

Numerical Solution of Delayed SIR Model of Tuberculosis with Combination of Runge Kutta Method and Taylor Series Approach

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Abstract.In this paper, the implementation of fourth-order Runge Kutta method with Taylor series approach for solving Susceptible Infected Remove (SIR) Model with including to system of delay differential equation (DDEs). The method obtained approximate solutions of the model using constant step size. The DDEs will be treated in expansion of Taylor Series i.e. $y(t - \tau) = y(t) + \tau \frac{dy}{dt}$. Numerical results are presented to show that the method is suitable for solving the System of DDEs and also time delay give the effect to Infected and recovered populations for $\tau \geq 0$.

Keywords: Runge Kutta Method; Taylor Series; SIR Model; Time Delay.

1 Introduction

Tuberculosis (TB) continues to be a health problem in the world, especially in developing countries. Tuberculosis is the top ten most common infectious diseases in the world. In 2016, as many as 10.4 million people worldwide had contracted tuberculosis and 1.7 million of them died.

Indonesia, India, China, the Philippines, Pakistan, Nigeria and South Africa are seven developing countries that contribute 64 percent of the total spread of tuberculosis worldwide. Data from the World Health Organization (WHO) recently released in January 2018 states that the number of TB cases in Indonesia reached 1.6 million in 2016, with an estimated 100,000 deaths per year. Thus, there are 273 cases of death per day or 11 deaths every hour due to TB (Putra (2018)). The average discovery for the city of Medan in that month could be 1000 people suspect TB with sufferers of 150 to 200 people every month found. TB patients who have been found in the past six years reached 7,600 people. Those who recovered were 3,078 people. The rest, currently in the process of processing and there are also many who die as many as 35 people (Dani, 2017). From the data submitted by Dani, (2017), there is an increasing TB cases in Medan, therefore finding a suitable mathematical model to control spread of TB disease is important.

Many researchers continue to build mathematical models for TB disease such as Zwerling et al., 2015, find new diagnoses, therapies, and develop algorithms to improve case discovery, diagnosis, and clinical management of patients with TB, making policies in making difficult decisions and choosing to use technology when working under heavy pressure. Then Taufik et

al. (2015) developed a mathematical model of vaccinated TB disease with exogenous infection and found that disease-free eigen values were always negative, meaning that the stable model was symptomatically disease-free. Rangkuti et. al. (2014) have been constructed an mathematical model of TB, VSEIR, to analysis the stability and the dynamical VSEIR populations for $0 \leq t \leq 25$. Iskandar et al. (2017) analyzed the stability of the mathematical model of SIR by adopting time delay during recuperation. The model was stabil.

Various numerical methodshave been solved SIR model. A more robust and intricate numerical technique is the Runge Kutta method (RK). This method is the most widely used one since it gives reliable starting values and is particularly suitable when the computation of higher derivatives is complicated Islam (2015).Based on previous statement, RK mothod will use to solve SIR model.

In skandar's paper, the solution of SIR with time delay numerically to show impact of time delay toward number of SIR populations.

2 Sir Model With Time Delay

The delay time in the model is described as the time needed by the virus that infects cells so that the virus develops, delays occur due to continuous treatment. In this paper the model of susciptible, infected and recovery (SIR) with time delay and without delay. This model was developed by Iskandar et al (2017) as

$$\frac{dS_v}{dt} = c\pi N + \gamma R - \frac{\beta_v S_v I}{N} - \mu \frac{S_v}{N} \quad (1)$$

$$\frac{dS_T}{dt} = (1 - c)\pi N - \frac{\beta_T S_T I}{N} - \mu S_T \quad (2)$$

$$\frac{dI}{dt} = \frac{\beta_v S_v I}{N} + \frac{\beta_T S_T I}{N} - \mu I - \vartheta I(t - \tau) \quad (3) \quad \frac{dR}{dt} = \vartheta I(t - \tau) - \mu R - \gamma R \quad (4)$$

$$N = S_v + S_T + E_v + E_T + I + R$$

Where N as the total population, S_v as vaccinated susceptible, S_T as unvaccinated susceptible, I , infected, R as recovered, c as vaccination rate parameter, π as birth rate parameter, β_v as transmission rate parameter of vaccinated susceptible, β_T as transmission rate parameter of unvaccinated susceptible, μ as natural mortality rate parameter, γ as the transfer rate of population from recovered to be susceptible, μ_T as parameter of death rate due to TB, v as recovery rate parameter, and τ as period recovery.

