An Empirical Study on the Vulnerability of Corporate Bond Industry Based on KMV Modified Model

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Abstract: Based on the existing concepts and calculation methods of Corporate Vulnerability (CVI), this paper constructs the measurement structure of the Corporate Bond Industry Vulnerability Index (CBVI). This paper uses the KMV model based on the Bootstrap method to correct the mapping relationship between the default distance and the default probability, and calculates the improved default probability based on the Bootstrap method. We apply the revised KMV model and the industry vulnerability index of corporate bonds to empirically study corporate bonds of 27 listed companies representing four industries: raw materials, industrials, consumer non-daily products, and utilities, then we compare the vulnerability of corporate bonds in these four industries. The conclusion shows that the industry vulnerability of corporate bonds is ranked from high to low as follows: Non-daily consumer goods industry, industrial industry, raw material industry, public utility industry. According to the measured industry tail vulnerability index of corporate bonds, it is found that there are several corporate bonds with high default probability in the utility industry.

Keywords: Corporate bonds, Vulnerability index, Bootstrap method, KMV model

1 Introduction

The sign of the gradual improvement of corporate bond issuance is that in 2007, the China Securities Regulatory Commission announced the "Pilot Measures for Corporate Bond Issuance". Since then, the most basic part of corporate bonds, that is, the initial issuance scale and quantity have continued to grow. The most concerned credit risk problem in corporate bond risk research is more and more worthy of in-depth analysis and solution with the passage of time [1]. And since the first substantial default event of my country's bonds occurred in 2014, the occurrence of default events in the bond market can no longer be "news", and the default trend has gradually increased. Especially in 2018, bond defaults reached a certain peak, and in 2019, the trend of high growth in 2018 continued. The default rate of private enterprises accounted for a high proportion of various default subjects, and the default risk was high [2].

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Studying the default risk degree of corporate bonds has become an urgent matter. For corporate bonds that are credit products, the expected default probability and the expected loss rate under various circumstances are the two core variables for in-depth research on the credit risk of enterprises. Domestic and foreign scholars have made relevant research on the influencing factors of corporate bond default risk, and foreign scholars have developed relatively mature theoretical research frameworks and models. Cheng-Few Lee uses financial ratio information combined with discriminant analysis and factor analysis to predict bond ratings, he discuss the comparison between Ohlson model and KMV-Merton model to estimate the probability of default, and use the empirical results to compare the results of two different probability estimation models [3]. Brian Barnard proves that the relationship between rating migration and default probability is complex, and the default probability of different rating classes is time-dependent rather than independent of each other. Rating migration is analogous to a delayed default process, affecting the probability of default in subsequent intervals [4]. Matthew Kurbat et al selected a large number of corporate debt data in the United States from 1991 to 2001, and used a more practical method to test the KMV model. The analysis results show that the KMV model is very empirical, and the calculated expected default probability can be applied to practice [5]. After careful study of various foreign and mature measurement methods, Chinese scholars have gradually opened up a theoretical path that is most suitable for the actual situation of the Chinese market. Che Xuehai uses the idea of corporate entities to represent corporate bonds empirically study whether the default distance calculated by the KMV model can accurately reflect the degree of corporate bond credit risk, the empirical conclusion shows that the effect is very good. The part worthy of in-depth discussion in the analysis process is that the author uses the Bootstrap-t test method to correct the KMV model [6]. Wei Qian, Li Liping, and Dong Zhe used the KMV method to study bond credit risk comparison, the final conclusion is: whether it is from the perspective of groups or from the perspective of a single company to analyze the credit risk of corporate bonds, the KMV model can well predict the risk trend [7]. Duan Defeng, Wang Jianhua, Song Hongfang calculated the KMV correction model improved by Bootstrap empirical distribution method, under the circumstance that the extractable part of valid data in the credit market is not sufficient, that is, the data sample set is small, the revised model can accurately assess the default risk by industry classification, it can further realize the comprehensive detection of credit market risks [8]. According to the literature collected and studied, most of the research ideas include that once the enterprise itself has problems, then the various unexpired corporate bonds issued by the company have the possibility of default [9]. According to the empirical results of existing articles, the default distance calculated based on various improved KMV models can be used to analyze the default risk of corporate bonds, it can be used to compare the credit risk of different corporate bonds. This paper also considers the remaining maturity of the debt, and applies the remaining maturity of the bond to the KMV model to obtain a more accurate default distance, which can better reflect the default risk of corporate bonds in my country.

