A Credible Mean-Skewness-Kurtosis Portfolio Selection Model with Chance-Constraint

Xiaolian Meng[1, *], Jing Ma[2]
[*] Corresponding author: mengxl66@126.com, Email: 120107010880@njust.edu.cn

School of Economics and Management Nanjing University of Science and Technology Nanjing, China

Abstract—This paper firstly suppose that the distribution of asset returns has the characteristics of heavy tail and high peak in the actual financial market, and the risky asset returns are set as triangular fuzzy numbers. Meanwhile, the third and fourth moments of the returns are used to express skewness and kurtosis. Based on the credibility theory, considering the degree of risk preference of investors, a credible multi-objective portfolio selection model with chance-constraints is built. Secondly, an improved multi-objective particle swarm algorithm is designed to solve the model, and an empirical analysis is conducted to prove the validity of the model by using historical trading data of 12 stocks from Shanghai Stock Exchange.

Keywords-portfolio selection; credibility theory; chance- constraint; improved multi-objective particle swarm optimization

1 INTRODUCTION

The securities market is an investment environment with great uncertainty. Investors need to consider possible asset types and portfolio changes in order to achieve a reasonable allocation of limited assets. The basic idea of investment portfolio theory is that the reasonable investment strategies is made to maximize the expected return of investors under a certain risk level, or minimize the risk of a given return, so that the optimal asset allocation goal is weighed.

In 1978, Zadeh [1] developed the possibility theory on the basis of fuzzy theory. Fuzziness has been emphasized in the research of portfolio selection model and used to describe the fuzzy and uncertain market environment. In the following decades, possibility theory has been widely used, but it does not possess self-dual property and lacks a strict mathematical foundation. Liu [2,3] first proposed the credibility measure in 2002, and created the credibility theory in 2004 as the axiomatized basis of fuzzy theory in order to overcome this shortcoming. Since then, some researchers have applied credibility theory to the research of portfolio selection model. For example, Zhang (2012) [4] proposed the mean-CVaR model under the credibility theory, and used performance measurement indicators such as Jensen Index to evaluate the established model. Mohebbi et al. (2018) [5] constructed a dual-objective mean-VaR portfolio selection model based on the combination of credibility theory and scenario trees, and used interactive dynamic programming to solve the model.

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Some scholars have conducted improved research on related assumptions in the classic portfolio model that do not conform to the reality of the securities market in addition to the description of uncertainty.

There is an asymmetrical peak and thick tail in the distribution of asset returns pointed out by Mandelbrot (1963) [6]. This view is in contradiction with the assumption that the return on assets follows a normal distribution in the mean-variance model proposed by Markowitz (1952) [7]. This makes the higher-order moments mean and variance of the return on assets be considered in the portfolio selection model. The higher-order moment overcomes the limitation that the variance measure is not sufficient to explain the non-normal distribution of returns. Therefore, the higher-order moments were introduced into the portfolio selection model, and the skewness and kurtosis of returns were taken as the objective function by many researchers. Yang and Lin (2014) [8] established a mean-variance-skewness-kurtosis portfolio model, taking into account realistic constraints such as transaction costs, and using genetic algorithms to solve the model. Their research work has verified the importance of higher-order moments. Deng and Liu (2020) [9] proposed a multi-objective high-order moment fuzzy stochastic portfolio model that considers background risks. The empirical results show the influence of skewness and kurtosis in the portfolio selection model.

The portfolio selection model has gradually become closer to reality with the continuous research of scholars. The chance-constraint theory is proposed considering that investors have different risk preferences under realistic investment scenarios, and hope the risk can be controlled within the expected risk threshold in order to realize the actual rate of return be greater than the given expected rate of return. Charnes and Cooper (1959) [10] proposed a chance-constrained programming to deal with uncertain variables. The probability of the random constraint condition be established is not less than a certain confidence level in this chance-constrained programming. This theory is subsequently used in the construction of the portfolio selection model. Gupta and Mehlawat (2014) [11] constructed a multi-objective portfolio selection model with chance constraints, and designed a hybrid intelligent algorithm to solve the model. Zhang and Huang et al. (2019) [12] established a multi-period portfolio selection model, which the influence of chance- constraints was taken into account so that the rate of return is not lower than a given confidence level.

