Optimal Portfolio Assessment Based on the Modern Portfolio Model

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Abstract—Investment in financial assets has two basic parameters, which are return and risk. These parameters have a positive relationship, and all investors face a trade-off between risk and return. Portfolio investment, a combination of financial assets, is widely used by investors because it can diversify risks while obtaining higher returns. Different investors have different preferences. Some investors prefer low risk, some prefer the high return, while others prefer high return per unit of risk. This paper used the modern investment portfolio model to construct the optimal portfolios based on three different principles: minimum risk, maximum return, utility, and maximum Sharpe ratio, and compare them. We proposed the portfolios with and without a risk-free rate. Based on our asset set, we found that the assets that make up the portfolios under the three principles above are different. The principle of risk diversification and the principle of maximum utility focus only on decreasing the risk and increasing the return, respectively. However, the principle of maximizing the Sharpe ratio better balances the relationship between risk and return, and the portfolio constructed under this principle pays the lowest per unit of risk. Furthermore, adding a risk-free asset to our portfolio did not raise the risk while increasing the return, and the unit return went up as well.

Keywords-Portfolio management; diversification effect; leveraging; Sharpe ratio

1 INTRODUCTION

Portfolio, a fundamental investing strategy, regards the investment target as an aggregation of various assets. By dividing the money into different assets, a portfolio can limit and reduce the unsystematic risk for the sake of customers. Because of the effect of diversifying risk, portfolio investment is widely used by investors. Pennacchi and Rastad [1] using pension funds data from 2000 to 2009 to obtain the risk choice of the optimal portfolio to maximize the benefit of taxpayers. Furthermore, the Sharpe Ratio of a diversified portfolio tends to be high, which is treated as a vital ratio influencing investors' decision. Since inappropriate portfolio investment will bring a low return for customers, the portfolio strategy is gradually completed in every aspect [2], such as contributing to the maximization of profit with risk under control.

For the words mentioned above, it is worthwhile to improve portfolio investment strategy by quantifying the estimated return and risk while evaluating the optimal portfolio based on related theories.

ICEMME 2022, November 18-20, Nanjing, People's Republic of China

DOI 10.4108/eai.18-11-2022.2326868

Portfolio management is of great importance for investors. Traditional analysis of portfolio was conducted according to the empirical rules on the regulation of stock market price volatility [3, 4]. Furthermore, the target for the portfolio is pursuing a high return regardless of risk. On the contrary, Markowitz [5] proposed that the portfolio be optimized based on the mean-variance model. The advantage of the theory is the groundbreaking progress about the attention on the correlation between assets, which has a positive effect on reducing the risk. Sharpe [6] used beta to represent the market sensitivity, using the single-ratio index to simplify various estimated variables. It decreases the operating cost and computation expenses compared to the original method. Sharpe [7] also proposed the capital asset pricing model based on the beta index, which becomes an appropriate input for the performance of the modern portfolio model.

Modern portfolio theory has attracted lots of study due to its controversiality. Tobin [8] extended the modern portfolio theory into the market with risk-free assets and cash and explored the risky asset and risk-free asset allocation. Ross [9] conducted the portfolio in the mutual fund market and found that the optimal portfolio can be achieved with a risk-less mutual fund. However, the assumption on keeping identical borrow and lending rates is not available in reality. Sharpe [10] established the Sharpe Ratio to evaluate the marginal excess return exchanged by taking the risk. Zakamouline and Koekebakker [11] illustrated that GSR overcomes the Sharpe Ratio's shortcomings by considering higher moments of distribution when evaluating portfolio performance.

This paper uses a modern portfolio model for forecasting the optimal portfolio in the capital market. We take the diversification effect, mean and variance utility effect, and leveraging effect into consideration while conducting the portfolio model. Furthermore, we consider the model with and without risk-free assets. We propose the portfolio model with maximum return and utility, the portfolio model with minimum variance, and the portfolio model with maximum Sharpe Ratio.

In-sample analysis result indicates that Sharpe ratio is more effective than mean and variance as a method for evaluating portfolio performance. In addition, the weight distribution should be accordance with the property of the asset. In detail, the Sharpe ratio of the portfolio with the higher return is 0.179, as a number 36% lower than the one of the optimal portfolio, showing the efficiency of the Sharpe ratio. Assets, such as scudinc, with low average return and generally low correlation with other assets, are appropriate for short-selling to inhibit the risk. Assets, such as windsor, with a high average return and universally high correlation with other assets should be allocated in a relatively inferior weight to elevate the overall return. Among the residual, assets, such as putnminic, with relatively low correlation with other assets, should be invested in a massive amount. The return of scudince, windsor and putnminic is 0.443%, 1.002%, 0.552%. The weight distribution of these three types of assets in an optimal portfolio is around -10:6:9.

The remainder of the paper is organized as follows: Section 2 describes the method and principle for establishing the portfolio model; Section 3 conducts the Markowitz model to generate the portfolio with lower variance, higher return, higher Sharpe Ratio than any funds and analyzes the result; Section 4 draws our conclusion.