The contribution of this paper is: Based on the Bootstrap method, we revised the mapping relationship between the default distance and the default probability in the KMV model. According to the revised model, the default distance of the corporate bond and the mapped default probability were calculated on the basis of the collected corporate bond data. According to Duan's [10] description of a group of corporate Vulnerability Index (Corporate Vulnerability Index), a group of Corporate Bond Vulnerability Index (Corporate Bond Vulnerability Index) was constructed. According to the industry vulnerability index of corporate bonds considered in the calculation of the calculated default probability, we compare the measured industry vulnerability index of corporate bonds in the third quarter of 2020. Through empirical comparison, we can get the overall vulnerability of corporate bonds in which industries are more vulnerable, and make preliminary preparations for mitigating corporate bond risks in individual industries in the future.

2 Methodology

Based on the existing KMV model modified by the Bootstrap method, this paper calculates the default distance and default probability of corporate bonds of listed companies. Then, according to the calculation formula of enterprise vulnerability proposed by Duan, the vulnerability index of enterprise debt that can reflect the actual situation of a certain industry is derived, we use the calculation method to obtain the quarterly vulnerability index of corporate bonds of listed companies in a certain industry.

2.1 Basic assumptions

Basic assumptions of the Black-Scholes-Merton model

- No transaction costs and short selling restrictions. Bottom;
- Shares bear no dividends;
- The price of the stock follows the Wiener process.

The value of the debts undertaken by the borrower must be less than the value of its assets, in which case the borrower will not default.

The borrower's capital structure consists only of owners' equity, short-term debt, long-term debt, and convertible preferred stock.

The change of the market value of the enterprise should follow the Wiener process, and the borrower's asset return obeys the normal distribution.

2.2 Variables

- Market risk-free interest rate: r
- Equity value: E
- Debt term: T
- Stock price volatility: σ_E
- Default point: DP
- Par value of corporate debt: D
- Enterprise asset value: Va
- Corporate asset volatility: σ_v
- Default distance: DD

- Expected probability of default: EDF

2.3 The calculation process of the KMV model

First, through the value of corporate liabilities, the market value of corporate stocks and the volatility of corporate stock prices, we calculate the enterprise asset value Va and the enterprise asset volatility σ_v. Through the enterprise equity value E and the enterprise stock price return volatility σ_E, according to the Black-Scholes-Merton option pricing formula to indirectly calculate the enterprise asset value Va and the enterprise asset volatility σ_v, we can get:

$$E = \mathrm{V}a * N(d_1) - D * e^{-rt} N(d_2) \tag{1}$$

$$d_1 = \frac{\left(\frac{\mathrm{V}a}{D}\right) + (r + \frac{1}{2}\sigma_v)T}{\sigma_v \sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma_v \sqrt{T} \tag{3}$$

where *r* is the market risk-free interest rate, *T* is the remaining maturity of the corporate debt, *N (d)* is the standard normal cumulative distribution function, and *D* is the face value of the corporate debt. And there is the following relationship between stock price volatility and asset volatility:

$$\sigma_E = \frac{\mathrm{V}a}{E} N(d_1) \sigma_v \tag{4}$$

By combining equations (1) - (4), the enterprise asset value Va and the enterprise asset volatility σ_v can be obtained.