In summary, the deficiencies of the classic portfolio selection theory had been perfected by the research on credibility theory and higher-order moments in terms of self-duality and the hypothesis of the distribution of asset returns. This has important research significance. In addition, the chance-constraints reflecting investors' subjective factors are used to build the portfolio selection model considering the risk appetite of investors will have an impact on investment strategies in the real trading environment of the financial market. A credible mean-skewness-kurtosis model with chance constraints (hereinafter referred to as the CMSK model) is proposed, and the multi-objective particle swarm algorithm is designed to solve the model through MATLAB programming so that the accepted confidence level and the provided appropriate security boundaries by investors can be pre-set.

2 RELATED THEORIES

Based on the credibility theory [13-14], some theoretical knowledge involved in the CMSK model is initially introduced, in which the credibility theory and the definition and theorems of credibility higher-order moments are included in this part. Fuzzy numbers ξ are a special kind of convex fuzzy sets whose membership functions μ_ξ are piecewise continuous, the relevant definitions and theorems are as follows:

Definition 1 Suppose that Θ is a non-empty set and $P(\Theta)$ is a power set of Θ . Pos is called as a possibility measure when the following three conditions are met:

$$Pos\{\Theta\}=1$$

$$Pos\{\varnothing\}=0$$

For any set family in $P(\Theta)$, there are

$$Pos\left\{\bigcup_i A_i\right\}=\sup_i Pos\{A_i\}$$

Definition 2 Record the possibility measure of a fuzzy event (such as $\{\xi \le x\}$) as $Pos\{\xi \le x\}=\sup_{t\le x}\mu_\xi(t)$, then its necessity measure is defined as follows:

$$Nec\{\xi \le x\}=1-\sup_{t>x}\mu_\xi(t)$$

The credibility measurement is defined as follows according to the concept of credibility measurement proposed by Liu in the credibility theory:

$$Cr\{\xi \ge x\}=\frac{1}{2}\left(Pos\{\xi \ge x\}+Nec\{\xi \ge x\}\right)$$

It is easy to prove that the credibility measure is self-dual, that is, it satisfies:

$$Cr\{\xi \ge x\}=1-Cr\{\xi < x\}$$

Definition 3 Suppose that the membership function of fuzzy variable is defined in the credibility space (Θ , $P(\Theta)$, Cr):

$$\mu_\xi(x)=1\wedge(2Cr\{\xi=x\})$$

Theorem 1 Reliability Inversion Theorem

Suppose ξ is a fuzzy variable whose membership functions is $\mu_\xi(x)$, then the membership function for any set of real numbers A can be used to express credibility as following:

$$Cr\{\xi \in A\}=\frac{1}{2}\left(\sup_{x\in A}\mu_\xi(x)+1-\sup_{x\in A^c}\mu_\xi(x)\right)$$

Definition 4 Suppose that the fuzzy variable ξ is a function from the credibility space $\left(\Theta, P(\Theta), Cr\right)$ to the set of real numbers, and the credibility distribution of Φ_ξ: $\Re \to [0,1]$ is defined as:

$$\Phi_\xi\left(u\right) = Cr\left\{\theta \in \Theta : \xi\left(\theta\right) \le u\right\}$$

Definition 5 ξ is set as a fuzzy variable. According to its credibility distribution, the credibility expectation value of can be defined as:

$$\begin{aligned} E(\xi) &= \int_0^{\infty}\left(1\text{-}\Phi_\xi\left(u\right)\right)du - \int_{-\infty}^{0}\Phi_\xi\left(u\right)du \\ &= \int_0^{+\infty}(1 - Cr\left\{\xi \le u\right\})\ du - \int_{-\infty}^{0} Cr\left\{\xi \le u\right\}du \end{aligned} \tag{1}$$

In where, the condition for the existence of the expected value is that at least one of the two integrals is finite.