2 METHOD

In this section, we introduce the method we used of selecting the optimal investment portfolio based on three different principles, one is diversification effect, one is leveraging effect, and the other is a higher Sharpe Ratio

2.1 Diversification Effect

Diversification is the act of spreading investment dollars across a range of assets to reduce investment risk, like putting eggs in different baskets. Owning a variety of assets minimizes the chances of anyone asset hurting your portfolio. The trade-off is that you never fully capture the startling achievements of a fantastic fund. The net effect of diversification is slow and steady performance and smoother returns, never moving up or down sharply. The reduction in volatility puts many investors at ease.

According to the Mean-Variance Model of Markowitz (1952), we assume that there are n types of risky assets in the capital market and the return of these assets are r_1, r_2, ..., r_n,and the investor allocation ratio of each risky asset are w_1,w_2,..., $w_n$respectively. Then, the return of the portfolio is $r_p = \sum_{i=1}^{n} w_i r_i$, where $\sum_{i=1}^{n} w_i = 1$.

Therefore, the expected return on the portfolio is the weighted average of the expected return of each individual asset:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{1}$$

The variance of the portfolio return is given by:

$$\mathrm{Var}(r_p) = \sum_{i=1}^{n} w_i^2 \mathrm{Var}(r_i) + \sum_{i \neq j} w_i w_j \mathrm{Cov}(r_i, r_j) \tag{2}$$

With the rationale above, we choose three risky assets with their own weight to construct an investment portfolio and the sum of the weights is 1, as in (3).

$$\begin{cases} y_1 = w_1 \times r_1 + w_2 \times r_2 + w_3 \times r_3 \\ w_1 + w_2 + w_3 = 1, w_1, w_2, w_3 \in R \end{cases} \tag{3}$$

where y_1 is the symbol of the portfolio we constructed, r_i is the symbol of asset i and w_i is the allocation ratios of each risk asset (i=1,2,3).

According to the equation mentioned above (2), the variance of the portfolio is shown below (4).

$$\begin{aligned} \mathrm{Var}(y_1) &= \mathrm{Var}(w_1 \times r_1 + w_2 \times r_2 + w_3 \times r_3) \\ &= \sum_{i=1}^{3} w_i^2 \sigma_i^2 + \sum_{i=1}^{3} \sum_{j=1, i \neq j}^{3} w_i w_j \sigma_{ij} \end{aligned} \tag{4}$$

where σ_i^2 is the variance on asset i and σ_{ij}^2 is the covariance between assets i and j.

Equation (4) indicates that $w_i^2 \geq 0, \sigma_i^2 \geq 0$, i=1,2,3 and $-1 \leq \sigma_{ij} \leq 1$. If we are going to minimize $\mathrm{Var}(y_1)$, we need to choose assets with lower variance, and the correlation between them should be low or even negative.

A risk-free rate is not considered in the above method. Now, let us consider the risk-free rate and add the risk-free rate as a random number to the portfolio (5).

$$\begin{cases} y_2 = w_1 \times (r_1 + r_{free}) + w_2 \times (r_2 + r_{free}) + w_3 \times (r_3 + r_{free}) \\ w_1 + w_2 + w_3 = 1, w_1, w_2, w_3 \in R \end{cases} \tag{5}$$

Equation (5) is equal to equation (6):

$$\begin{cases} y_2 = w_1 \times r_1 + w_2 \times r_2 + w_3 \times r_3 + r_{free} \\ w_1 + w_2 + w_3 = 1, \; w_1, w_2, w_3 \in R \end{cases} \tag{6}$$

Since the rate is a risk-free rate, the standard deviation is 0, and it is not correlated with other assets. Therefore, the risk-free rate does not correlate with portfolio y_1. The variance of y_2 in equation (6) is equal to y_1 in equation (3), and the derivation formula is shown in equation (7). In other words, whether or not a risk-free rate is added to the portfolio, its standard deviation remains constant. But the expected return is different. It is equal to the expected return of a portfolio without the risk-free rate and the expected return of the risk-free rate (8).

$$\begin{aligned} Var(y_2) &= Var(w_1 \times r_1 + w_2 \times r_2 + w_3 \times r_3 + r_{free}) \\ &= Var(y_1 + r_{free}) \\ &= Var(y_1) \end{aligned} \tag{7}$$

$$\begin{aligned} E(y_2) &= E(w_1 \times r_1 + w_2 \times r_2 + w_3 \times r_3 + r_{free}) \\ &= E(y_1) + E(r_{free}) \\ &= E(y_1) + E(r_{free}) \end{aligned} \tag{8}$$

Therefore, we can construct investment portfolios based on the methods above and explore the diversification effect, including risk-free rate and excluding risk-free rate.