The core of the KMV model is to measure the future value of the enterprise according to the existing data of the present value, and calculate the distance to default (DD) of the enterprise by calculating the debt situation. The distance to default (DD) refers to the relative distance from the current level to the default point (DP) in the value of a company's assets during the risk period. The default distance is a good measure of default risk, and the default distance DD is a standardized indicator, it can be used to compare and obtain the corresponding credit risk status of different companies. The larger the DD value, the smaller the ratio of the company's liabilities and assets, the stronger the ability of the company to repay its debts. It shows that the possibility of corporate default is small. If it is assumed that the value of the company's assets obeys a normal distribution, the calculation formula of the company's default distance can be expressed as:

$$DD = \frac{Va - DP}{Va * \sigma_v} \tag{5}$$

KMV has empirically analyzed various default cases, and concluded that the maximum possible value of default is the value obtained by adding half of the company's short-term debt and long-term debt. Therefore, there are:

$$DP = short - term\ debt + 0.5 * long - term\ debt \quad (6)$$

Finally, it is necessary to analyze the mapping relationship between the default distance *DD* and the expected default probability *EDF* according to the specific situation. Since the distance to default, *DD*, is an estimate of the degree to which the company has reached default. If it is assumed that the actual trend of the company's asset value satisfies a normal distribution, the expected default probability function is obtained by setting:

$$EDF=1-F(DD) \quad (7)$$

The distribution of *F* satisfied needs to be analyzed on a case-by-case basis. Because according to various situations and empirical evidence, the probability of default calculated from the mapping relationship designed from the perspective of normal distribution is not convincing to measure the risk of corporate debt.

So far, combined with the model assumptions, according to the above three steps, the KMV model has been solved to measure and predict the expected default probability of the listed company's credit risk. Because the distribution to which *F* obeys is unknown, further exploration is required.

2.4 The construction principle of Bootstrap method

The Bootstrap method builds the theory: when the distribution of the model is uncertain, simply collect sample data and repeat sampling with the help of programming language functions to obtain Bootstrap samples, and then estimate statistics based on the Bootstrap samples to describe the characteristics of the overall distribution. The Bootstrap method can be roughly divided into the following two methods:

- Nonparametric Bootstrap method: For the unknown *F*, randomly select *n* subsamples from it to obtain the sample $\chi=(X1, X2, X3,...,Xn)$, $R(X,F)$ represents a statistic in the mean, variance, density function, etc. of F. The Bootstrap method estimates $R(\chi,F)$ in the following ways:

First, use the sampled $\chi=(X1, X2, X3,...,Xn)$ to analyze the empirical distribution function, $X1=x1, X2=x2,...Xn=xn$ The probability of extracting each observation is the same.

Then draw a simple subsample $\chi^*=(X1^*, X2^*, X3^*,...,Xn^*)$ as the Bootstrap sample. From the perspective of nonparametric statistics, it is the maximum likelihood estimate of *F*, χ^* obeys, and the possible values are $\{x1, x2, x3,...,xn\}$, and its mean and variance respectively are:

$$E(\chi^*) = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x} \quad (8)$$

$$\mathrm{Var}(\chi^*)=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=S^2 \quad (9)$$

Repeating the above steps *M* times to calculate the *M* values of *R*(*χ, F*).

Finally, the distribution of *R**= *R*(*χ**,) is used to approximate the *R(χ,F)* distribution, and the *R** distribution is called the Bootstrap distribution.

- Parametric Bootstrap method: The parametric Bootstrap method is to find a certain statistical characteristic *R*(*X, F*) when the distribution *F* is known. In this case, the parametric Bootstrap method extracts n data from the known *F* The other steps are the same as the nonparametric Bootstrap method.

2.5 Calculation of corporate debt vulnerability index

According to the concept of corporate vulnerability index proposed by Duan, it can be obtained that corporate vulnerability CVIs is a new set of indicators to measure the economic and financial environment. They can be viewed as stress indicators, reflecting elevated credit risk in the corporate sector.

Value-weighted CVI (CVIvw): The value-weighted CVI is weighted by the market capitalization of each company and then takes into account the size of each company.

$$CVI_{VW}(t)=\sum_{i=1}^{I}w_{it}p_i\,(t,\ 12) \quad (10)$$

where *i* represents the *i-th* company, represents the number of companies included in the enterprise group, and represents the ratio of the assets of enterprise *i* to the total assets of the company group at time *t*. *Pi(t, 12)* represents the expected default probability of firm i within 12 months after time *t*.