Theorem 2 Assuming that ξ and η are two mutually independent fuzzy variables, then the credible expectation for any real numbers m and n ,satisfies the following theorem:

$$E(m\xi + n) = mE(\xi) + n$$
$$E(m\xi + n\eta) = mE(\xi) + nE(\eta)$$

Definition 6 The third and fourth-order credibility moments of the expected value $E(\xi)$ are used to measure the skewness and kurtosis of the fuzzy variable ξ .The obtained third central moment and fourth central moment, namely skewness and kurtosis, are defined as follows:

$$E^{(3)}(\xi) = E\left[\left(\xi - E\left(\xi\right)\right)^3\right] \tag{2}$$

$$E^{(4)}(\xi) = E\left[\left(\xi - E\left(\xi\right)\right)^4\right] \tag{3}$$

3 MODEL

The uncertainty of the securities market can be described more by introducing the third-order moment-bias of returns and degree and fourth moment-kurtosis [15] , using the mean, skewness and kurtosis based on credibility theory as the target of the model so as to overcome the non-self-dual defect of the possibility measurement during the process of constructing the CMSK model .Meanwhile, a confidence level represented by the credibility value is set in advance as a chance-constraint to establish an effective boundary for the portfolio to realize that the actual rate of return is greater than a certain expected rate of return .Taking into account the different risk preferences of investors, In addition, three realistic constraints, including budget constraint, investment ratio constraint, and base approximation, are also added to the model in order to make the constructed model closer to the realistic investment environment. The definitions of relevant variables and parameters in the model are as follows:

$\bar{R}_i = (\alpha_i, a_i, \beta_i)$: A triangular fuzzy variable representing the uncertain rate of return of the i-th asset, its center value is $\mathrm{a_i}$, and the left-right spread is α_i and β_i;

$Cr\{\bar{R}_i \geq u\}$: Represents the reliability of $\bar{R}_i \geq u$;

x_i: Indicates the investment ratio of the i-th asset;

z_i: Indicates whether the i-th asset is invested. It is a binary variable. When $z_i = 1$, it is invested, otherwise it is not invested;

$\tilde{p}_x$: Represents a portfolio of multiple assets;

$CE(\tilde{p}_x)$: Represents the credibility mean of the portfolio's return rate;

$CS(\tilde{p}_x)$: Indicates the credibility skewness of the portfolio return rate;

$CK(\tilde{p}_x)$: Indicates the credibility kurtosis of the portfolio's return rate;

R_e: Indicates the expected rate of return on investment;

θ: Indicates the confidence level;

L: Indicates the lower limit of investment for a single investment portfolio;

u: Indicates the investment upper limit of a single investment portfolio;

m_l: Indicates the lower limit of the portfolio base constraint, that is, the minimum number of risky assets in the portfolio;

m_u: Indicates the upper limit of the portfolio base constraint, that is, the maximum number of risky assets in the portfolio;

The triangular fuzzy number is used as a measure of the uncertainty of the rate of return in the following model. Suppose $\bar{R}$ is a triangular fuzzy variable, the center value is a, and the left and right diffusions are α and β respectively in the credibility space $(\Theta, P(\Theta), Cr)$. The $\bar{R} = (\alpha, a, \beta)$, the membership function $\mu_{\bar{R}}(x)$ and its inverse function $\mu^{-1}_{\bar{R}}(x)$ can be expressed as:

$$\mu_{\bar{R}}(x) = \begin{cases} 1\text{-}(a-x)/\alpha & a-\alpha \leq x \leq a \\ 1\text{-}(x-a)/\beta & a \leq x \leq a+\beta \\ 0 & \text{otherwise} \end{cases}$$

$$\mu^{-1}_{\bar{R}}(x) = \begin{cases} (a-\alpha)(1\text{-}2x) + 2ax & x \leq 1/2 \\ a(2\text{-}2x) + (a+\beta)(2x-1) & x > 1/2 \end{cases} \tag{4}$$

