Portfolio Models and Stock Price Forecasts Based on Mean-Variance Theory

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Abstract-This article selected the performance coefficients of ten securities stocks and combined the portfolio theory and model solution results to get a reasonable portfolio plan: P7, P8, P9, and the stock selection plan was: focus on the three stocks abc006, abc007, and abc008. Analyzing the performance indicators of the investment portfolio further, it was concluded that the investment portfolio on the required effective frontier could achieve the smallest risk standard deviation when the expected return rate was equal; when the return standard deviation was fixed, the risk was minimized. Finally, predicted the volatility of the future stock index and gave reasonable suggestions.

Keywords-markowitz mean-variance model, nonlinear least squares method, effective frontier

1 Introduction

With the rapid development of the domestic economy in recent years, China's securities market has also achieved a considerable scale, with outstanding achievements in many aspects. As the most essential feature of the securities market, volatility plays a vital role in the decision-making analysis of investors' risk and return, the maximization of shareholders' equity, and the effective supervision of supervisory authorities. Therefore, analyzing the law of volatility in the securities market and the causes of market volatility can help people understand the law and provide a traceable basis for investors, regulators, and listed companies.

This article first selected ten securities stocks, and used the Markowitz mean-variance model to comprehensively consider returns and risks to make reasonable choices. Subsequently, a single index method based on the Markowitz mean-variance model was used to measure portfolio performance using risk-reward ratios (including Treynor ratio, Sharpe ratio, and Jense index). Finally, a nonlinear fitting method was used to fit and predict the stock index fluctuations in the data.

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2 Materials and Methods

2.1 Materials

The data in this article comes from question B of the 2020 3rd China Youth Cup National College Students Mathematical Contest in Modeling. As of the end of 2019, there are 4,000 stocks traded on the Shanghai and Shenzhen stock exchanges. This article selects ten constituent stocks as the object to study the regularity of stock market fluctuations.

2.2 Solve the optimal stock selection plan and investment portfolio plan

(1) Establish a multi-objective programming model based on Markowitz's mean-variance theory, express the return with the expected rate of return(formula ①), and the standard deviation of the rate of return represents the total risk(formula ②);

The Markowitz mean-variance model is as follows:

Maximize revenue max $[E(R_P)] = W^t \cdot R$ ①

Minimize variance min $[Var(R_P)] = W^t \cdot Q \cdot W$ ②

$$\text{s.t.} \begin{cases} F^t \cdot W = 1 \\ \text{other constraints} \end{cases}$$

Among them, R_p is the rate of return of the p-th portfolio[1].

Model assumptions:

·All investors are rational investors and prefer effective frontiers. A rational investor is aversion to risk and prefers returns, and will choose the investment portfolio with the largest return and the smallest return, that is, the effective frontier [2];

·The stocks given in the attachment are all risky stocks, and there are no zero-risk stocks;

·Investment in ten stocks can be subdivided infinitely;

·Take the three-year Treasury bill interest rate as the risk-free interest rate: $r_f = 2.748\%$

(2) Using Matlab to solve the model, obtain the effective frontier composed of ten effective investment portfolios [3].

2.3 Select reasonable evaluation indicators

First, by solving the Markowitz mean-variance model, the effective investment frontier is obtained . Second, calculate the performance coefficients of ten randomly generated portfolios, including the expected rate of return and the standard deviation of the expected rate of return. Third, find the standard deviation of the combination on the effective frontier of the expected return and the expected return of the combination on the effective frontier of the standard deviation of the equal expected return.

2.4 Analyze the current volatility of the stock index and predict the volatility of the stock index in the next year

First of all, the closing price is regarded as a parameter of stock index volatility. Then the attached data is simplified and summarized with the plot function [4]. Next, comprehensively considering several large models, and finally decided to use the nonlinear least squares fitting method to fit the known function and use it to predict the stock price in the next year.