Equally-weighted CVI (CVIew): The equal-weighted CVI setting makes the probability of default for each company sum equal weighted.

$$CVI_{ew}=\frac{1}{I}\sum_{i=1}^{I}p_i\,(t,12) \quad (11)$$

Tail CVI ($\mathrm{CVI_{tail}}$): Tail CVI provides a metric to measure a relatively dispersed group of companies, measuring the most vulnerable companies in a group by taking this the top 5% of the companies with the highest probability of default in the group, and the total probability of default of these companies is averaged to obtain the tail CVI.

Based on this concept, this paper sets out to establish a quarterly vulnerability index of corporate bonds that can immediately reflect the environmental characteristics of corporate bonds in a certain industry. Considering the three calculation methods, three specific calculation formulas for measuring corporate debt vulnerability are designed:

Value-weighted CBVI (CBVIvw): The value-weighted CBVI is weighted according to the issuance amount of each bond, and then the size of the issuance of each bond is calculated.

$$CBVI_{VW}(t) = \sum_{i=1}^{I} w_i PD_i\,(t) \tag{12}$$

$$w_i = \frac{g_i}{G_i} \tag{13}$$

gi is the bond issuance amount of corporate bond i, and G_i is the total bond issuance amount of the industry in which the corporate bond is located. represents the number of corporate bonds issued, and $PD_i(t)$ is the default probability of corporate bond *i* at time *t*.

Equally-weighted CBVI (CBVIew): The equal-weight CBVI setting makes the default probability of each corporate bond equal-weighted and summed up.

$$CBVI_{ew}(t) = \frac{1}{I}\sum_{i=1}^{I} PD_i\,(t) \tag{14}$$

Tail CBVI ($\mathrm{CBVI_{tail}}$): Tail CBVI provides a metric to measure the most vulnerable bonds in an industry by taking the top 3 companies with the highest quarterly corporate bond default probability in this industry The tail CBVI is obtained by summing and averaging the default probabilities of these corporate bonds.

3 Empirical analysis

3.1 Determination of model parameters

According to the above sample selection, combined with the actual situation in my country, the parameters in the model can be determined as:

- The one-year treasury bond yield published by ChinaBond Information Network on August 31, 2020 can be regarded as the risk-free interest rate: r= 2.416%.
- Due to the complex situation of my country's securities market, we can add up the value of tradable shares and the value of non-tradable shares when calculating the equity value of a company. Use the closing price of the stock on August 31, 2020 to calculate the value of tradable shares, and use the product of net assets per share and the number of non-tradable shares to obtain the value of non-tradable shares. The value of tradable shares is equal to the product of the number of tradable shares and the price. Share value is equal to the product of net assets per share and the number of non-tradable shares.
- The remaining maturity of the debt is the collected real data of the remaining maturity of the bond on August 31, 2020.
- Stock price return volatility: $\sigma_E = \sqrt{\frac{n}{n-1}\sum_{i=1}^{n}(\mu_i - \bar{\mu}_i)^2}$, where μ_i is the logarithmic rate of return of the stock on the day i, and n is the actual number of trading days.
- The face value of corporate debt *D*, is the face value of the total liability in the collected corporate balance sheet.

- Default point DP:

$$DP = short - term\ debt + 0.5\ long - term\ debt \tag{15}$$

- Default distance and default probability can be obtained from formulas (5) and (7).

3.2 Empirical results

Data sources: The data in this article was obtained from the Choice financial terminal. We collected data from June 1 to August 31, 2020 on 27 corporate bonds of representative listed companies in the four primary industries of raw materials, industrials, non-daily consumer goods and utilities. The data includes daily data of bonds and stocks of the issuer and quarterly financial statements of listed companies, including bond prices, the balance sheet of the issuer of bonds, and information about stocks issued by the issuer. The observation data that the model needs to use are the one-year treasury bond yield, the remaining maturity of corporate debt, the value of current liabilities of the enterprise, the value of non-current liabilities of the enterprise, The total debt value of the company, the total number of shares issued by the company, the number of outstanding shares, the closing price of the stock, and the net asset per share, as shown in table 1.

Table 1 Basic information of corporate bonds.