From Eq. (1) and the credibility inversion theorem, the credibility expectation value of $\bar{R}$ can be expressed as:

$$E(\overline{R})=\int_0^{+\infty}(1\text{-}Cr\left\{\overline{R}\le u\right\})\ du-\int_{-\infty}^{0}Cr\left\{\overline{R}\le u\right\}du$$
$$=\int_0^{+\infty}(1-Cr\left\{\overline{R}\le u\right\})du$$
$$=a+\frac{\beta-\alpha}{4} \tag{5}$$

From Eq. (2-3), the kurtosis and skewness of $\overline{R}$ can be expressed as:

$$S(\overline{R})=(\alpha+\beta)^2(\beta-\alpha)/32 \tag{6}$$

$$K(\overline{R})=\frac{5}{432}(\alpha^4+\beta^4)+\frac{(\alpha\beta)^2}{72}+\frac{2}{135}(\alpha^2+\beta^2) \tag{7}$$

3.1 Objective function of CMSK model

From Eq. (5-7), the objective function of trustworthy mean, skewness and kurtosis of the portfolio is constructed as shown in Eq. (8-10):

$$CE\left(\tilde{x}\right)=\sum_{i=1}^{n}\left(a_i+\frac{\beta_i-\alpha_i}{4}\right)x_i \tag{8}$$

$$CS\left(\tilde{x}\right)=\sum_{i=1}^{n}\frac{\left(\alpha_i+\beta_i\right)^2\left(\beta_i-\alpha_i\right)}{32}x_i \tag{9}$$

$$CK\left(\tilde{x}\right)=\sum_{i=1}^{n}\left(\frac{5}{432}\left(\alpha_i^4+\beta_i^4\right)+\frac{\left(\alpha_i\beta_i\right)^2}{72}+\frac{2}{135}\left(\alpha_i^2+\beta_i^2\right)\right)x_i \tag{10}$$

3.2 Constraints of the CMSK model

Different investors will require the actual rate of return to be higher than a given expected one according to their own degree of reducing risk appetite. The introduction of chance- constraints can make the probability of this condition not lower than a certain confidence level, and generate different investment strategies that meet the characteristics of investor risk aversion for different confidence levels. In addition, three basic practical constraints, including budget constraints, investment ratio constraints, and base constraints, have also been added for the actual trading constraints of the stock market. The constraints of the model are described as follows:

1) Chance constraints: The portfolio Selection model with chance-constraints can enable investors to pre-set the confidence level they wish to accept and play a better role in risk control [16-17]. Assuming that the investor's expected rate of return is R_e and the confidence level θ is the credible chance-constraint is as follows:

$$Cr\left\{\sum_{i=1}^{n}\overline{R}_i x_i\ge R_e\right\}\ge\theta$$

According to Eq. (4) and credibility theory, the above formula is transformed into:

$$\sum_{i=1}^{n} \mu^{-1}_{\overline{R}_i}(\theta) x_i \geq R_e$$

when $\theta \geq 0.5$, $\sum_{i=1}^{n} \mu^{-1}_{\overline{R}_i}(\theta) x_i = \sum_{i=1}^{n} \left\{ a_i (2\text{-}2\theta) + (a_i + \beta_i)(2\theta - 1) \right\} x_i \geq R_e$

2) Budget constraints: When the proportion of each asset in the portfolio is x_i, in order to keep the sum of the investment proportions at 1, the budget constraint requires:

$$\sum_{i=1}^{n} x_i = 1$$

3) Investment ratio constraint: According to the requirements of the financial market, the investment ratio of a single asset needs to be within a certain range, that is, meet the following conditions:

$$0 \leq l \leq x_i \leq u, \quad i = 1, 2, \ldots, n$$

4) Cardinality constraints: The number of assets in the portfolio is restricted through the base number constraint to control the transaction cost in the process of securities investment. This can be expressed as:

