Game Theory Applied in Strike Problems

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Abstract—This paper aims to provide some key reference data for the quick solution of the strike problem. It mainly includes the following contents:
1. The need to provide a basis for the settlement of the strike.
2. Through the analysis of the characteristics and advantages of Nash equilibrium theory and linear programming theory, we clarify the scientific nature of combining these three methods
3. According to these ideas and methods, by analyzing the characteristics of strikes, we establish three MODLES: M1: Natural Strike Game Theory; M2: Strike Without Government; and M3: Trade Union Strike. By strictly following modeling steps and using reasonable assumptions and inputting appropriate and key data, we can obtain reliable reference values. It provides decision-making basis for solving the problem of strike game.

Keyword—strike, Nash equilibrium, linear origramming, MATLAB, model

1 Introduction

Tens of thousands of strikes take place around the world every year. In 2016, for example, there were 801 strikes in France; On January 8, 2020, more than 200 million workers took part in India's largest-ever strike. The impact of these strikes on politics, economy, society, technology, and people's lives is enormous. A 10-day strike by dockworkers on October 3, 2002, had already cost the American economy $10 billion. Therefore, some sociologists even think that strikes are no less harmful than war in today's world.

Therefore, it is significant to choose a topic related to the strategy of quelling strikes quickly.

The methods to pacify and deal with strikes include political legislation, policy guidance, third-party mediation, negotiation, and compromise. All these methods need to be backed up by some hard data. This project is about providing that data. According to the characteristics of different types of strikes, we use the idea of Nash equilibrium and the method of linear programming to build models, input key values, solve and obtain decision-making reference data or matrix.

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2 Methodology

2.1 Problem restatement and the crux

The strike is the expression of contradiction and conflict between the two parties. It is a signal that shows a rational and emotional interweaving of both sides directly related to the strike. If the two sides can be provided some data related to the game results, such as the balance point of interest between the two sides of the strike, these data are the basis of the objective data for the rational persons, and the emotional person's tranquilizer. This is of great significance for both sides to avoid risks, eliminate misunderstandings and reach consensus as soon as possible.

2.2 Application of game theory and Nash Equilibrium

In the 1950s, The American mathematician John. Nash proposed the "Nash equilibrium" theory of non-cooperative games. This theory has been widely used in economy, politics, military, society, and other fields, and even occupies a dominant position in the way of thinking. John Nash won the 1994 Nobel Prize in economics for his work. Our using the Nash Equilibrium to solve the strike problem is to borrow its thinking method. In the Nash equilibrium point, all the choices are balanced, which means that all the players are stick to that choice without switching to other optimal choices. Moreover, in mixed Nash equilibrium situation, the utility for different choices is equal to make it no difference between two choices. Through these three MODLES, the Nash equilibrium point of double inverse interest is solved [1-3].

2.3 Application of linear programming with

Matrix and Laboratory can better match with Nash equilibrium theory, so as to provide decision data quickly, accurately and objectively. At the same time, it is easy to implement the idea of "Nash equilibrium".

Based on the above analysis, we believe that the combination of Nash equilibrium theory and MATLAB or OCTAVE is appropriate.

2.4 Theories of models established in the report

Through analysis, we believe that the combination of Nash equilibrium, mathematical modeling methods and steps, linear algebra (matrix) theory and MATLAB can provide reliable and key reference data for solving strike problems. Obviously, the theory used in modeling is the comprehensive application of the above theoretical methods. Based on this, the report establishes three corresponding mathematical models according to the three basic forms of strike, carries on the assignment and the solution, so as to get the data we need. The three basic models are:

Model 1: Natural Strike

Model 2: Strike Without Government

Model 3: Trade Union Strike

3 Assumption of Models

In these three models, we made the following five assumptions:

1.All symbols are absolute values which are nonnegative, and we use signs to represent the positive or negative characteristic.

2.All symbols are unit variable, and we use coefficient to represent the relative magnitude of values.

3.The reputation of trade union is proportional to the benefits gained by labours

4.Even though the variable N is not determinant, to make the model more realistic we still add that variable.

5.In model 3, since the probability of revenge approaches to 0, assume the probability of revenge equals 0.

6. For employer, we suppose all cost to run the company is the cost of salary.

4 Model 1: Natural Strike

4.1 Symbol explanation

Table 1 symbol of variables in Model 1

Symbol	Description
C_s	Cost of strike.
L_r	Labour's loss because of the revenge of the employer.
C_r	Cost of revenge.
L_s	Employer's loss in strike.
S_r	Increased salary received.
S_p	Increased salary paid.

Table 1 describes the variables we use in Model 1.

4.2Game theory graph and payoff matrix

Table 2 payoff matrix of Model 1.