Name of securities	**Stock code**	**Primary industry**
111069.SZ	002340.SZ	raw materials
111071.SZ	000709.SZ	raw materials
111078.SZ	000778.SZ	raw materials
111076.SZ	000778.SZ	raw materials
1880236.IB	002074.SZ	industry
1880001.IB	002074.SZ	industry
111072.SZ	300495.SZ	industry
078037.IB	00548.HK	industry
139456.SH	600089.SH	industry
1180096.IB	601669.SH	industry
127495.SH	01958.HK	non-daily consumer goods
127429.SH	01958.HK	non-daily consumer goods
111083.SZ	002594.SZ	non-daily consumer goods
111075.SZ	002594.SZ	non-daily consumer goods
124044.SH	03396.HK	non-daily consumer goods
111087.SZ	000598.SZ	Utilities
111081.SZ	000598.SZ	Utilities
127616.SH	600098.SH	Utilities
127347.SH	03768.HK	Utilities
127792.SH	00916.HK	Utilities
127540.SH	00916.HK	Utilities

111084.SZ	000027.SZ	Utilities
111077.SZ	000027.SZ	Utilities
152407.SH	600168.SH	Utilities
078087.IB	01816.HK	Utilities
7103.IB	600900.SH	Utilities
038006.00	600900.SH	Utilities

Empirical results based on the traditional KMV model: According to the selected data and the determined model parameters, using MATLAB software to calculate formulas (1)-(5), we get the calculation results based on traditional KMV modelas shown in table 2:

Table 2. Calculation results based on traditional KMV model.

Securities name	Default distance	default probability
16 Green G1	1.8346	0.03328
18Hegang G1	2.0736	0.01906
19 Emerging G2	2.9391	0.00165
19 Emerging G1	2.9685	0.00150
18 Guoxuan Green Bond 02	1.1222	0.13089
18 Guoxuan Green Bond 01	1.1654	0.12193
18 Meishang 01	1.8836	0.02981
07 Shenzhen high-speed debt	2.8394	0.00226
20 Special change Y1	1.5745	0.05769
11 China Hydropower Debt	2.2411	0.01251
G17 Beijing Auto 1	1.1733	0.12033
G16 Beijing Auto 1	1.4888	0.06826
19 Yadi G1	1.4260	0.07694
18 Yadi G1	1.4832	0.06901
12 Lenovo Debt	0.4210	0.33687
19 Xingrong G2	2.9347	0.00167
19 Xingrong G1	2.9892	0.00140
G17 Development 1	3.7024	0.00011
15Kun Water	3.1998	0.00069
G18 Longyuan 1	1.3802	0.08377
G17 Longyuan 2	1.5346	0.06244
19 Shenneng G2	0.6436	0.25990
19 Shenneng G1	0.7092	0.23911
G20 arms control	0.7198	0.23581
07 Guangdong Nuclear Power Bonds	2.6996	0.00347
02 Three Gorges Debt	5.0159	2.64*10-7
03 Three Gorges Debt	4.5522	2.65*10-7

Empirical results of the KMV model revised based on the Bootstrap method: Since the final calculation process of the KMV model is to use the default distance mapping to obtain the default probability, when the number of samples is not enough to fully explain the problem, the results of the probability of default calculated by us are not accurate, nor can we accurately determine the true distribution that the sample data obeys. This paper uses the Bootstrap sampling method to obtain the empirical distribution of the sample according to the existing literature, and then calculates the default probability according to the distribution column value of the empirical distribution. The generated default probability can be used to compare the results to better explain the problem. The specific method is to first randomly sample 27 times in the default distance set, which ensures the randomness to a great extent. According to this process, 1000 times are continuously extracted, and finally a large Bootstrap sample set needs to be generated. According to the empirical distribution of Bootstrap samples, the probability density of the default distance is calculated. According to the discrete distribution theory, the discrete distribution columns of Bootstrap samples are calculated. Finally, the probability of default is calculated according to formula (5). The result is as follows in table 3:

Table 3 Empirical results of KMV model modified based on Bootstrap method.

