

# A Game Model and Simulation Analysis of Emission Reduction Problem Between Enterprises and Third-Party Verifiers

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**Abstract**—Under the government subsidy mechanism, this paper investigates the behavioral choices and evolutionary dynamics of carbon emission reduction enterprises and third-party verification agencies using an evolutionary game model. The model is also simulated and analyzed by using MATLAB software. The results show that the final behavioral choice of carbon emission reduction enterprises is to reduce carbon emissions under different amounts of government subsidies, fines for excess carbon emissions and fines for negligence, the probability of unscheduled government spot checks, and the price of carbon trading market. The decision of the third-party verification agency is determined by the amount of government financial support. Therefore, the government should formulate appropriate subsidy policies and reward and punishment mechanisms to promote the environmental monitoring behavior of third-party verification agencies, in order to achieve the purpose of reducing social carbon emissions and protecting the environment.

**Keywords:** carbon emission reduction; government subsidies; evolutionary game model; Regression analysis

## 1 INTRODUCTION

Since 1990, the issue of greenhouse gas emissions control has become one of the more relevant topics of discussion today. In the implementation of the strategic goal of low carbon emission reduction, the government plays a key role as a leader in the monitoring process of industrial production. China has always been a firm supporter of carbon emission solutions, adopting policies such as subsidies [1] and cap-and-trade [2]. And the government is paying more and more attention to carbon emission reduction, while consumers' awareness of energy saving and emission reduction is also gradually increasing. This is an important driving force for the development of the domestic low-carbon economy in recent years. Therefore, the products produced by enterprises are also transforming into low-carbon products. Behind this requires technical improvements and increased investment in carbon emission reduction costs. The supervision of third-party verification agencies should also be improved [3]. In this context, manufacturers are faced with huge technical costs and the cost of carbon reduction patterns vary from industry to industry [4], so it is important to determine the most appropriate carbon emissions and carbon allowances to purchase [5]. Through the research and analysis in this paper, we hope to help enterprises make the most reasonable choice [6]. It also helps the government to make the decision on subsidies and the third-party verification agency to

monitor the decision [7].

## 2 MODEL FORMULATION

### 2.1 Problem description

According to the emission reduction regulations introduced by the country [8], enterprises make technological improvements to reduce carbon emissions and thus accelerate the achievement of carbon reduction targets [9]. Generally speaking, there is a higher cost in this industry, making enterprises less motivated. From the perspective of carbon emission reduction incentive policies [10], the government should give certain subsidies and support to emission reduction enterprises to reduce the cost of emission reduction [11]. In addition, it can also rely on third-party environmental protection agencies to monitor the carbon emissions of enterprises through the purchase of social services [12]. The government should also give some additional financial support to the third-party monitoring agencies [13], which can enhance the role of the third-party monitoring agencies in the implementation and thus improve the social credibility of environmental management.

### 2.2 Parameter assumptions and payment matrix

To formulate the problem, we make the following assumptions. The game subjects of both carbon emission reduction enterprises and third-party verification agencies are finite rational and choose the strategy of maximizing benefits. The basic benefits of enterprises and third-party verification agencies are  $E_1$  and  $E_2$ . The cost of carbon reduction for enterprises is  $C_2$ . Oversight costs of third-party verification agencies is  $C_4$ . Under the supervision of a third-party verification agencies, the cost of carbon trading required when a enterprise does not reduce emissions is  $C_1$ . The government's financial support for enterprises and third-party verification agencies are  $S$  and  $R$ . The probability of government sampling of third-party verification agencies is  $\alpha$ . When the government finds that the third-party verification agencies does not do its duty to supervise, the government's penalty to the third-party verification agencies is  $S_3$  if the enterprise reduces carbon emissions; if the enterprise does not reduce carbon emissions, the government's penalty to the third-party verification agencies is  $S_2(S_2 > S_3)$ , and the penalty to the enterprise is  $S_1$ . The payment matrix of the game between the two parties is shown in the following table.