3 Results & Discussion

3.1 Optimal stock selection plan and investment portfolio plan

Table 1 Average return, standard deviation of return, and covariance matrix of ten stocks

	Average rate of return/%	Standard deviation of yield/%	Covariance matrix（×0.0001）									
abc001	0.0793%	0.13%	4.61	1.97	0.98	1.42	1.46	0.77	0.85	0.73	2.38	1.21
abc002	0.1018%	0.12%	1.97	4.09	0.47	1.16	2.11	1.75	1.72	1.22	1.27	0.71
abc003	-0.0849%	0.21%	0.98	4.70	1.20	1.06	0.99	0.58	0.86	0.52	0.47	0.48
abc004	0.0354%	0.12%	1.42	1.16	1.06	3.84	1.23	0.88	0.89	0.96	0.89	0.62
abc005	0.0534%	0.14%	1.46	2.11	0.99	1.23	5.19	1.06	1.56	1.43	0.94	0.66
abc006	0.1364%	0.14%	0.77	1.75	0.58	0.88	1.06	5.58	1.89	0.70	0.69	0.54
abc007	0.3931%	0.16%	0.85	1.72	0.86	0.89	1.56	1.89	6.84	0.60	0.47	0.46
abc008	0.1590%	0.14%	0.73	1.22	0.52	0.96	1.43	0.70	0.60	5.15	0.67	0.31
abc009	-0.0240%	0.09%	2.38	1.27	0.47	0.89	0.94	0.69	0.47	0.67	2.35	0.82
abc010	-0.1412%	0.07%	1.21	0.71	0.48	0.62	0.66	0.54	0.46	0.31	0.82	1.19

Data source: Question of the 3rd China Youth Cup Mathematics Model Undergraduate Group-Stock Selection and Investment Portfolio Plan

By observing the above Table 1, the average return of the three stocks abc003, abc009, and abc010 is negative, indicating that investing in these three stocks is likely to lose money. In addition, the standard deviation of the returns of the above ten stocks fluctuates in the interval [0.07~0.21], and the standard deviation of the overall returns is small, that is, the difference between the daily returns and the average daily returns of the ten stocks during a given data period is small, and the fluctuations Less sex. The covariance matrix shows that the expected returns of ten stocks are positively correlated, and are completely positively correlated, indicating that it is difficult for these combinations to produce risk diversification effects. At the same time, because the correlation coefficients in the covariance matrix of these ten stocks are very small, the combination may also have a dispersion effect. This shows that the impact of non-systematic risks can be reduced by diversifying investment[5].

Table 2 The mean-variance effective frontier data sheet for the asset portfolio

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
abc001	0.0000	0.0000	0.0000	0.0112	0.0608	0.1224	0.1425	0.0578	0.0000	0.0000
abc002	0.0000	0.0000	0.0048	0.0115	0.0101	0.0017	0.0000	0.0000	0.0000	0.0000
abc003	0.0331	0.0271	0.0212	0.0153	0.0095	0.0000	0.0000	0.0000	0.0000	0.0000
abc004	0.0613	0.0729	0.0842	0.0942	0.1003	0.0913	0.0209	0.0000	0.0000	0.0000
abc005	0.0104	0.0091	0.0066	0.0031	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
abc006	0.0506	0.0651	0.0787	0.0922	0.1076	0.1193	0.1048	0.0466	0.0000	0.0000
abc007	0.0519	0.1096	0.1666	0.2215	0.2713	0.3358	0.4494	0.6107	0.0000	0.0000
abc008	0.1011	0.1312	0.1608	0.1900	0.2190	0.2518	0.2824	0.2849	0.7932	1.0000
abc009	0.1016	0.1278	0.1525	0.1670	0.1507	0.0779	0.0000	0.0000	0.2068	0.0000
abc010	0.5900	0.4573	0.3246	0.1940	0.0707	0.0000	0.0000	0.0000	0.0000	0.0000
PortRisk	0.0096	0.0099	0.0105	0.0115	0.0127	0.0142	0.0160	0.0185	0.0217	0.0262
PortReturn	-0.0004	0.0001	0.0005	0.0010	0.0015	0.0020	0.0025	0.0030	0.0035	0.0039

Table 2 and Figure 1 give the mean-variance efficient frontier data and graphs of the asset portfolio, respectively. Combined with portfolio theory and model solution results to obtain a reasonable portfolio solution is: P7, P8, P9, stock selection scheme is: concentrated in abc006, abc007, abc008 these three stocks.