Labours \ Employers	Revenge	Do Nothing	Compensate (pay rise)
Strike	$(-C_s - L_r, -C_r - L_s)$	$(-C_s, -L_s)$	$(-C_s + S_r, -L_s - S_p)$
No Strike	$(-L_r, -C_r)$	$(0,0)$	$(S_r, -S_p)$

Table two shows the payoff of each player in different scenario [4-6].

4.3 Pure Nash Equilibrium strategy

According to the payoff matrix （Table 2）, when labours choose to strike, comparing the corresponding rewards for the employers, we can see that the strategy for the employers is to Do Nothing $(-L_s \geq -C_r - L_s, (-L_s \geq -L_s - S_p)$; similarly, when labours choose not to strike, comparing the corresponding rewards for the employers, we can see that the strategy for the employers is to Do Nothing as well. Therefore, the dominant strategy for the employers is to Do Nothing. Then we can eliminate the payoff matrix to the following.

Table 3 payoff matrix of Model 1 when employers do nothing

Employers Labours	Do Nothing
Strike	$(-C_s, -L_s)$
No Strike	$(0,0)$

From table 3, a part of Table 2 showing the payoff in the scenario of employers' "do nothing", it is trivial to see that the dominant strategy for labours is not to strike since $-L_s \leq 0$.

Therefore, the Nash-equilibrium for the initial game is No Strike – Do Nothing. (Table 3)

However, this Nash equilibrium is different from the real-life case. The main flaws of the model lie in the absence of some critical variables (that will be included in the new model) and the strategy of employers, which necessitates the construction of a new model. The following explains the math behind this model (entitled General Case (non-symmetric)).

5 Model 2: Strike without trade union

5.1 General Case (non-symmetric)

Table 4 payoff matrix of Model 2

Player 1 Player 2	Strategy A1	Strategy A2
Strategy B1	(a_{11}, b_{11})	(a_{12}, b_{12})
Strategy B2	(a_{21}, b_{21})	(a_{22}, b_{22})

According to Table 4, let probability of Strategy A1 be p and probability of Strategy B1 be q

Let the expected utility of player 2 when player 1 chooses Strategy A1 be EU_{A1} and the expected utility of player 2 when player 1 chooses Strategy A2 be EU_{A2}

$EU_{A1} = qb_{11} + (1-q)b_{21}$ (1)

$EU_{A2} = qb_{12} + (1-q)b_{22}$ (2)

Set $EU_{A1} = EU_{A2}$, we have

$qb_{11} + (1-q)b_{21} = qb_{12} + (1-q)b_{22}$ (3)

Then if $(b_{22} - b_{21}) \cdot (b_{11} - b_{12}) > 0$

$q = \frac{b_{22}-b_{21}}{(b_{22}-b_{21})+(b_{11}-b_{12})}$ (4)

Let the expected utility of player 1 when player 2 chooses Strategy B1 be EU_{B1} and the expected utility of player 1 when player 2 chooses Strategy B2 be EU_{B2}

Similarly,

$EU_{B1} = pa_{11} + (1-p)a_{21}$ (5)

$EU_{B2} = pa_{12} + (1-p)a_{22}$ (6)

Set $EU_{B1} = EU_{B2}$, we have

$pa_{11} + (1-p)a_{21} = a_{12} + (1-p)a_{22}$ (7)

Then if $(a_{22} - a_{21}) \cdot (a_{11} - a_{12}) > 0$

$p = \frac{a_{22}-a_{21}}{(a_{22}-a_{21})+(a_{11}-a_{12})}$ (8)

5.2 Strike model without trade union

1) symbol description and assumption

Table 5 symbols of variables of Model 2

Symbol	Description
N	The amount of labor involved in strike.
M	The length of strike days.
O	The value that each labor created for the employer per day.
H	The increased part of labor's salary per labor per day.
J	The length of strike day that could be reduced by compensation.
K	The salary paid for the labor per day before compensation.
L	The length of day of the whole game lasting.
C	The cost of living for each labor per day.

Table 5 demonstrates the variables applied in Model 2 under the assumption that we consider that the game will last for certain time, L. Even the labor chooses to strike, and the employers choose not to compensate, there will be powerful external force to stop the strike (government).