$$m_l \leq \sum_{i=1}^{n} z_i \leq m_u, \quad z_i \in \{0,1\}$$

3.3 The concrete expression of CMSK model

$$CMSK \begin{cases} \max z_1 = CE\left(\overline{P}_X\right) \\ \max z_2 = CS\left(P_X\right) \\ \min z_3 = CK\left(P_X\right) \\ s.t. \\ \mathrm{Cr}\left\{CE\left(P_X\right) \geq R_e\right\} \geq \theta & (a) \\ \sum_{i=1}^{n} x_i = 1, x_i \geq 0 & (b) \\ l \leq x_i \leq u, i = 1, 2, \ldots, n & (c) \\ m_l \leq \sum_{i}^{n} z_i \leq m_u, z_i \in \{0,1\} & (d) \end{cases}$$

In which, the model objectives Z_1, Z_2, Z_3 respectively represent the credible mean, credible skewness and credible kurtosis of the expected return of the portfolio; constraint (*a*) represents the model's chance-constraint; constraint (*b*) the model's budget constraint; constraint condition (*c*) represents the investment ratio constraint of the model; the constraint condition (*d*) represents the cardinal number constraint of the model.

4 CMOPSO

The model constructed above is to solve a multi-objective optimization problem, in which multiple conflicting and related objectives need to be weighed. The multi-objective particle swarm optimization algorithm (MOPSO) can be used to solve the proposed model because of its few computing resources as possible to obtain a non-inferior solution set covering the entire search space, uniformly distributed, and close to the real Pareto front. In addition, the MOPSO algorithm is improved, and a new constrained multi-objective particle swarm optimization algorithm (CMOPSO) is designed, in which dynamic infeasibility constraint dominance is used as a constraint processing method to avoid premature Fall into the local optimum and improve the global search ability of the algorithm [18-19]. The key operator design of the constrained multi-objective particle swarm algorithm is as follows:

1)Dynamic ε infeasibility constraint dominance relationship:

A certain mediation method is needed to make the control of infeasible solutions have a dynamic adaptive process in order to control the proportion of infeasible solutions in the iterative process. The infeasibility of the candidate set X is defined as:

$$dc(X) = \begin{cases} \max\left\{g_i(X), 0\right\}, & i = 1, 2, \dots p \\ \max\left\{|h_j(X)| - \delta, 0\right\}, & j = 1, 2, \dots q \end{cases}$$

In which, $dc(X)$ is the distance from solution X_i to the feasible space. The infeasibility is 0 when X_i is a feasible solution; $g_i(X)$ is an equality constraint; $h_j(X)$ is an inequality constraint; δ is a tolerance coefficient, usually 0.001 or 0.0001.

In order to further satisfy the constraint requirements and make the search approach the Pareto optimal solution, the dynamic infeasibility threshold ε is defined as follows:

$$\varepsilon = \begin{cases} \varepsilon_0 \times (1 - 5t / 4M), t \le 0.8M \\ 0, \quad t > 0.8M \end{cases}$$

In which, ε_0 is the initial value of infeasibility; t is the current evolutionary algebra; M is the maximum algebra of population evolution, used to control the number of iterations of the algorithm. ε decreases with the increase of the number of iterations. When the infeasibility threshold of the solution X_i is less than ε , it is called a feasible solution.

2)The speed and position update method of particles: A linearly decreasing weight is used when updating the speed and position of the particle swarm, and the update method is as follows:

$$\begin{cases} V_i(t+1) = w \cdot V_i(t) + r1 \cdot c1 \cdot (pbest_i(t) - X_i(t)) \\ +r2 \cdot c2 \cdot (gbest(t) - X_i(t)) \\ X_i(t+1) = X_i(t) + V_i(t+1) \end{cases}$$

In which, $w = w_{\max} - \frac{t*(w_{\max} - w_{\min})}{M}$ is a linearly decreasing weight, $w_{\max}$ and $w_{\min}$ are artificially set; $r1$ and $r2$ are random numbers in the interval [0,1]; $c1$ and $c2$ are learning factors.

