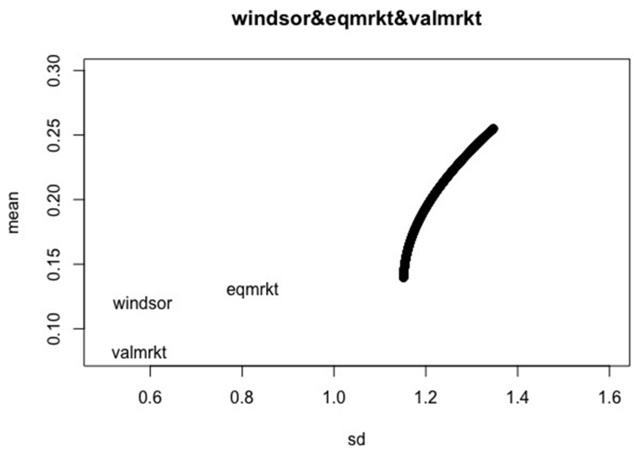

Figure 5: efficient frontier of Valmrkt & Windsor & Eqmrkt

The last combination we tried is the portfolio of equity mrkt, Putnminc, and Windsor. We choose these three assets since Equity mrkt is the asset with the highest return, Putminic is one with the second-lowest risk, and Windsor is the one that gives a high Sharpe ratio. Since all three individual asset points are below the efficient frontier, purchasing them together can make investors earn a higher return and lower risk (Figure 6). It indicates that the hypothesis is verified, a portfolio of equity mrkt, Putnminc, and Windsor is better than all individual assets.

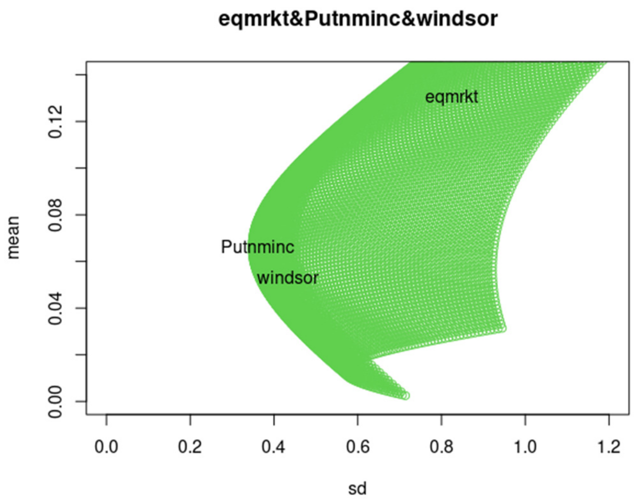

Figure 6: efficient frontier of Putnminc & Windsor & Eqmrkt

3.2 Portfolio establishment

After deciding which assets should be included in the portfolio, we begin to compute how to manage the weights of 3 portfolios. We aim to establish six kinds of portfolios.

(1) with the risk-free asset, establish a portfolio with a return that is higher than any individual asset

(2) with the risk-free asset, establish a portfolio with the risk that is lower than any individual asset

(3) with the risk-free asset, establish a portfolio with a Sharpe ratio that is higher than any individual asset

(4) without risk-free assets, establish a portfolio with a return that is higher than any individual asset

(5) without risk-free assets, establish a portfolio with the risk that is lower than any individual asset

(6) without risk-free assets, establish a portfolio with aSharpe ratio that is higher than any individual asset

4. RESULT

This section indicates the results with 3 parts: efficient frontiers, optimal portfolios without risk-free assets, and optimal portfolios with the risk-free asset.

4.1 Efficient frontiers

As mentioned above, we finally decided to use the asset eqmrkt, Putnminc, and Windsor to create portfolios. There are two efficient frontiers, where one contains risk-free assets, and the other is not. The points on the efficient frontiers are the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. Also, the portfolios located below the efficient frontier are suboptimal because they do not provide adequate returns for the risk levels. Portfolios clustered on the right side of the efficient frontier are suboptimal because they have higher levels of risk for the defined rate of return. Without risk-free assets, when the portfolio can have a maximum expected return rate, its mean is around 0.013, and the standard deviation is around 0.06. When the portfolio has a maximum Sharpe Ratio, its mean is around 0.012, and the standard deviation is around 0.05. Also, when the portfolio has a minimum variance, its mean is around 0.08, and the standard deviation is around 0.03.(figure7)