L≥M≥J O≥H+K≥K

2) game theory graph and payoff matrix

Table 6 payoff matrix of Model 2

Employer / Labour	Compensate	No Compensate
Strike	$((L-M+J)(H+K)-LC,-(L-M+J)N(H+K)+N(L-M+J)O)$	$(K(L-M)-LC,-NK(L-M)+N(L-M)O)$
No Strike	$(L(H+K)-LC,-LN(H+K)+NLO)$	$(KL-LC,-LNK+NLO)$

As it is shown in Table 6 (the demonstration of payoff of different players in each scenario), for the labor, they have a constant cost of living every day, which is shown in the table as -LC. And the income for the labor equals to the days multiple the everyday salary. The days depend on whether the labor choose to strike since there is no salary during the strike days. The everyday salary depends on whether the employer choose to compensate. For the employer, based on our assumption, all cost is the salary paid to the labor. And the company will contribute their value to the company, which is shown in the table as NO multiple the days. The days depend on whether the labor choose to strike since the employer pays no salary to labor during the strike days. The everyday salary depends on whether the employer choose to compensate.

3) pure Nash Equilibrium strategy calculation

For example, we suppose that N=10 K=5 C=2 J=10 L=60 M=50 H=2 O=8

Therefore,

Table 7 payoff matrix1 of Model 2 calculation

Employer / Labour	Compensate	No Compensate
Strike	(20,200)	(-70,300)
No Strike	(300,600)	(180,1800)

From the Table 7 which exhibits an example of Model2, if the variables equal to the value provided above (Table 7), the Nash Equilibrium will go to No Compensate for employer and No strike for labor.

And, if we suppose that N=10 K=5 C=6 J=40 L=60 M=50 H=3 O=10

Therefore,

Table 8 payoff matrix2 of Model 2 calculation

Employer / Labour	Compensate	No Compensate
Strike	(40,1000)	(-310,500)
No Strike	(80,1200)	(-60,3000)

As Table 8 exhibits another circumstance in Model 2, if the variables equal to the value provided above (Table 8), the Nash Equilibrium will go to Compensate for employer and No strike for labor.

6 Model 3: Trade Union Strike

6.1 Symbol description

Table 9 symbol pf variables in Model 3

Symbol	Description
C_s	Cost of strike.
L_s	Employer's loss in strike.
S_r	Increased salary received.
S_p	Increased salary paid.
R	The change of reputation of trade union

In Model 3, we use the variables shown in Table 9, which is similar to Model 1, to simplify the trade union strike model, assuming that the cost of strike and employer's loss is different is different scenarios.

In the strike model with trade union shown in Table 10, we introduce a new variable, R, to represent the change of the reputation of the trade union in different scenarios. The variable, R, is proportional to the power of the trade union and how much benefits that the trade union provides the labours with. I assume R to be positive change in this model, which means that if the reputation decreases, the sign before R should be negative.

Scenario 1 "strike and no compensate": Since there is no effect on the salary of labours even when the labours strike, the change of reputation should be negative, referring to -R.

Scenario 2 "strike and compensate": The increase of the salary after the strike implies that labours' strikes exert pressure on the employers, indicating a positive change of reputation of trade union.

Scenario 3 "no strike and no compensate": Because no one is involved or influenced in this scenario, referring to non-changing for the reputation of trade union.

Scenario 4 "no strike and compensate": Even without the strike of the labour, the employers still compensate the labours, which exhibit the strong power and influence of trade union. Therefore, the change of the reputation of trade union is positive R with a coefficient of two.

6.2 Game theory graph and payoff matrix

Table10 payoff matrix of Model 3

Employers **Labours**	**No Compensate**	**Compensate (pay rise)**
Strike	$\left(-C_{s1}, -L_{s1},\ -\mathrm{R}\right)$	$\left(-C_{s}2 + S_{r}1, -L_{s2} - S_{p1,}\mathrm{R}\right)$
No Strike	$(0{,}0,\ 0)$	$(S_{r2}, -S_{p2,}2\mathrm{R})$

6.3 Mixed Nash Equilibrium strategy

Like what Model 1 suggests, there is pure Nash equilibrium ("No strike and No compensate") in the game theory model without the interference of trade union. However, in Model 3, the change of reputation of trade union results in the non-dominant strategy in the model because trade union are more likely to switch to the scenario where they can get positive reputation scenario. Therefore, the strategy is mixed with different possibilities for different options in this model.

The definition of Nash Equilibrium states that a mixed-strategy Nash equilibrium is a mixed strategy action profile with the property that no single player can obtain a higher expected payoff (utility) according to the player's preference over all such lotteries [7-10]. This definition indicates that the expected utility for different options in the game theory should be the same for each player.

Therefore, we intend to assume the possibility of each action and figure out the value of each possibility which could make the utility equal for each player.

The assumption for the possibility is demonstrated as the table 11 below.