2.2 Leveraging

The second portfolio aims to find a way to maximize the overall returns based on the adjusted allocation weights. It is associated with the definition of leveraging. Based on the MV utility function [12], this formula demonstrates the mean variance utility function, and α is the risk tolerance, σ is the variance, and τ is the investor's risk tolerance. It shows of the likelihood of the leverage caused by different factors. For example, highly volatile stocks with a lower utility number end up in a more leverage risk, assuming that the investor is risk neutral so that τ remains unchanged.

$$U = \alpha - \frac{1}{2\tau}\sigma^2 \tag{9}$$

Inside the portfolio data, the estimation of the future return is based on the average expected returns from 1968 to 1982 to avoid potential forecasting errors. Whereas the risks are represented by the standard deviation, which is the volatility of the asset price change. P is the estimated probability that stock price change, which is also seen as the asset’s risk tolerance.

Based on the theoretical statistical analysis, there is a correlation between return and standard deviation. In the Markowitz Portfolio Optimization models (1955), the assumption is that the market investors are risk averse, which means that the portfolio needs to reach out its maximum returns by taking the same amounts of risks. The formula below represents the theoretical calculation based on the probability (p) when the states occur. It shows off the correlation between estimated returns and standard deviation that is combined by P.

By applying the estimation results into the utility formula, the utility score is the highest when applying the investment assets with the highest returns and standard deviation. The utility ratio below combines those three formulae, which refers to our final utility formula.

$$\mathrm{E(P)}=\left(\frac{1}{\mathrm{n}}\right)\mathrm{np}=\mathrm{p} \tag{10}$$

$$\mathrm{Var(P)}=\frac{\mathrm{P(1-P)}}{\mathrm{N}} \tag{11}$$

$$\mathrm{U}=\mathrm{P}-\left(\frac{1}{2}\right)\times\frac{\mathrm{P(1-P)}}{\mathrm{N}} \tag{12}$$

It is the reason why the portfolio chooses windsor, eqmrkt and valmrkt as the three assets portfolio, based on the calculation of the utility showed in Table 1. Those three assets have the highest utility scores.

Table 1 Utility Measure

	drefus	**fidel**	**keystne**	**putnminc**
P (Return)	0.68%	0.47%	0.65%	0.55%
Utility	0.34%	0.24%	0.33%	0.28%
	scudinc	**windsor**	**eqmrkt**	**valmrkt**
P (Return)	0.44%	1.00%	1.08%	0.68%
Utility	0.22%	0.51%	0.55%	0.34%

2.3 Higher Sharpe Ratio

In this section, we investigate the optimal investment portfolio to maximize Sharpe Ratio based on past data.

Sharpe Ratio is the return earned at the cost of per unit of risk, which is used to evaluate the profitability of certain investment strategy.

To begin with, when single assets are invested, the return of each single assets is r_{ij} and the Sharpe Ratio is S_i (13)

$$\mathrm{S_i}=\frac{E(\mathrm{r_i})}{\sqrt{\frac{1}{\mathrm{m-1}}\sum(\mathrm{r_{ij}}-\mathrm{r_i})^2}} \tag{13}$$

$$E(\mathrm{r_i})=\frac{1}{\mathrm{n}}\sum \mathrm{r_{ij}}\ ,\mathrm{i}=1,2,\ldots\mathrm{n}\ ;\ \mathrm{j}=1,2,\ldots\mathrm{m} \tag{14}$$

In equation (13), $E(r_i)$ represents the average return of single assets (14). Meanwhile, the denominator of S_i is the volatility of return.

When people invest on portfolio, for n types of assets within the portfolio, r_1, r_2, $r_3 \ldots r_n$ are the return of each assets. The expected return and variance of portfolio are shown in (1) and (2). Then, the Sharpe Ratio is S_p (15)

$$S_p = \frac{E(r_p)}{\sigma(r_p)} \tag{15}$$

$$\sigma(r_p) = \sqrt{Var(r_p)} \tag{16}$$

In equation (15), $\sigma(r_p)$ is the standard deviation of the return of portfolio, which is gained according to equation (16). It signifies the fluctuation of portfolio and the extent of which expected return deviates from actual return.

Furthermore, market can be divided into with and without risk-free assets. Such assets have no probability of loss in the market, including T-bills, notes and bonds. Because of the property, in market with risk free asset, the Sharpe Ratio should be altered into S_f (17)

$$S_f = \frac{E(r_p - r_f)}{\sigma(r_p)} \tag{17}$$

In equation (17), $r_f$is the risk-free rate, r_p-r_f is the excess return, indicating the return earned by taking risk. Assuming the r_f is normally stable, $\sigma(r_p)$ remains the same.

Finally, because of the equation (18), the Sharpe Ratio of portfolio without risk free asset is larger than the one with risk free asset. (19)

$$E(r_p) > E(r_p - r_f) \tag{18}$$

$$S_p > S_f \tag{19}$$

By using R to calculate the Sharpe Ratio of all single assets and portfolios in market with and without risk-free asset, a more optimal portfolio will be derived with the Sharpe Ratio higher than any single assets. The empirical test and results will be elaborated next section.