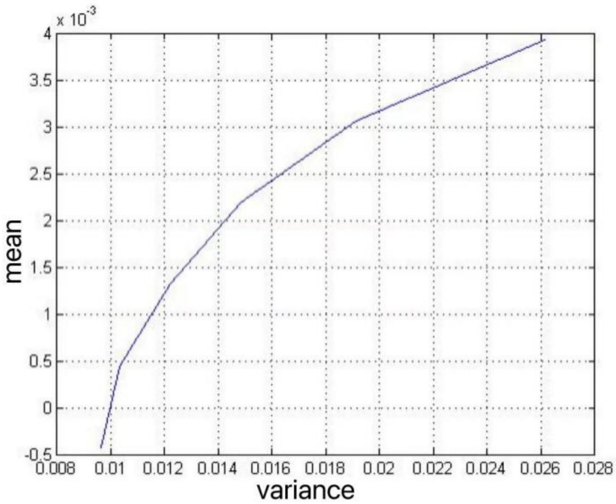

Figure 1 The mean-variance effective cutting-edge chart of the asset portfolio

3.2 Comprehensive evaluation of portfolio performance and analysis results

By solving the Markowitz mean-variance model, an effective investment frontier is obtained. Next, calculate the performance coefficients of ten randomly generated portfolios, including the expected rate of return and the standard deviation of the expected rate of return. Then, find the standard deviation of the combination on the effective frontier of the expected return and the expected return of the combination on the effective frontier of the standard deviation of the equal expected return, and obtain that the portfolio on the effective frontier can be equal in the expected return When the risk standard deviation is minimized. When the return standard deviation is fixed, the risk is minimized. Therefore, the Markowitz mean-variance model established in the problem is effective and reasonable. The effective frontiers of a portfolio are:

$$\sigma_P^2 = 1.2707u^2 + 2.1350e - 04u + 0.8942$$

Sharpe Ratio(formula ③):

$$\mathrm{SR} = \frac{\mathrm{R_P - r_f}}{\sigma_\mathrm{p}} \quad ③$$

Results:

$$SR_{P7} = \frac{\mathrm{R_{P7} - r_f}}{\sigma_{\mathrm{p7}}} = -0.1347 \qquad SR_{P8} = \frac{\mathrm{R_{P8} - r_f}}{\sigma_{\mathrm{p8}}} = -1.4990 \qquad SR_{P9} = \frac{\mathrm{R_{P9} - r_f}}{\sigma_{\mathrm{p9}}} = -1.1009$$

Finally, based on the single index method of the Markowitz mean-variance model, the Sharpe ratio is introduced as a performance indicator to measure the portfolio, and get P9> P8 > P7.

3.3 The current volatility of the stock index and the volatility of the stock index in the coming year

3.3.1 The current volatility of the stock index

Treat the closing price as a parameter of stock index fluctuations, then simplify the data and use the plot function to summarize it. By observing the above Figure 2, it is obvious that the index of stock 7 fluctuates the most and has a strong rise. Therefore, in the future investment in stocks should pay attention to the pursuit of income at the same time, pay attention to the risk of volatility.

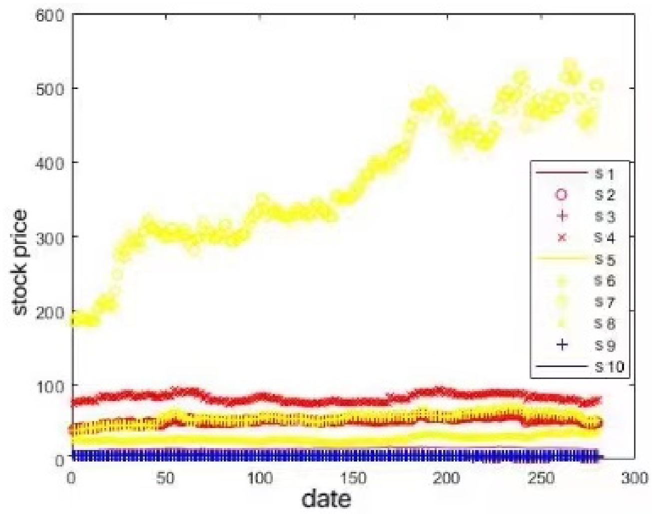

Figure 2 Volatility chart of stock values

Non-linear least squares modeling principle(formula ④):

$$Q = \sum_{k=1}^{N} [y_k = f(X_k, \theta)]^2 \quad ④$$

The nonlinear least squares method is a parameter estimation method that estimates the parameters of the nonlinear static model based on the minimum sum of squares of the error. Where y is the output of the system, x is the input, and θ is the parameter.