3)Selection strategy of local best position pbest: When the new solution $X_i(t)$ constraint obtained in the t-th iteration dominates the current local optimal position $pbest_i(t)$, take $pbest_i(t+1) = X_i(t)$; when the two do not dominate each other, randomly select one of the two as the new local optimal position; otherwise, take $pbest_i(t+1)=pbest_i(t)$.

According to the above key operators, the designed constrained multi-objective particle swarm algorithm is shown in Fig.1.

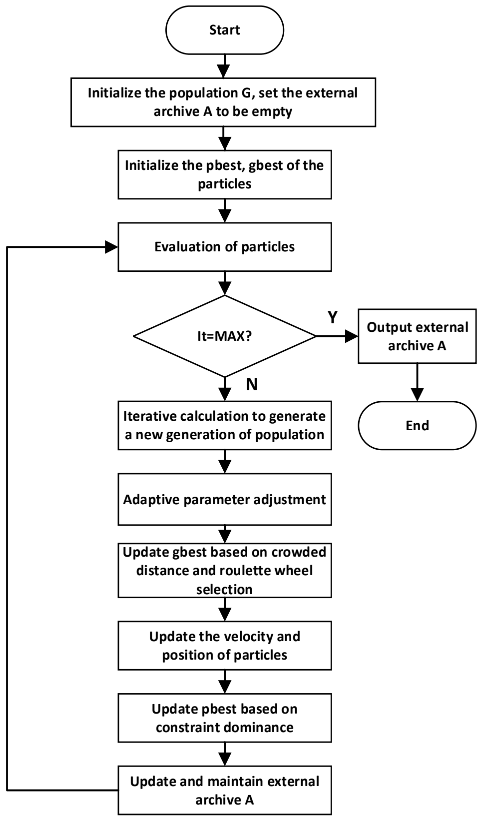

Figure.1 Basic flow chart of CMOPSO algorithm

The specific algorithm steps in Fig.1 are as follows:

Step*1*: Generate the initial population G randomly, set the external file A to be empty, and set the evolution algebra t=0;

***Step*2:** Initialize the local optimal position and the global optimal position of each particle;

***Step*3:** Let t = t+1, the iterative calculation starts to loop;

***Step*4:** Calculate the infeasibility threshold of the current evolutionary algebra, and adaptively adjust the inertia weight parameters;

***Step*5:** Combine dense distance and roulette selection to select the best global position for each particle in the population;

***Step*6:** Use linearly decreasing weights to update the speed and position of each particle in the population G;

***Step*7:** Update the local best position of each particle according to the constraint domination relationship;

***Step*8:** Update the external file A with the non-inferior solution of the particle population, and maintain the external file;

***Step*9:** Judge whether it has reached the maximum evolutionary algebra M of the population. If it does not reach M, return to Step3; otherwise, end the loop and output the external file collection.

5 EMPIRICAL RESEARCH

The historical data of 12 representative stocks (n=12) in the SSE 50 Index are selected as empirical data samples for empirical analysis from January 2019 to February 2021 for a total of 508 trading days in the following. The selected stocks, the codes and names are shown in Tab.1.

Table 1 Selected stock codes and names

Stock Code	Stock Name	Stock Code	Stock Name
600570	Hundsun	601818	China Everbright Bank
603160	Goodix	600104	Saic Motor
603259	WuXi AppTec	600031	Sany Heavy Industry
601336	New China Life Insurance	600887	Yili Group
601668	China State Construction Engineering	600196	Fosun Pharma
601066	China Securities	601166	Industrial Bank

Suppose the daily rate of return of the i-th risk asset is $r_{i,t} = (p_{i,t} - p_{i,t-1}) / p_{i,t-1}$, where $p_{i,t}$ is the closing price of security i in time t. The frequency estimation method of literature [19] are used to fuzzify the historical data of daily return rate. Hundsun is taken as an example. Firstly, the historical yields are sorted to magnitude for finding the maximum yield and minimum yield of the security and calculating the group distance. Secondly, the yields are grouped, the interval containing most of the yields are recorded as $[d_1, d_2], \ldots, [d_m, d_{m+1}]$, and let n_j be the frequency of the *j*-th interval, $j=1, 2, \ldots, m$,and record $n_k = \max_{1 \leqslant j \leqslant m}(n_j)$.The frequency statistics of Hundsun obtained are shown in Fig.2 below.