3 EMPIRICAL TEST

In this section, we collected data through the Internet. And then, we organized and analyzed the data, including descriptive statistics and normality tests. Afterward, we used the data we collected to make an empirical analysis of the method in Section 2. Finally, we discussed all the portfolios we have constructed through their parameters, mean return, standard deviation, and Sharpe ratio.

3.1 Data collecting

For the empirical analysis, we use data from Yahoo Finance. The asset set contains monthly return data for 9 risky assets and a risk-free rate in the period from 1968 to 1982.

Table 1 is the descriptive statistics of this asset set, including minimum vaData collectinglue, maximum value, median, mean, first quartile, third quartile and standard deviation. From Table 1, we find that the standard deviation of the risk-free rate tbill is 0.003, which is extremely close to 0. Therefore, we can consider tbill to be risk-free. In the following, we assume that tbill is a risk-free rate, whose standard deviation is 0. For these nine risky assets, "eqmekt" has the highest mean return (10.8%) and "putnmic" has the lowest standard deviation. Generally, the minimum return of an asset is negative, which means the loss of property, and the maximum return is higher than 0, which means that the existing property is greater than the original property. As we can see in Table 1, "keystne" has the lowest minimum return (-0.332) while "putnmic" has the highest (-0.079). "Eqmekt" has the highest maximum return (0.333) while "putnmic" has the lowest (0.115). "putnmic" has slighter volatility than any other assets. There is not big difference between the median values and mean values of some assets (drefus, keystne, putnmic, scudinc, windsor and mkt), indicating that the returns of these assets fluctuates smoothly and there is no extreme value.

Table 2 Descriptive statistics of assets

	Min	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Standard deviation**
drefus	-0.126	-0.023	0.007	0.007	0.041	0.171	0.047
fidel	-0.159	-0.034	0.002	0.005	0.046	0.21	0.057
keystne	-0.332	-0.041	0.006	0.007	0.06	0.22	0.084
putnmic	-0.079	-0.011	0.005	0.006	0.02	0.115	0.03
scudinc	-0.106	-0.018	0.004	0.004	0.022	0.122	0.036
windsor	-0.145	-0.022	0.008	0.01	0.045	0.187	0.049
eqmrkt	-0.192	-0.027	0.004	0.011	0.054	0.333	0.069
valmrkt	-0.118	-0.023	0.004	0.007	0.044	0.164	0.048
mkt	-0.121	-0.024	0.006	0.007	0.045	0.166	0.049
tbill	0.003	0.004	0.005	0.006	0.007	0.014	0.003

In the Mean-Variance Model of Markowitz (1952), the return is assumed to be normally distributed. We performed the Shapiro-Wilk normality test on 9 risky assets, and 0shows the p-value result of the test. The result table shows that 7 assets pass the test with a confidence level of 0.01, respectively drefus, fidel, keystne, scudinc, windso, valmrkt and mkt 2 assets fail the test. That is, their return cannot be assumed to be a normal distribution. These two assets were therefore excluded from the selection.

Table 3 Shapiro-Wilk normality test

asset	**drefus**	**fidel**	**keystne**	**putnmic**	**scudinc**
p-value	0.538	0.305	0.023	0.008	0.024
asset	**windsor**	**eqmrkt**	**valmrkt**	**mkt**	

p-value	0.034	0.0004	0.119	0.109

0shows that the return of drefus fluctuated sharply from 1968 to 1974, and then the volatility further increased in 1975, though it decreased and became relatively stable the next year. However, in 1977, the return of drefus volatility started fluctuating severely again.

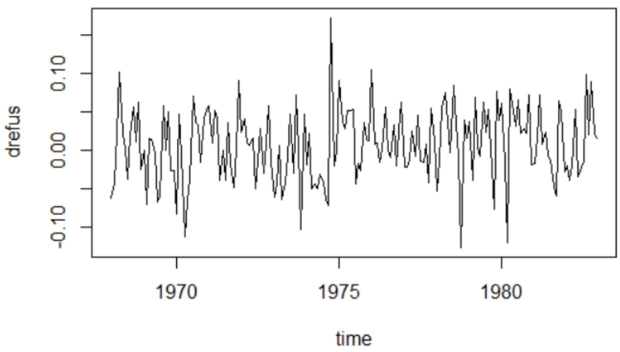

Figure 1 Return of drefus

As we can see from Fig. 2, the return of windsor fluctuated sharply from 1968 to 1975, and the fluctuations were similar from year to year. Though the volatility decreased in 1976 and 1977, it started fluctuating severely again in 1978.