Securities name	Probability frequency Distribution	Default probability
16 Green G1	0.03755556	0.6342
18Hegang G1	0.03685185	0.5866
19 Emerging G2	0.03533333	0.4141
19 Emerging G1	0.03792593	0.4082
18 Guoxuan Green Bond 02	0.03925926	0.7763
18 Guoxuan Green Bond 01	0.03588889	0.7677
18 Meishang 01	0.03737037	0.6245
07 Shenzhen high-speed debt	0.03700000	0.4339
20 Special change Y1	0.03711111	0.6861
11 China Hydropower Debt	0.03781481	0.5532
G17 Beijing Auto 1	0.03940741	0.7661
G16 Beijing Auto 1	0.03640741	0.7032
19 Yadi G1	0.03570370	0.7157
18 Yadi G1	0.03744444	0.7043
12 Lenovo Debt	0.03503704	0.9161
19 Xingrong G2	0.03940741	0.4149
19 Xingrong G1	0.03714815	0.4041
G17 Development 1	0.03744444	0.2619
15Kun Water	0.03970370	0.3621
G18 Longyuan 1	0.03600000	0.7248
G17 Longyuan 2	0.03588889	0.6941
19 Shenneng G2	0.03655556	0.8717
19 Shenneng G1	0.03418519	0.8586
G20 arms control	0.03700000	0.8565

07 Guangdong Nuclear Power Bonds	0.03740741	0.4618
02 Three Gorges Debt	0.03603704	0.0000
03 Three Gorges Debt	0.03711111	0.924

Calculation and comparison of corporate debt vulnerability indices in various industries: Using formulas (10), (11) and the measurement method of the corporate bond tail vulnerability index, the overall bond vulnerability of the industry in which the bond is located can be obtained according to the measured default probability. When we apply a certain corporate bond weight, we calculate the weight as the percentage of the issuance of such bonds to the total issuance of corporate bonds by listed companies in the primary industry in which the bond is located. Therefore, we can obtain the corporate debt vulnerability index of each industry corresponding to the following four first-level industries in table 4-6 and figure 1-6:

Table 4 Industry Weighted Vulnerability Index of Corporate Bonds.

Primary industry	**(Normal Distribution Based) Industry-Weighted Vulnerability Index of Corporate Bonds**	**(Based on Bootstrap empirical distribution) Industry-weighted vulnerability index of corporate bonds**
raw materials	0.01035115	0.48436367
industry	0.04958781	0.61185913
non-daily consumer goods	0.15149367	0.77576388
Utilities	0.05485136	0.41684886

Note: This table is derived from the calculation results of the MATLAB programming language

Table 5 Industry Equal-Weighted Vulnerability Index of Corporate Bonds.

Primary industry	**(Normal Distribution Based) Industry Equal-Weighted Vulnerability Index of Corporate Bonds**	**(Based on Bootstrap empirical distribution) Industry Equal-Weighted Vulnerability Index of Corporate Bonds**
raw materials	0.01387	0.510856
industry	0.05918	0.640273
non-daily consumer goods	0.13428	0.761065
Utilities	0.07403	0.500234

Note: This table is derived from the calculation results of the MATLAB programming language

Table 6 Industry Tail Vulnerability Index of Corporate Bonds.

Primary industry	**(Normal Distribution Based) Industry Tail Vulnerability Index of Corporate Bonds**	**(Based on Bootstrap empirical distribution) Industry Tail Vulnerability Index of Corporate Bonds**
raw materials	0.017995292	0.544962
industry	0.103504255	0.743346
non-daily consumer goods	0.178048356	0.799285
Utilities	0.244942012	0.862261

Note: This table is derived from the calculation results of the MATLAB programming language

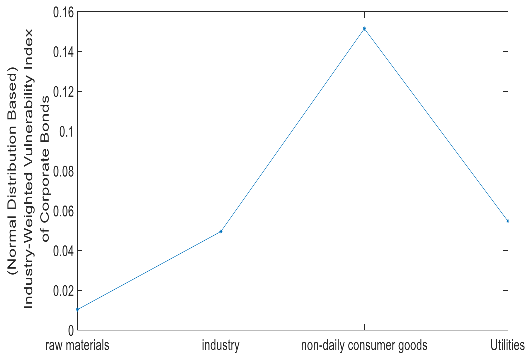

figure 1 (Normal Distribution Based) Industry-Weighted Vulnerability Index Line Chart for Corporate Bonds.