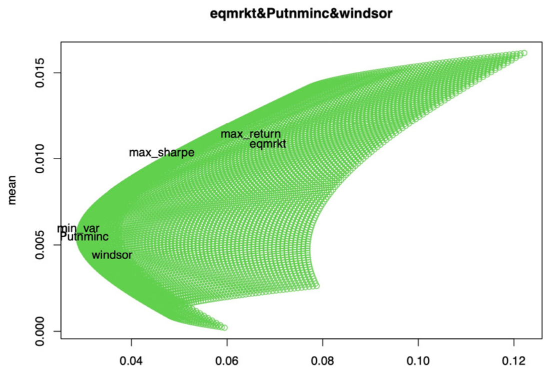

Figure 7: Efficient Frontier Without Risk-free Asset

However, if we consider a risk-free asset, the portfolio can have a maximum expected return rate while its mean is around 0.0105 and variance is around 0.003. When the portfolio has amaximum Sharpe Ratio, its mean is around 0.0085, and variance is around 0.002. Also, when the portfolio has a minimum variance, its mean is around 0.0065, and the variance is around 0.0002. (figure 8)

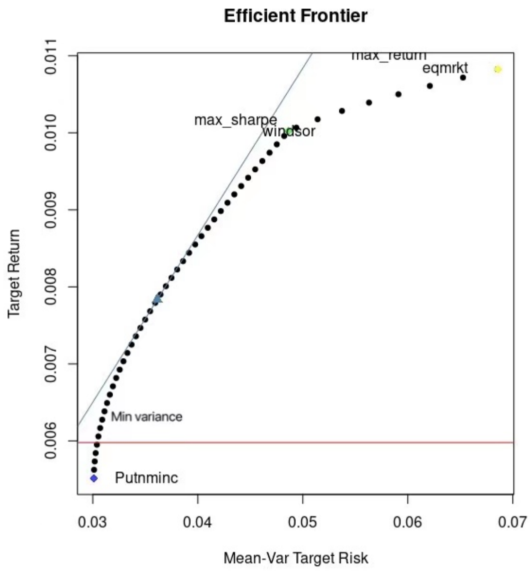

Figure 8: Efficient Frontier with Risk-free Asset

4.2 Optimal Portfolios Without Risk-free Asset

Now we conduct six portfolios and two for each type. The first is the portfolio with the variance of the return lower than the variance of return for any other funds without considering risk-free assets. In this case, the variance of return for this portfolio is approximately 0.0008. The variance of return for eqmrkt is 0.0047, the variance of return for Putnminc is 0.0009, and the variance of return for Windsor is 0.0024. To obtain the optimal portfolio, we should short sell 0.25 units stocks from eqmrkt, long 0.96 units stocks from Putnminc, and then long 0.3 units stocks from Windsor. (figure 9)

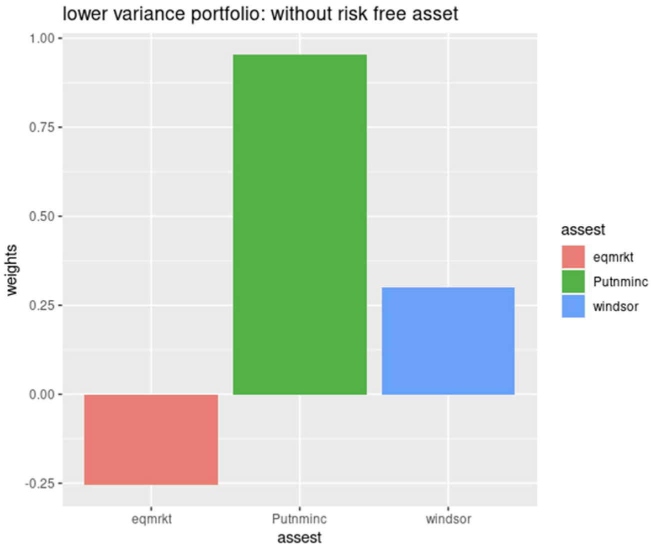

Figure 9: Lower variance portfolio (without risk-free asset)

The second is the portfolio with an expected return higher than the expected return of any other funds. As mentioned before, the expected return of eqmrkt is 0.0108, 0.0055 for Putnminc, and 0.01 for Windsor. In this case, we should long 0.6 units stocks from eqmarket, and 0.6 units stocks from Windsor. Then, we short sell 0.2 units of stocks from Putnminc. So, the expected return of this portfolio is 0.0114, which is lower than each individual fund. (figure 10)

Figure 10: higher expected return portfolio(without risk-free asset)

In addition, for the portfolio with a Sharpe Ratio that is higher than the Sharpe Ratio of any funds, we can obtain the optimal option by longing for 0.03 units stocks from eqmrkt, 0.09 units stocks Putnminc, and 0.88 units stocks from Windsor. The Sharpe Ratio of this portfolio is 0.2224, and the Sharpe Ratio of eqmrkt is 0.158, 0.183 for Putnminc, and 0.146 for Windsor. (figure 11)

Figure 11: Higher Sharpe Ratio portfolio (without risk-free asset)