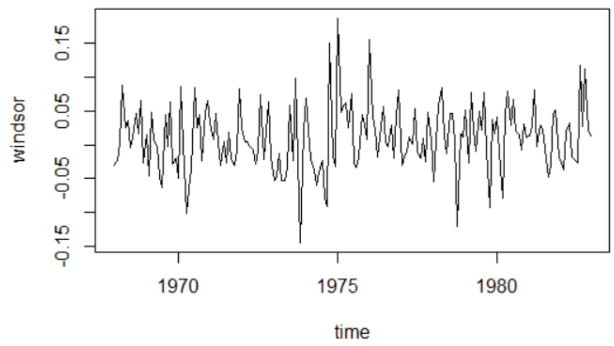

Figure 2 Return of windsor

3.2 Result Analysis

3.2.1 Diversification effect

According to (4) and what we know that $w_i^2 \geq 0, \sigma_i^2 \geq 0$ (i=1,2,3) and $-1 \leq \sigma_{ij} \leq 1$, if we are looking for a portfolio composed of three assets with minimum standard deviation, we need to choose assets with lower variance and lower or negative correlation. In fact, there is a strong correlation between each of these nine assets, and their correlation coefficient are shown in 0 From 0 we can see that the lowest correlation coefficient is 0.56 (between keysrne and putnmic), and the highest is nearly 1 (between mkt and valmrkt). Also, we can find that drefus is strongly correlated with all other assets, especially about 95% with fidel and valmrkt, and about 80% with other assets expect for putnmic (less than 70%). However, the correlation coefficient between scudinc and other assets are most about 80% approximately. In this case, we select 3 assets with the lowest standard deviation from the dataset. Therefore, the portfolio constructed by these 3 assets may have a standard deviation lower than any assets.

Table 4 Correlation with 9 assets

	drefus	**fidel**	**keystne**	**putnmic**	**scudinc**	**windsor**	**eqmrkt**	**valmrkt**	**mkt**
drefus	1	0.942	0.865	0.664	0.798	0.912	0.843	0.958	0.959
fidel	0.942	1	0.876	0.572	0.760	0.876	0.823	0.954	0.957
keystne	0.865	0.876	1	0.557	0.697	0.810	0.831	0.863	0.869
putnmic	0.664	0.572	0.557	1	0.833	0.638	0.580	0.630	0.632
scudinc	0.798	0.760	0.697	0.833	1	0.817	0.732	0.813	0.814
windsor	0.912	0.876	0.810	0.638	0.817	1	0.896	0.923	0.928
Eqmrkt	0.843	0.823	0.831	0.580	0.732	0.896	1	0.872	0.879
valmrkt	0.958	0.954	0.863	0.630	0.813	0.923	0.872	1	0.999
mkt	0.959	0.957	0.869	0.632	0.814	0.928	0.879	0.999	1

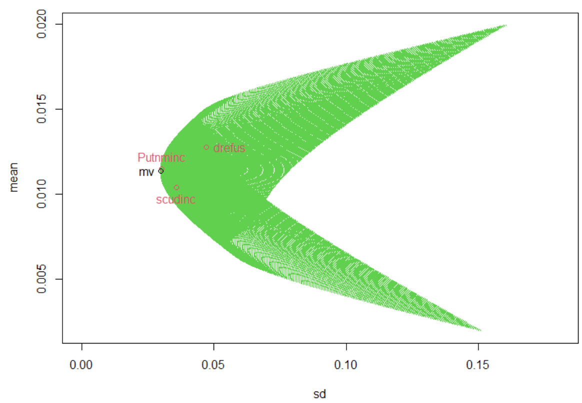

Figure 3 Portfolios constructed by the three assets based on principle 1

Assuming that short selling is allowed and the allocation ratio of asset investment ranging from -2 to 3, the green area in 0shows all the portfolio constructed by allocating different investment ratios to the three assets (putnminc, scudinc, and drefus). The leftmost point is the point with the lowest standard deviation which named portfolio "mv". Through calculation, we found that portfolio "mv" invests 99%, 7%, and -6% in putnminc, scudinc and drefus respectively (20). The standard deviation of portfolio "mv" is 3.005% which is slightly lower than putnminc (3.008%), the asset with the lowest standard deviation among the nine assets. Therefore, portfolio "mv" satisfies the condition that its standard deviation is lower than any funds in the asset set, with a mean value of 0.537%. The parameters of the appropriate assets we selected and portfolio "mv" are shown in 0

$$\mathrm{mv}=\mathrm{w}_1\times\mathrm{r}_{\mathrm{put}}+\mathrm{w}_2\times\mathrm{r}_{\mathrm{scu}}+\mathrm{w}_3\times\mathrm{r}_{\mathrm{dre}} \tag{20}$$

where w_1=0.99, w_2=0.07, w_3=-0.06.