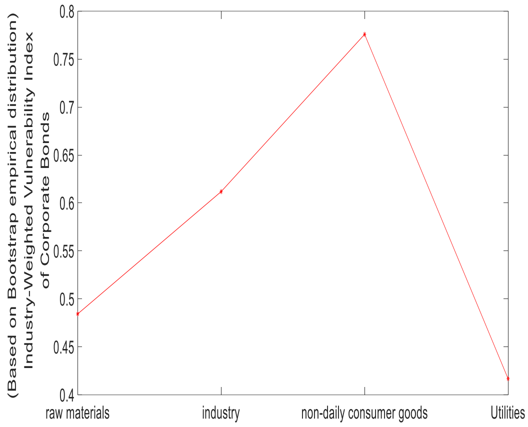

figure 2 (Based on Bootstrap empirical distribution) Industry-Weighted Vulnerability Index Line Chart for Corporate Bonds.

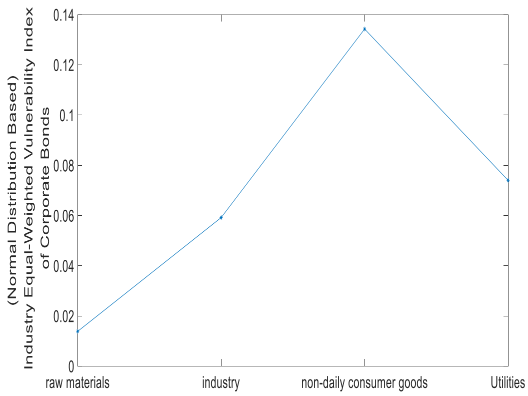

figure 3 (Normal Distribution Based) Industry Equal-Weighted Vulnerability Index Line Chart for Corporate Bonds.

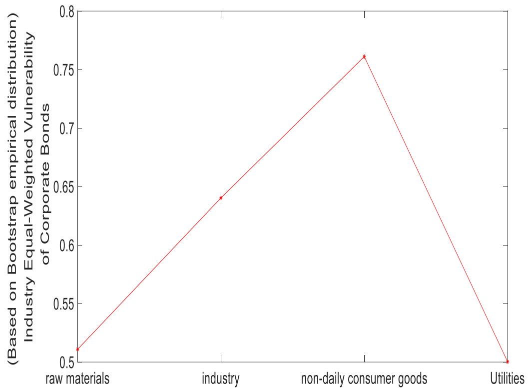

figure 4 (Based on Bootstrap empirical distribution) Industry Equal-Weighted Vulnerability Index Line Chart for Corporate Bonds.

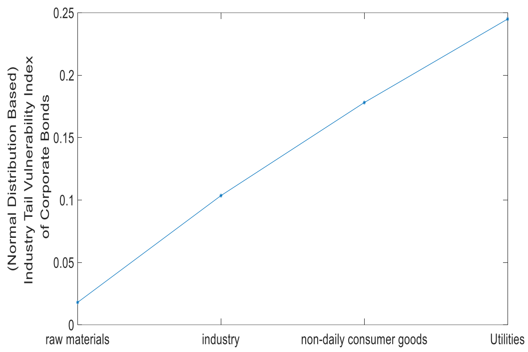

figure 5 (Normal Distribution Based) Industry Tail Vulnerability Index of Corporate Bonds Line Chart for Corporate Bonds.

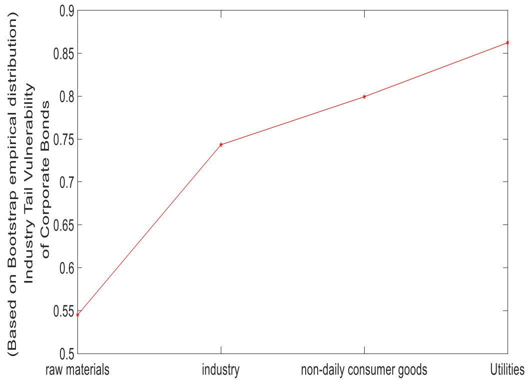

Figure 6 (Based on Bootstrap empirical distribution) Line chart of the industry tail vulnerability index of corporate bonds.