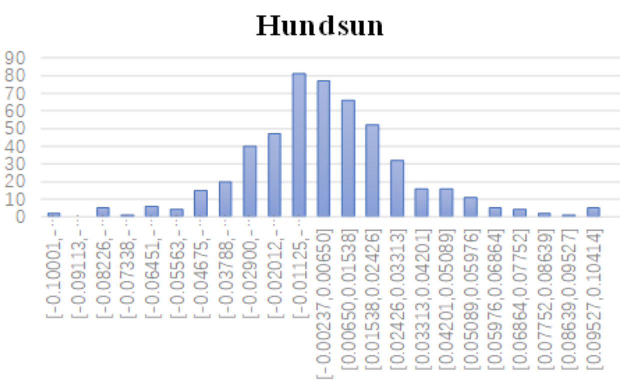

Figure.2 Frequency distribution of daily yield of Hundsun from January 2019 to February 2021

Then, let $a_i = \frac{d_k + d_{k+1}}{2}$, $\beta_i = \frac{d_m + d_{m+1}}{2} - a_i$, $\alpha_i = a_i - \frac{d_1 + d_2}{2}$, get the triangular fuzzy number $\overline{R_1} = (0.08867, -0.00681, 0.10652)$.

The fuzzy rate of return of the other 11 stocks can be obtained in the same way, and the specific data are shown in Tab.2.

Table 2 Fuzzy rate of return of 12 stocks

Stock	**Fuzzy rate of return**		
	α_i	a_i	β_i
S_1	0.08876	-0.00681	0.10652
S_2	0.09114	-0.04440	0.10413
S_3	0.08488	-0.00220	0.10187
S_4	-0.08157	0.00607	0.90370
S_5	-0.08049	0.00163	0.08374
S_6	0.08896	-0.00677	0.10675
S_7	0.06092	-0.00741	0.10662
S_8	0.08474	-0.00429	0.10396
S_9	-0.09708	-0.00154	0.09400
S_{10}	0.09757	0.00209	0.09546

S_{11}	0.09338	0.00212	0.09761
S_{12}	0.07872	0.00305	0.09447

The improved constrained multi-objective particle swarm algorithm is used to solve the model and the effectiveness of the established model and algorithm are proved. The number of evolutions $M = 300$, Population size $G = 100$, External archive size $A = 50$. The specific parameters used in the CMSK model and algorithm are shown in Tab.3.

Table 3 Main parameter settings

Parameter	Parameter settings
θ	0.80,0.85,0.90
R_e	0.
m_u	5
m_l	5
u	0.3
l	0.01
ε	0.001
M	300
G	100
A	50
w_{max}	0.9
w_{min}	0.4

Run the CMOPSO algorithm through MATLAB to get the confidence level θ=0.80,0.85, 0.90 . The approximate Pareto frontiers in three different situations are shown in Fig.3-5:

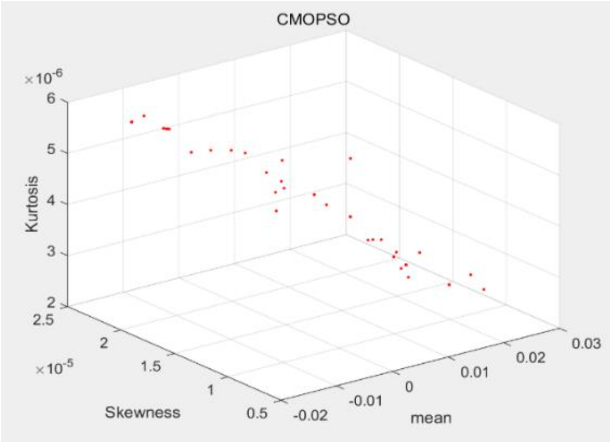

Figure.3 The three-dimensional Pareto chart obtained by the CMSK model when the confidence level $\theta = 0.8$