Table 5 The parameters of 3 assets and the portfolio "mv"

Asset	Weight	Mean	Standard Deviation	Sharpe Ratio
putnminc	0.99	0.552%	3.008%	0.183%
Scudinc	0.07	0.443%	3.597%	0.123%
Drefus	-0.06	0.677%	4.724%	0.123%
"mv"	-	0.537%	3.005%	0.178%

Risk-free assets are not included in the above-mentioned portfolio "mv". Taking risk-free rate tbill into consideration:

$$\mathrm{mv}_2=\mathrm{w}_1\times(\mathrm{r}_{\mathrm{put}}+\mathrm{r}_{\mathrm{free}})+\mathrm{w}_2\times(\mathrm{r}_{\mathrm{scu}}+\mathrm{r}_{\mathrm{free}})+\mathrm{w}_2\times(\mathrm{r}_{\mathrm{dre}}+\mathrm{r}_{\mathrm{free}}) \tag{21}$$

where w_1=0.99, w_2=0.07, w_3=-0.06.

The risk-free asset tbill is added to the portfolio as a random number as in (21) and we assume that tbill does not correlate with any assets. The expected return of portfolio "mv_2" is equal to the expected return of portfolio "mv" plus the mean return of risk-free rate (8), which is 1.135%, twice as large as that of "mv". Since tbill is a risk-free rate, the standard deviation of tbill is 0, the standard deviation of "mv_2" is the same as "mv", shown in (7), which is still 3.005%. Table 6 shows the parameters of 3 assets, a risk-free rate tbill, and the portfolio "mv_2".

Table 6 The parameters of 3 assets, risk-free rate tbill and the portfolio "mv_2"

Asset	Weight	Mean	Standard Deviation	Sharpe Ratio
Putnminc	0.99	0.552%	3.008%	0.183%
Scudinc	0.07	0.443%	3.597%	0.123%
Drefus	-0.06	0.677%	4.724%	0.123%
tbill	1	0.598%	0	-
"mv_2"	-	1.135%	3.005%	0.378%

3.2.2 Leveraging

The investment scenario in this portfolio aims to discover an asset allocation method that is more risk neutral whereas maintaining the same efficiency as the previous portfolio, which refers to a better Sharpe ratio.

Based on the Markowitz Portfolio Theory (1959), as the hypothesis, in this case, is that investors are risk averse, the upper curve of the efficient front tier is infinite, which refers to the unlimited returns and risks. The upper curve indicates a positive relationship between risks and return, which refers to the unlimited risk and return. The portfolio range is set by -1 to 1, and it is calculated by the formulation below. The calculation results in short one portion of Valmrkt, it is because Valmrkt has a mean return that is way below than the average. In the max leveraging portfolio, it is ideal to maximum the use in risky assets that have the higher returns.

$$mv = w_1 \times r_{win} + w_2 \times r_{eqm} + w_3 \times r_{val} \tag{22}$$

After calculating the weights of the portfolio using the portfolio weights, the portfolio (mv) satisfied that the efficiency that is higher than the minimum variance portfolio (18%). As shown in Table 7, the combined portfolio has a mean and standard deviation that is higher than all the other risky assets. The combination of the risky assets is not as efficient as investing in windsor, as it does not take advantage of diversification effects.

Table 7 Three assets portfolio with highest return

Asset	Weight	Mean	Standard Deviation	Sharpe Ratio
windsor	1	1.002 %	4.864 %	21%
eqmrkt	1	1.082%	6.586%	16%
valmrkt	-1	0.681 %	4.800 %	14%
"*mv*"	1	1.396 %	7.259 %	19%

The efficiency frontier shows in Figure 4 is based on the formula above. The portfolio lies on the highest point on the upper curve compared to the minimum variance ratio.

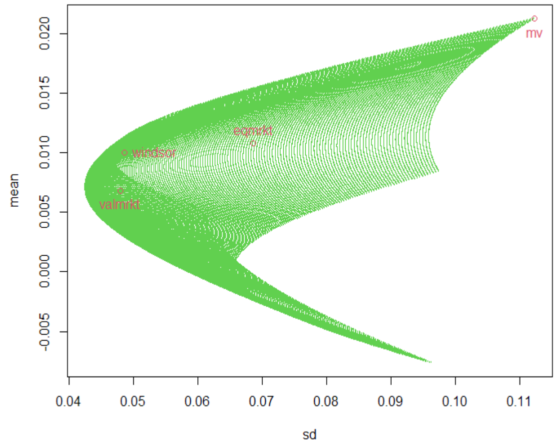

Figure 4 Portfolios constructed by the three assets based on principle 2

After considering the risky assets (Table 8), the new portfolio (mv_2) has a higher return of 2.001% with a higher Sharpe ratio of 27%. In terms of Rf, it is adding up the T-bill's daily returns regardless of its effect on the standard deviation. It is defined as the market random errors, which are the market returns that the Markowitz portfolio theory cannot explain.

$$mv = w_1 \times r_{win} + w_2 \times r_{eqm} + w_3 \times r_{val} + r_{free} \tag{23}$$

Table 8 Three assets portfolio with without risk free assets and with risk free assets

Asset	Mean	Standard Deviation	Sharpe Ratio
"mv"	1.396 %	7.259 %	19%
"mv_2"	2.001%	7.259 %	27%

3.2.3 Sharpe Ratio

The dataset collected shows the monthly return of 9 risky funds and risk-free funds from 1968 to 1982. We choose the fund (windsor) with the largest Sharpe ratio among all the funds (Table 9) and the other two randomly chosen funds (scudinc, putnminc) to form the portfolio with larger Sharpe Ratio than any funds.