Note: Figures 1, 2, 3, 4, 5 and 6 are all derived from the calculation results of the MATLAB programming language

We can analyze from the above table and the line chart: from the analysis in Figure 1, 2, 3 and Figure 4, whether the mapping relationship between default distance and default probability obeys the normal distribution or the Bootstrap empirical distribution. Among these four industries, the corporate debt-weighted vulnerability index and the corporate debt equal-weighted vulnerability index of the non-daily consumer goods industry are relatively higher than those of the other three industries. The difference is that if the mapping between default distance and default probability satisfies a normal distribution, the corporate debt-weighted vulnera-bility index and the corporate debt-weighted vulnerability index of the raw materials industry are the lowest compared with the other three industries. Compared with the public utility industry, the corporate debt-weighted vulnerability index and the corporate debt-weighted vulnerability index are smaller in the industrial sector. The order from high to low of the corporate debt-weighted vulnerability index and corporate debt-weighted vulnerability index of the four industries is: non-daily consumer goods industry, public utility industry, industrial industry, and raw material industry. If the vulnerability index derived from the Bootstrap empirical distribution is used, the corporate debt-weighted vulnerability index and the corporate debt equal-weighted vulnerability index of utilities are the lowest compared with the other three industries. Compared with the industrial industry, the corporate debt-weighted vulnerability index and the corporate debt-weighted vulnerability index of the raw material industry are smaller, the corporate debt-weighted vulnerability index and corporate debt-weighted vulnerability index of the four industries are in descending order: non-daily consumer goods, industry, raw materials, and public utilities. Compared with reality, public utilities are a kind of basic service industry, which refers to the general term of various undertakings that meet the basic requirements shared by social enterprises and residents, and serve the daily life of urban residents, urban circulation and infrastructure. As a basic service, its stocks and bonds as a whole are less likely to default. Compared with the industrial industry and the raw material industry, the overall vulnerability of companies in the industry is actually lower. Therefore, from a practical point of view, the mapping between the default distance and the default probability satisfies the Bootstrap empirical distribution, which can better reflect the default of corporate bonds.

According to Figure 5 and Figure 6, no matter what distribution the mapping between default distance and default probability satisfies, utilities have extreme default. The tail of corporate bonds of public utilities is very vulnerable, indicating that the extreme probability of default of corporate bonds in this industry is very high, and there are one or several corporate bonds with relatively high default probability. The extreme situation of this industry should be more worthy of attention. The order of corporate debt vulnerability in extreme situations of the other three industries is from high to low: non-daily consumer goods, industry, and raw materials.

4 Conclusions

Based on the KMV model modified by the Bootstrap method and a set of corporate vulnerability indices proposed by Duan, this paper constructs a series of corporate debt vulnerability index measurement methods in a certain industry. We construct this index to predict the overall default of corporate bonds in various industries, and more importantly, to compare the overall vulnerability of corporate bonds in various industries. The comparative analysis shows which industries corporate bonds are more prone to default, which can be prevented in advance. This paper compares the the weighted vulnerability index of corporate bonds, the equal-weighted vulnerability index of corporate bonds and the tail vulnerability index of corporate bonds for these four industries(non-daily consumer goods industries, Industrial industry, raw material industry, utility industry) with the mapping of default distance to default probability based on normal distribution and the default distance to default probability mapping based on Bootstrap empirical distribution. After the comparison, it is concluded that the order of the corporate bond industry vulnerability index from high to low is: corporate debt vulnerability in non-daily consumer goods industry, corporate debt vulnerability in industrial industry, corporate debt vulnerability in raw material industry, and corporate debt vulnerability in public utility industry. We also found that several corporate bonds in the utility sector have relatively high default probabilities. It has paved the way for the subsequent research on the measurement method of accurate industry or individual corporate debt vulnerability index, it also makes an empirical premise for mitigating the tail risk of corporate bonds. The method used in this paper still needs to be improved, and the default probability of corporate bonds measured by the improved KMV model still has limitations and needs further development.

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