Indicators of good or bad fit(formula ⑤):

$$R - square = \frac{SSR}{SST} = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST} \quad ⑤$$

When the fitted function is a linear function to the parameters, the goodness of fit R-square is used to evaluate the quality of the fit. The normal value range of the value of the goodness of fit by the formula is [0,1]. The closer to 1, the stronger the explanatory power of the variance variable for y, and the better the model fits the data.

The known data is fitted with matlab's cftool fit toolbox. After multiple fitting results, the 8 polynomials of the Fourier approximation is finally selected. The fit is as follows:

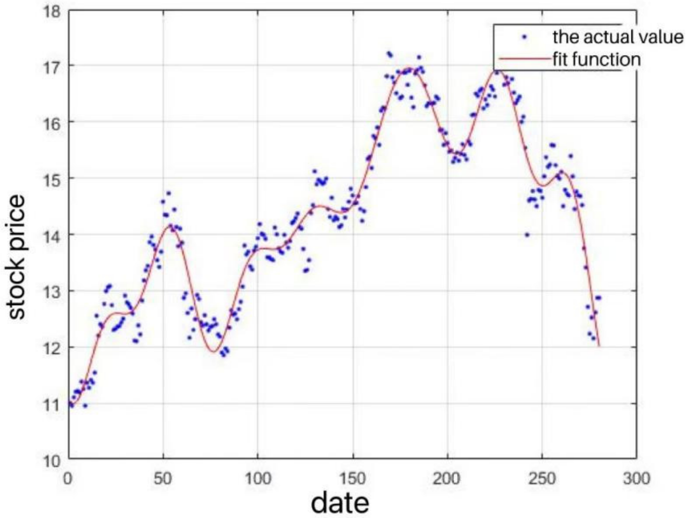

Figure 3 Fits the figure

From the Figure 3, we can see that the curve of Fourier approximation (horizontal coordinates are the number of days given data, the ordinate is the stock value) can roughly fit the actual value, and R-square is 0.9533, close to 1, indicating that the fitting effect is better.

3.3.2 Volatility in stock indices over the next year

The model is predicted as follows:

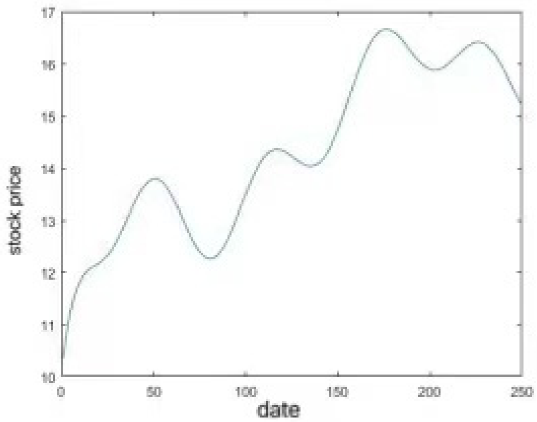

Figure 4 Forecast Map

As can be seen from Figure 4, the stock price will generally fluctuate over time over time.

4 Conclusions

This article selected the performance coefficient of ten securities stocks, combined with the portfolio theory and model solution results to get a reasonable portfolio solution was: P7, P8, P9, stock selection scheme was: concentrated in abc006, abc007, abc008 three stocks. Further analysis of the performance indicators of the portfolio, it was concluded that the portfolio on the effective frontier could achieve the minimum risk standard deviation when the expected rate of return was equal, and when the yield standard deviation was certain, the risk minimization could be achieved.

Through modeling, give reasonable investment advice and strategies: diversified portfolio, through investment of 3 to 5 stocks to spread non-systemic risk.

Disadvantages: The Markowitz mean-variance model is more complicated, and there are certain difficulties in practical application. In addition, this article only uses the standard deviation of returns to measure the total risk of stock investment. The nonlinear fitting algorithm can find the best function match of the data by minimizing the square sum of the error. The data can be easily obtained by using the least square method. However, as the number of fittings increases, over-fitting may occur, which in turn affects the goodness of fitting.

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