## 3 EVOLUTIONARY GAME MODEL DISCUSSION

### 3.1 Stability analysis of equilibrium point

The replication dynamic equation of the system(S) is:

Table 1 Payment Matrix

Carbon reduction enterprises	Third party verification agency	
	Supervision	Non-supervision

Emission reduction	$S + E_1 - C_2$ $R + E_2 - C_4$	$S + E_1 - C_2$ $(1 - \alpha)(E_2 + R) + \alpha(E_2 - S_3)$
No emission reduction	$E_1 - C_1$ $R + E_2 - C_4$	$(1 - \alpha)E_1 + \alpha(E_1 - S_1)$ $(1 - \alpha)(E_2 + R) + \alpha(E_2 - S_2)$

$$\begin{cases} \frac{dx}{dt} = x(1-x)[y(C_1 - \alpha S_1) + \alpha S_1 + S - C_2] \\ \frac{dy}{dt} = y(1-y)[x\alpha(S_3 - S_2) - C_4 + \alpha(S_2 + R)] \end{cases}$$

It is easy to obtain the following Proposition 1.

**Proposition 1** The equilibrium point of the system (S) is (0,0), (0,1), (1,0) (1,1). When  $C_1 > \alpha S_1$ ,  $C_4 > \alpha(S_3 + R)$ ,  $\alpha S_1 + S < C_2 < C_1 + S$ ,  $(X_D, Y_D)$  is also the equilibrium point of the system (S), where  $X_D = \frac{C_4 - \alpha(S_2 + R)}{\alpha(S_3 - S_2)}$ ,  $Y_D = \frac{C_2 - \alpha S_1 - S}{C_1 - \alpha S_1}$ .

### 3.2 Analysis of evolution results

The Jacobi matrix of this game system for equation (3) is

$$J = \begin{bmatrix} \frac{\partial F(X)}{\partial x} & \frac{\partial F(X)}{\partial y} \\ \frac{\partial F(Y)}{\partial x} & \frac{\partial F(Y)}{\partial y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

where  $a_{11} = (1 - 2x)[y(C_1 - \alpha S_1) + \alpha S_1 + S - C_2]$ ,

$a_{12} = x(1 - x)[C_1 - \alpha S_1]$ ,  $a_{21} = y(1 - y)[\alpha(S_3 - S_2)]$ ,

$a_{22} = (1 - 2y)[x\alpha(S_3 - S_2) - C_4 + \alpha(S_2 + R)]$ .

The determinant of the matrix J is

$$\begin{aligned} \det J &= (1 - 2x)[y(C_1 - \alpha S_1) + \alpha S_1 + S - C_2] \\ &\times (1 - 2y)[x\alpha(S_3 - S_2) - C_4 + \alpha(S_2 + R)] \\ &- x(1 - x)[-\alpha(S_1 + A) + C_3 + A] \\ &\times y(1 - y)[\alpha(S_3 - S_2)] \end{aligned}$$

The trace of the matrix J is

$$\det J = (1-2x)[y(C_1 - \alpha S_1) + \alpha S_1 + S - C_2]$$

$$\times (1-2y)[x\alpha(S_3 - S_2) - C_4 + \alpha(S_2 + R)]$$

$$-x(1-x)[- \alpha(S_1 + A) + C_3 + A]$$

$$\times y(1-y)[\alpha(S_3 - S_2)]$$

When  $\det J > 0$  and  $\text{tr}J < 0$ , the system evolutionary has an stabilization strategy (ESS). Substituting each of the five equalization point into  $\det J > 0$  and  $\text{tr}J < 0$ . Let  $A = \alpha S_1 + S - C_2$ ,  $B = \alpha(S_2 + R) - C_4$ ,  $C = C_1 + S - C_2$ ,  $D = \alpha(S_3 + R) - C_4$ . The evolutionary stabilization strategy of the system (S) can be obtained as shown in following tables.

**Table 2** Evolutionary results for the corresponding conditions

Point	$A < 0, B < 0, C < 0, D < 0$			$A > 0, B > 0, C > 0, D > 0$			$A > 0, B > 0, C < 0, D < 0$		
	detJ	trJ	result	detJ	trJ	result	detJ	trJ	result
0,0		-	ESS	+	+	UP	+	+	UP
0,1	-	N	SP	-	N	SP	+	-	ESS
1,0	-	N	SP	-	N	SP	+	-	ESS
1,1	+	+	SP	+	-	ESS	+	+	UP
$X_D, Y_D$							+	N	SP

Note: SP represents Saddle point; UP represents Unstable point

**Table 3** Evolutionary results for the corresponding conditions

Point	$A > 0, B > 0, C < 0, D > 0$			$A < 0, B > 0, C < 0, D > 0$			$A < 0, B > 0, C < 0, D < 0$		
	detJ	trJ	result	detJ	trJ	result	detJ	trJ	result
0,0	+	+	UP	-	N	SP	-	N	SP
0,1	+	-	ESS	+	-	ESS	+	-	ESS
1,0	-	N	SP	+	+	UP	-	N	SP
1,1	-	N	SP	-	N	SP	+	+	UP