The model can be simplified by assuming the following fractions

$$w = \frac{S_v}{N}, \quad x = \frac{S_T}{N}, \quad y = \frac{I}{N}, \quad z = \frac{R}{N}$$

Thus, the model can be simplified as follows

$$\frac{dw}{dt} = c\pi + \gamma z - \beta_v xy - \mu w \quad (5)$$

$$\frac{dx}{dt} = (1 - c)\pi - \beta_T xy - \mu x \quad (6)$$

$$\frac{dy}{dt} = (\beta_v + \beta_T)xy - \mu y - \vartheta y(t - \tau) \quad (7)$$

$$\frac{dz}{dt} = \vartheta y(t - \tau) - (\mu + \gamma)z \quad (8)$$

Here, the time delay variable $y(t-\tau)$ is expanded by Taylor series in form $y(t) - \tau \frac{dy}{dt}$. The SIR model can be written as

$$\frac{dw}{dt} = c\pi + \gamma z - \beta_V xy - \mu w \quad (9)$$

$$\frac{dx}{dt} = (1 - c)\pi - \beta_T xy - \mu x \quad (10)$$

$$\frac{dy}{dt} = (\beta_V + \beta_T)xy - \mu y - \vartheta \left(y(t) - \tau \frac{dy}{dt} \right) \quad (11)$$

$$\frac{dz}{dt} = \vartheta \left(y(t) - \tau \frac{dy}{dt} \right) - (\mu + \gamma)z \quad (12)$$

3 Runge-Kutta Method

Fourth order Runge Kutta Method is the best method for solving system of differential equations. The following formula normally used is written as

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (13)$$

Where

$$\begin{aligned} k_1 &= f(x_i, y_i) \\ k_2 &= f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1h\right) \\ k_3 &= f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2h\right) \\ k_4 &= f(x_i + h, y_i + k_3h) \end{aligned} \quad (14)$$

To overcome the difficulty in solving delay form, the delay form i.e. $y(t-\tau)$ changed to $y(t) + \tau \frac{dy}{dt}$ which was proposed by Toaha (2009). They $y(t) + \tau \frac{dy}{dt}$ was obtained by Taylor series.

4 Result And Discussion

Several investigations have done for SIR model of Tuberculosis with time delay in this paper. This model is suitable for the Medan City. Some parameters are taken from department of Health. The parameter is known as $c=0.05$, $\pi=0.03$, $\gamma=0.067$, $\beta_V=0.1$, $\beta_T=0.675$, $\mu=0.03$, $\mu_T=0.032$, $\vartheta=0.7389$. The initial populations in probability are

$$w(0) = \frac{25400}{2229408}, \quad x(0) = 2080288/2229408, \quad y(0) = 92470/2229408, \quad z(0) = 31250/2229408$$

Reported by department of health of Medan city. The simulation was carried out using MAPLE by adopted data of Tuberculosis cases in the Medan city.

Behaviour dynamic of SIR model without delay time can be seen from figure 1.

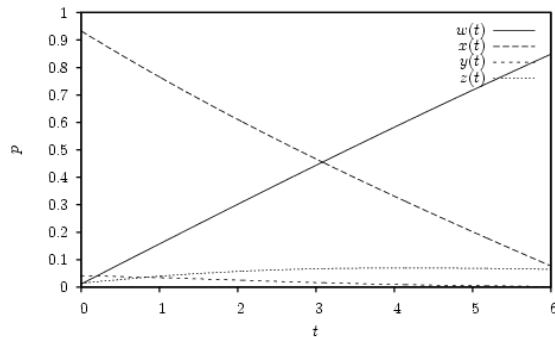
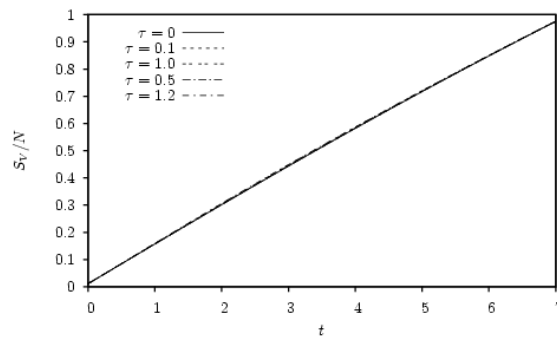
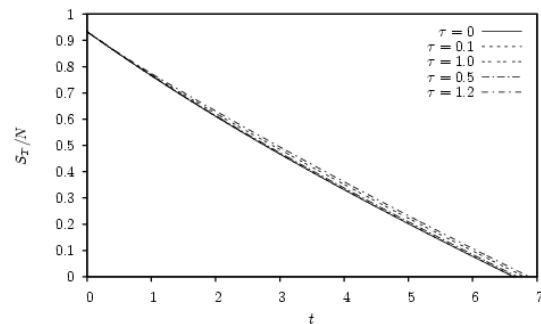