Table 9 Sharpe Ratio of all single funds

	drecfus	fidel	keystne	Putnminc	scudinc
Sharpe Ratio	0.143	0.083	0.078	0.183	0.123
	windsor	**eqmrkt**	**valmrkt**	**mkt**	
Sharpe Ratio	0.206	0.158	0.142	0.145	

First, we constructed portfolio models without risk-free fund. According to the modern portfolio theory, we generate portfolio models without risk-free funds using these three funds.

$$Y = w_1 r_{put} + w_2 r_{scu} + (1 - w_1 - w_2) r_{win} \tag{24}$$

To form portfolios with different weights, we individually generate 250 points with an equally spaced value between -2 to 3 as w_1 and w_2. The possible combination of portfolio is showed below (Table 10).

Table 10 All possible combination of portfolios without risk-free fund

obs	w_1	w_2	Mean	Standard Deviation	Sharpe Ratio
1	-2.000	-2.000	0.030	0.167	0.181
2	-2.000	-1.980	0.030	0.166	0.181
3	-2.000	-1.960	0.030	0.170	0.181
4	-2.000	-1.940	0.030	0.165	0.181
5	-2.000	-1.920	0.030	0.165	0.180

…	…	…	…	…	…
62496	3.000	2.920	-0.020	0.154	-0.129
62497	3.000	2.940	-0.020	0.154	-0.129
62498	3.000	2.960	-0.020	0.155	-0.130
62499	3.000	2.980	-0.020	0.155	-0.130
62500	3.000	3.000	-0.020	0.156	-0.130

Then, we pick up the portfolio with the largest Sharpe Ratio (Table 11).

Table 11 Portfolio with the largest Sharpe Ratio without risk-free fund

obs	w_1	w_2	w_3	Mean	Standard Deviation	Sharpe Ratio
47001	1.775	-2.000	-1.225	0.013	0.047	0.280

In terms of the result, Putminic occupies the largest weight in the portfolio, which is 1.775. On the contrary, scudinc is sold-off with the weight of -2. The largest Sharpe Ratio is 0.280.

This time, we constructed portfolio models with a risk-free fund. According to the modern portfolio theory, we can also generate portfolio models with the risk-free fund.

$$Y = w_1 r_{put} + w_2 r_{scu} + (1 - w_1 - w_2) r_{win} + rf \tag{25}$$

By using the same method to generate w_1 and w_2, we can get the result showed below (Table 12).

Table 12 All possible combination of portfolios with risk-free fund

obs	w_1	w_2	Mean	Standard Deviation	Sharpe Ratio
1	-2.000	-2.000	0.036	0.167	0.181
2	-2.000	-1.980	0.036	0.166	0.181
3	-2.000	-1.960	0.036	0.166	0.181
4	-2.000	-1.940	0.036	0.165	0.181
5	-2.000	-1.920	0.036	0.165	0.180
…	…	…	…	…	…
62496	3.000	2.920	-0.014	0.154	-0.129
62497	3.000	2.940	-0.014	0.154	-0.129
62498	3.000	2.960	-0.014	0.155	-0.130
62499	3.000	2.980	-0.014	0.155	-0.130
62500	3.000	3.000	-0.014	0.156	-0.130

The portfolio with the largest Sharpe Ratio is presented below (Table 13).

Table 13 Portfolio with the largest Sharpe Ratio with risk-free fund

obs	w_1	w_2	w_3	Mean	Standard Deviation	Sharpe Ratio
47001	1.775	-2.000	-1.225	0.019	0.047	0.280

According to the result, the chosen portfolio is the same as the one without risk-free asset. The Sharpe Ratio is also the same as the former one. Putnminic is invested in the largest share, which is 1.775. Scudinc is sold-off for diversifying the risk.

3.3 Discussion

In this part, we combined all the chosen portfolios together (Table 14). On one hand, by comparing the mean of return and Sharpe Ratio, we find that the one with a higher Sharpe Ratio than any funds with risk-free asset shows the highest return among six portfolios. In addition, the two portfolios with higher Sharpe Ratio also own the highest Sharpe Ratio among six portfolios. Therefore, an investor needs to pay attention to both risks and return performance and treat Sharpe Ratio as an effective ratio to predict the future behavior of any funds.

Table 14 Mean and Sharpe Ratio of six chosen portfolio

Ratio	Port (lower variance)	Port with rf (lower variance)	Port (higher return)	Port with rf (higher return)	Port (higher Sharpe Ratio)	Port with rf (higher Sharpe Ratio)
Mean of return	0.005	0.011	0.009	0.014	0.013	0.019
Sharpe Ratio	0.179	0.179	0.275	0.275	0.280	0.280

On the other hand, there is an obvious enhancement in mean of return after adding risk-free asset in the portfolio.