**Table 4** Evolutionary results for the corresponding conditions

Point	$A > 0, B > 0, C > 0, D < 0$			$A > 0, B < 0, C > 0, D < 0$			$A > 0, B < 0, C < 0, D < 0$		
	detJ	trJ	result	detJ	trJ	result	detJ	trJ	result
0,0	+	+	UP	-	N	SP	-	N	SP
0,1	-	N	SP	+	+	UP	-	N	SP
1,0	+	-	ESS	+	-	ESS	+	-	ESS
1,1	-	N	SP	-	N	SP	+	+	UP

As shown in the table2 to table4, we can obtain the ESS of the system (S).

**Proposition 2**

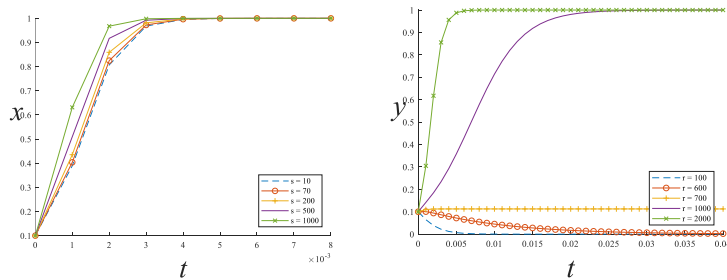
- (1) When  $A < 0, B < 0, C < 0, D < 0$ , the ESS is  $(0,0)$ .
- (2) When  $A > 0, B > 0, C < 0, D > 0$  or  $A < 0, B > 0, C < 0, D > 0$  or  $A < 0, B > 0, C < 0, D < 0$ , the ESS is  $(0,1)$ .
- (3) When  $A > 0, B > 0, C > 0, D < 0$  or  $A > 0, B < 0, C > 0, D < 0$  or  $A > 0, B < 0, C < 0, D < 0$ , the ESS is  $(1,0)$ .
- (4) When  $A > 0, B > 0, C < 0, D < 0$ , the ESS is  $(0,1)$  or  $(1,0)$ .
- (5) When  $A > 0, B > 0, C > 0, D > 0$ , the ESS is  $(1,1)$ .

From Proposition 2, when  $A < 0, B < 0, C < 0, D < 0$ , the strategy chosen by enterprises and third-party verifiers is non-abatement and non-regulation. When  $A > 0, B > 0, C < 0, D > 0$  or  $A < 0, B > 0, C < 0, D > 0$  or  $A < 0, B > 0, C < 0, D < 0$ , the strategy chosen by enterprises and third-party verifiers is not to reduce emissions and regulate. When  $A > 0, B > 0, C > 0, D < 0$  or  $A > 0, B < 0, C > 0, D < 0$  or  $A > 0, B < 0, C < 0, D < 0$ , the strategy chosen by enterprises and third-party verifiers is to reduce emissions and not to regulate. When  $A > 0, B > 0, C < 0, D < 0$ , the strategy chosen by enterprises and third-party verifiers is no abatement and regulation or abatement and no regulation. When  $A > 0, B > 0, C > 0, D > 0$ , the strategy chosen by enterprises and third-party verifiers is to reduce emissions and regulate.

**4 SIMULATION**

In order to present the comparison results more intuitively and to verify the obtained theorems, the game model is studied in depth using MATLAB software. The behavior of both sides of the game is considered as influenced by the variation of parameters. According to the actual economic situation of carbon emission reduction enterprises and third-party verifiers, it is reasonable to assume that  $C_1=400, E_1=1200, E_2=1000, C_2=600, C_4=700, A=300, \alpha=0.5, C_3=500, S=1100, R=1300, S_1=800, S_2=700$ .

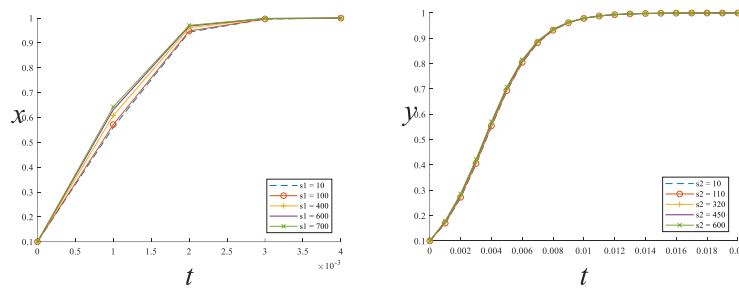
**Scenario 1** The effects of different government subsidies  $S$  and  $R$  on the behavior of carbon reduction companies and third-party verifiers, respectively.