Table 11 possibility matrix of Model 3

Employers **Labours**	**No Compensate**	**Compensate (pay rise)**
Strike	$(Z1,\ X2)$	$(Z1,\ X3)$

No Strike	$(Z2,\ X2)$	(Z2, X3)

The utility of labours' "strike" is calculated as

$U = X2\ \times\ (-Cs1) + X3\ \times\ (-Cs2 + Sr1)$ (9)

The utility of labours' "No strike" is calculated as

$U = X2\ \times 0 + X3 \times (Sr2)$ (10)

The utility of employers' "no compensate" is calculated as

$U = Z1\ \times (-Ls1) + Z2\ \times\ 0$ (11)

The utility of employers "compensate" is calculated as

$U = Z1\ \times\ (-Ls2 - Sp1) + Z2\ \times (-Sp2)$ (12)

Since X3 + X2 = 1 and Z1 + Z2 = 1, eTm X = 1 and eTm Z = 1.

With these four equations, it is easy to get the solution for the four variables (X3, X2, Z1, Z2).

6.4 Utility of players

Payoff matrix of Labour: $\begin{bmatrix} -C_{s1} & -C_{s2} + S_{r1} \\ 0 & S_{r2} \end{bmatrix}$

Payoff matrix of employer: $\begin{bmatrix} -L_{s1} & -L_{s2} - S_{p1} \\ 0 & -S_{p2} \end{bmatrix}$

Payoff matrix of trade union: $\begin{bmatrix} -R & R \\ 0 & 2R \end{bmatrix}$

Possibility matrix of labour: $\begin{bmatrix} Z1 \\ Z2 \end{bmatrix}$

Possibility matrix of employer: $[\ X2\ \ X3]$

The utility of Labours is calculated as

$Ul = \sum_{i=1}^{2} xj \times \sum_{i=1}^{2} zi\ \times\ Plij = Z^{T} * Pl * x$ (13)

The utility of Employer is calculated as

$Ue = \sum_{i=1}^{2} xj \times \sum_{i=1}^{2} zi\ \times\ Peij = X^{T} \times Pe^{T} \times z$ (14)

The utility of trade union is calculated as

$Utu = \sum_{i=1}^{2} xj \times \sum_{i=1}^{2} zi\ \times\ Ptuij = Z^{T} \times Ptu \times x$ (15)

The net utility of labours is calculated as

$\Delta Ul = Ul - Utu$ (16)

7 Limitations and improvements

7.1 restriction of our models

In the three models we investigated, we only involve limited players in game theory (labours, employees, and trade union). However, the strike issues are more complicated, which are influenced by a large number of factors. Hence, our models do not comprehensive enough to reveal the whole mechanism behind strikes and the accurate reaction of labours and employees as well as the trade-off or benefit or loss of these players. Without the consideration of other factors besides the variables we involved, our models are trying to simulate the real-life strike issues by analyzing all the three players, but these models still could not fully represent all the strike cases in all kinds of scenarios. Our models are effective and useful under the condition we assume.

In addition, labour laws are different in countries and some assumption in our models may not be applied in the real life. Therefore, our models may not be used for consideration if the country's labour law have more important restrictions and requirements for the companies and employers and trade union.

7.2 lack of data

Our models use symbols to stand for the benefits or losses for each player in different scenarios. However, those symbols are theoretical and, hence, must be confirmed with the data in real life strike issues. However, the official websites of some strike unions do not publish any data or numbers about their loss or gain, which may not correspond to the model and assumption we suggest.

7.3 optimization of models (more variables)

Because of the restriction of our models for the assumption, in order to get a more comprehensive and advanced model for the game theory of strikes, we had better introduce more relevant players or variables that may affect the establishment of models to a large extent. Instead of calculation the strategy in the two-people game theory in a two-dimension plane, we should use three-dimension plane to figure out the equilibrium point along those variables.

8 Conclusion

In all, model 1 states a general and theoretical strike game theory situation where there should be no strike and no compensation while model 2 and model 3 demonstrates the different Nash equilibrium in different strike scenarios by considering different players involved and the specific costs and benefits for each scenario. When there are only employees and the labours in model 2, there will be two different pure Nash equilibrium ("No compensate No Strike" and "Strike Compensate") depending on the value of the variables we assume in the model 2 analysis above. When trade union intervenes the strike problems, because the reputation influences the optimal choice for the players, the strategy in the model 3 should be a mixed equilibrium and, therefore, we calculate the possibility for each action by applying linear algebra to balance the utility for each choice. In conclusion, we identify a theoretical natural strike game theory model (general but not realistic) and realistic strike model (considering more details for costs and benefits: life costs and productivity, etc.) as well as the strike model with trade union. For the future work, we aim to figure out realistic numbers and combine MATLAB to calculate the existing Nash equilibrium. In addition, we may introduce more variables or players involved in the strike problems.

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