Table 15 Weight of six chosen portfolios

Type of portfolio	W1	W2	W3
Port (lower variance)	0.99	0.07	-0.06
Port with rf (lower variance)	0.99	0.07	-0.06
Port (higher return)	1.00	1.00	-1.00
Port with rf (higher return)	1.00	1.00	-1.00
Port (higher Sharpe Ratio)	1.77	-2.00	1.23
Port with rf (higher Sharpe Ratio)	1.77	-2.00	1.23

According to the data (Table 15), most of the portfolio contains the minus weight, which indicates that people can short sell the asset with a low return to diversify the risk while minimizing the sacrifice on return. Besides, the scale of weight is various among the six portfolios. As we can see, the portfolio with a higher Sharpe Ratio owns the largest scale of weight from -2 to 1.77. The scale of the portfolio with lower variance is from -0.06 to 0.99. Therefore, narrowing the scale of weight have little impact on the low risk of the portfolio, but a broad weight scale makes it possible to pursue for a higher return.

Table 16 Funds of six chosen portfolio

Type of portfolio	Fund1	Fund2	Fund3
Port (lower variance)	Putminic	scudinc	Drefus
Port with rf (lower variance)	Putminic	scudinc	Drefus
Port (higher return)	Eqmrkt	windsor	Valkmrkt
Port with rf (higher return)	Eqmrkt	windsor	Valkmrkt
Port (higher Sharpe Ratio)	Putminic	scudinc	windsor
Port with rf (higher Sharpe Ratio)	Putminic	scudinc	windsor

From the perspective of chosen funds (Table 16), windsor, Putminic, scudince exist in four of the portfolios. The weight of windsor in portfolio with higher returns is a little bit lower than the weight in a port with a higher Sharpe Ratio, the gap is 0.23. The weight of Putminic in port with lower variance is significantly lower than the weight in port with a higher Sharpe Ratio, which is 0.78. It shows that when people chase both high return and low risk, investing assets with medium effect on diversifying the risk is more appropriate and effective for the portfolio to reach the goal. The weight of scudinc in port with lower variance is distinctively lower than the weight in port with higher Sharpe Ratio, showing the gap of 2.07. It indicates that asset having the best effect on reducing the risk is also a proper choice to play the role of short-selling in the portfolio to decrease the risk with little influence on the return.

To sum up, when people want to invest portfolio to improve the return without taking too much risk, they should establish a portfolio containing both funds with high returns to increase the overall return of portfolio and funds with relatively good effect on diversifying the risk. The funds with bad performance on return can be short sold. Risk-free asset should be added into the portfolio for improving the overall return of portfolios.

4 CONCLUSION

Nowadays, an investor's fundamental skill is to construct a portfolio based on inefficient market information. It needs to take into consideration of hedging strategies, future price estimation, and economic shocks prediction. The article aims to construct the optimal portfolio using Markowitz's Portfolio theory, giving basic insights into allocating weights inside the market. The optimal investment portfolio of risky assets is based on discovering how to reach the highest Sharpe ratio from using 3 risky assets. It also demonstrates which of the assets are more likely to apply based on the methodology. The estimation is supported by the diversification and leveraging method to adjust the portfolio risk and return. The diversification strategy aims to target assets with low or negative correlation so that the portfolio can severely lowering down its risks. Whereas leveraging strategy focusing more on higher returns of the assets regardless of its overall risks and efficiency. Furthermore, it discusses the effect of applying risk free assets to our portfolio and how it affects the Sharpe ratios. The result in the third portfolio chosen three assets has the highest sharp ratio in combination. It has the highest sharp ratio to the portfolio that only taken leveraging or diversification effect individually. The third portfolio is also the most risk averse portfolio that is close to our 6 assets portfolio. Adding up risk free rate is to consider the returns that are affected by different factors, that extra return is the estimation errors. It overall benefits the sharp ratio as it increases the measurement of return without adjusting the risk. By taking into consideration of all 6 risky assets that pass the test, the Sharpe ratio can reach to 30% by applying the weights of -2.19, -0.3, 0.88, -0.71, 2.33, 0.98 in fidel, keystne, drefus, scudinc, windsor, and vlamrkt

There are some limitations when applying the Markowitz model itself. It has the assumption as the financial markets are efficient and investors are risk averse. Thus it ignores the effect of the financial market crisis. It also completely ignores how the number of quantities being traded correlates with the assets market price. Thus, applying this theory to the model is not a perfect prediction of future price changes. There is also a limitation to the data collection as its concerns ranged from 1968 to 1982, which could be too old for the present market research. During the test, Putnminc and Equimrkt do not pass the empirical test rules. It could lead to an inappropriate estimation of the asset's future returns when we are conducting the three assets portfolio. Further research could investigate the systematical error in the market by analyzing large datasets to better predict future return.

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