4.3 Optimal Portfolios With Risk-free Asset

Moreover, we can combine the risky asset, but we can use the risk-free fund to hedge risk. To obtain the portfolio with lower variance than any other funds', we need to short sell 0.13 units stocks from eqmrkt, long 0.48 units stocks from Putnminc, long 0.5 units stocks from risk-free fund, and long 0.15 units stocks from Windsor. The variance of the portfolio is 0.0002098, which is lower than each individual fund. (figure 12)

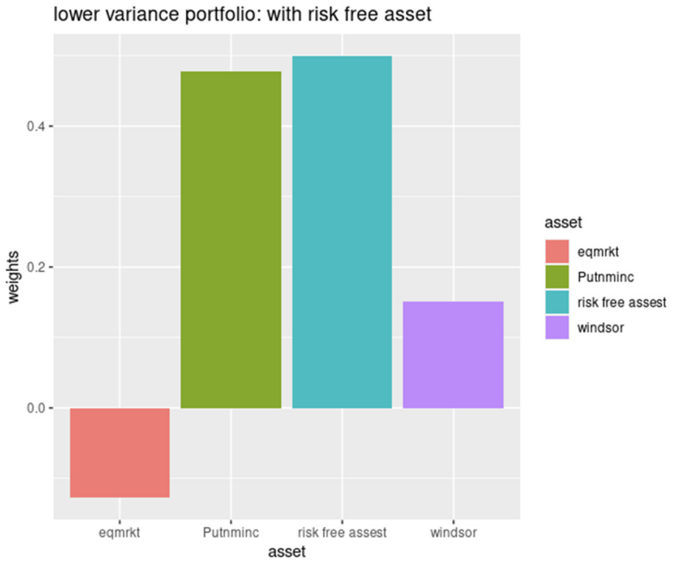

Figure 12: lower variance portfolio (with the risk-free asset)

Also, when we want to obtain a portfolio with a higher expected return than any other fund, we can long 0.5 units stocks from eqmrkt, short 0.2 units stocks from Putnminc, long 0.1 units stocks from risk-free fund, and long 0.6 units stocks from Windsor. The expected return of this portfolio is 0.011, which is higher than each individual fund. (figure 13)

Figure 13: higher expected return portfolio (with the risk-free asset)

Lastly, to get the portfolio with a higher Sharpe Ratio than any other funds', we can long 0.07 units stocks from eqmrkt, 0.007 units stocks from Putnminc, 0.12 units risk-free asset, and 0.8 units stocks from Windsor. In this case, the Sharpe Ratio of the portfolio is 0.2326, while Sharpe Ratio of eqmarkt is 0.0707, Putnminc's is -0.0153, Windsor's is 0.0831. Thus, the Sharpe Ratio of the portfolio is obviously higher than each individual fund. (figure 14)

Figure 14: higher Sharpe Ratio portfolio (with the risk-free asset)

5. CONCLUSION

In this paper, we research how to find the portfolio (with and without risk-free asset) with lower variance, higher expected return, and higher Sharpe ratio by using the monthly data of 9 different assets we collected from Yahoo finance. Based on normal distribution independent and identically distributed tests, we assume all the data are in normal distribution and find all the funds are independent and identically distributed. By the figure of the efficient frontier, we choose the suitable portfolios.

By constructing different weights on each fund, we can gain the optimal portfolios we want. On the one hand, when we only consider the risky assets, we can have the portfolio with a low variance if we short sell 0.25 units eqmrkt, long 0.96 units Putnminc, and 0.3 units Windsor. Also, we can have the portfolio with the high expected return if we long 0.6 units eqmrkt and 0.6 units Windsor, and short sell 0.2 units Putnminc. If we want to gain a high Sharpe Ratio portfolio, we can long 0.03 units eqmrkt, 0.09 units Putnminc, and 0.88 units Windsor. On the other hand, when we consider the risk-free asset, we can have the portfolio with a low variance if we are short 0.13 units eqmrkt, long 0.48 units Putnminc, 0.15 units Windsor, and 0.5 units risk-free asset. To get the high-expected-return portfolio, we can long 0.5 units eqmrkt, 0.1 units risk-free asset, 0.6 units Windsor, and short sell 0.2 units Putnminc. Lastly, if we want to gain the portfolio with a high Sharpe Ratio, we can long 0.07 units eqmrkt, 0.007 units Putnminc, 0.12 units risk-free asset, and 0.8 units Windsor.

Although the results of the study are meaningful, there are several limitations to our study. Firstly, we only consider three assets in the portfolio. Since there were way more assets in the market, the combination of other assets may lead to a higher return. Secondly, for simplicity of calculation, we assumed the risk-free rate would not change in fifteen years. However, it is not realistic as the risk-free rate changes continuously. Lastly, the data obtained is from a few decades ago, which cannot be used in current investing decisions. Since combinations of more assets in the portfolio can reduce risk, in the future, we wish to investigate portfolios with combinations of more assets, such as five assets. Then, we will compute the weights of each asset to find portfolios with the highest return, lowest standard deviation, and largest Sharpe ratio.

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