Figure 1 Dynamic behavior of SIR model of Tuberculosis in Medan City

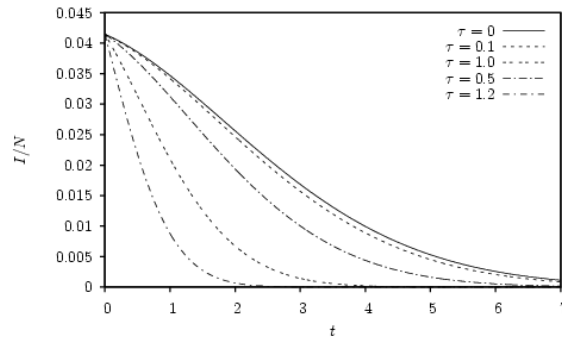
Here, x-axis is time (months) and the y-axis is the probability of SIR model. Above results showed that probability of vaccinated susceptible increased sharply, whereas unvaccinated susceptible dropped dramatically toward the x-axis. For probability of infection, decreased slightly toward the horizontal axis, it means, by this model, infection population will be small, while recovered population increase slowly because, number of infection also slight. The following figure showed dynamical result of SIR with various time delays which were described in Figure 2.



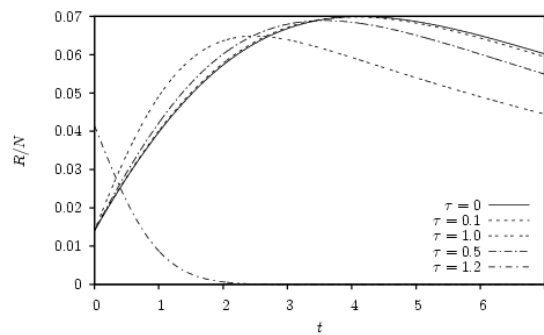
(a)



(b)



(c)



(d)

Figure 2 Graphics of Probability of all variables with Time Delay $\tau=0.1,0.5,1.0$ and $\tau=1.2$ and without time delay.

Figure 2 was divided by four time delay value (τ): (a) for $\tau=0.1,0.5,1.0$ and $\tau=1.2$, impact of time delay is small. Probability of susceptible without and with time delay almost same. (b) for different τ was given, the impact of time delay also small to susceptible population. (c) the impact of time delay is to big. By time delay, population of infection drop sharply close to x axis, as long the time delay, infected population getting smaller. (d) the last graphic showed that there is no impact of τ to development of recovery population.

5 Conclusions

Numerical solution of SIR with delay time has been obtained. Some parameters are taken from department of Health. The parameter is known as $c:=005$, $\uparrow\pi=0.03$, $\gamma:=0.333$, $\beta_v=0.1$,

$\beta_T=0.675$, $\mu:=0.03$, $\mu_T=0.032$, $\theta=0.37$ The initial populations in probability are $w(0) = \frac{25400}{2229408}$, $x(0) = 2080288/2229408$, $y(0) = 92470/2229408$, $z(0) = 31250/2229408$

reported by department of health of Medan city. Calculation for numerical method used fourth order Runge-kutta Method and assisted by MAPLE software. Time delay τ , i.e. $\tau=0.1, 0.5, 1.0$ and $\tau=1.2$, very gave big impact to number of infected population. The population of infection drop sharply close to x-axis, as long the time delay, infected population getting smaller otherwise no impact of time delay to other populations. RK method can be an alternative method for solving another system of delay differential equation.

Acknowledgments. The heading should be treated as a 3rd level heading and should not be assigned a number. The financial support received from BOPTN UNIMED is gratefully acknowledged.

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