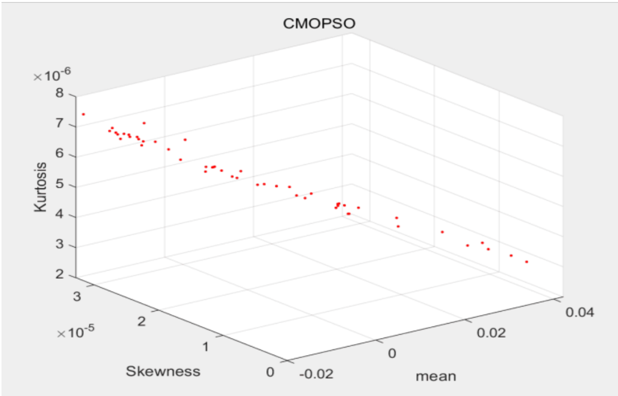

Figure.4 The three-dimensional Pareto chart obtained by the CMSK model when the confidence level $\theta = 0.85$

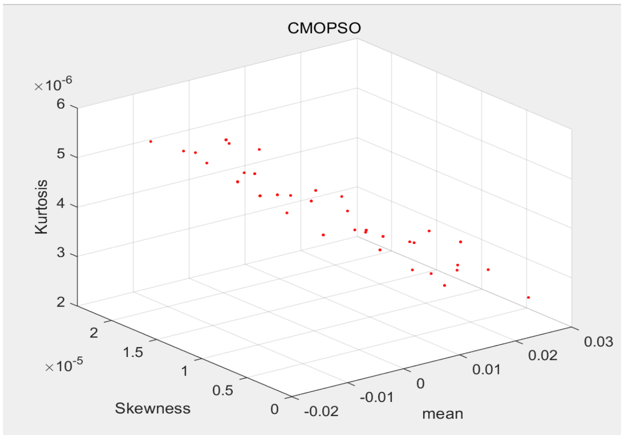

Figure.5 The three-dimensional Pareto chart obtained by the CMSK model when the confidence level $\theta = 0.9$

It can be seen from Fig.3 that these non-dominated solutions obtained by the CMOPSO algorithm have better distribution and convergence, which indicates that the Pareto optimal front has a greater relationship with the mean, skewness, and kurtosis. Take them as The model objective is used to weigh and optimize the approximate Pareto frontier, which shortens the distance from the actual Pareto frontier, thus proving the validity of the constructed CMSK model.

With confidence levels $\theta = 0.8, 0.85, 0.9$, the selection results of the investment portfolio are shown in Tab.4 below:

Table 4 Portfolio asset allocation under different confidence levels

	Stock investment ratio and target value									
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
θ =0.80	0	0.221	0	0	0.159	0	0	0	0.268	0.251
θ =0.85	0	0.156	0	0.225	0	0	0	0	0.137	0.245
θ =0.90	0.034	0.191	0	0.261	0	0	0	0	0	0.235
	S_{11}	S_{12}	Z_1		Z_2		Z_3			
θ =0.80	0	0.101	0.02545		8.4051e-06		4.0649e-06			
θ =0.85	0.237	0	0.01155		5.3349e-05		3.4325e-06			
θ =0.90	0.279	0	0.00875		2.6484e-05		4.3376e-06			

6 CONCLUSION

In this paper, how to build the high-order moment portfolio selection model based on credibility theory is researched. The chance-constraint is used to reflect the degree of investor's risk preference under different confidence levels, which has certain practical significance. In addition, the proposed CMSK model is non-convex, non-linear, and non-differentiable. Therefore, a constrained multi-objective particle swarm optimization algorithm is designed to solve it with MATLAB software. The edge search capability is improved and the non-inferior solution set covering the entire search space. Meanwhile, uniformly distributed and close to the real Pareto front is obtained with as little computing resources as possible by using the improved algorithm. The effectiveness of the target portfolio selection model and algorithm is illustrated.

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