**Figure 1** The impact of  $S$  on enterprises and  $R$  on Verification Agency

From Figure 1, the different values of  $S$  do not affect the behavioral choices of carbon abatement firms, simply that as the government subsidies increase, carbon abatement firms are able to approach the equilibrium decision more quickly. And as seen in Figure 2, there exists a value of  $R$  as an intermediate value for the decision choice of the third-party verifier. When the government subsidy to the third-party verifier is higher than this value the third-party verifier will choose to supervise, while when the government subsidy is lower than this value the third-party verifier will choose not to supervise. This indicates that government financial support influences the behavior of third-party verifiers, and the higher the subsidy amount is the more incentive for the third-party verifier's supervision behavior.

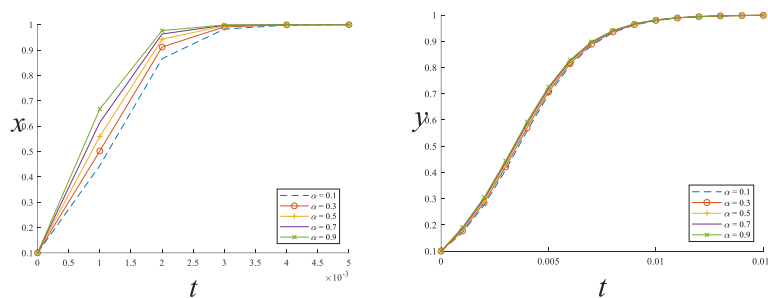
**Scenario 2** The effects of different government penalties  $S_1$  and  $S_2$  on the behavior of carbon reduction enterprises and third-party verifiers, respectively.



**Figure 2** The impact of  $S_1$  on enterprises and  $S_2$  on Verification Agency

It is obvious from Figures 2 that the different values of the government's penalty amounts  $S_1$  and  $S_2$  do not affect the final behavioral choices of carbon abatement enterprises and third-party verifiers. The higher the government penalty amount  $S_1$ , the faster the carbon abatement enterprises will choose their abatement behaviors, while the government's penalty intensity to the third-party verifier has almost no effect on their behavioral choices.

**Scenario 3** The impact of government sampling probability  $\alpha$  on the behavior of carbon emission reduction enterprises and third-party verifiers, respectively.



**Figure 3** The impact of  $\alpha$  on enterprises and Verification Agency

As can be seen in Figures 3, taking different values of the government sampling probability  $\alpha$  does not affect the behavioral choices of carbon reduction enterprises and third-party verifiers.

It only affects the speed of stabilization of both parties. The results show that the higher the probability of government spot-checking, the faster the speed of stabilization of carbon emission reduction enterprises and third-party verifiers. The impact on the speed of third-party verifiers is a bit more obvious.

## 5 CONCLUSIONS

(1) The decisions of carbon abatement companies and third-party verifiers are influenced by the cost of abatement, government incentives and penalties, and the probability of spot checks. When the abatement cost and the amount of subsidies or penalties vary within a certain range, the ESS will change accordingly.

(2) When the amount of government subsidies or penalties is within a specific range, the system may converge to two different ESS, and then, external factors become important factors affecting the evolutionary stability of the system. The government should set the subsidy level with the actual development of the specific situation of the enterprise, but the government's financial support for third-party verification agencies should not be too low.

(3) The higher the probability of spot checks by local governments, the higher the willingness of carbon abatement firms to adopt abatement strategies. It can be seen that carbon abatement enterprises will be motivated to promote emission reduction efforts due to the increased supervision by local governments. But ultimately, none of them will affect the behavioral choice of carbon abatement enterprises, whose behavioral choice is still to reduce emissions. The stronger the government's supervision, the stronger the willingness of the third-party verifier to supervise. The final behavioral choice of the third-party verifier is still to conduct supervision.

(4) An increase in the market price of carbon trading helps to increase the willingness of carbon abatement firms to reduce emissions, and the higher the market price, the more efficient the carbon abatement firms are toward their ultimate stabilization strategy. But the fluctuation of carbon trading market price will not affect the final behavior choice of enterprises. The ultimate stabilization strategy of enterprises is to adopt carbon emission reduction behaviors.

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