Optimization of Leased Equipment Maintenance Strategy Considering Maintenance Time Limit

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Abstract: Aiming at the maintenance problem of leased equipment with limited maintenance time, two maintenance strategy models of full refund and proportional refund are proposed. From the perspective of the lessor, the model comprehensively considers factors such as minor repair costs, failure penalty costs, repair time limits and refunds during the rental period, and establishes a full refund with the goal of minimizing the total cost rate expected by the lessor maintenance and proportional refund maintenance are two strategic models, and the relevant theoretical methods for model optimization are given. Numerical analysis verifies the validity and feasibility of the model. The strategy model can provide theoretical support for the lessor to make equipment maintenance decisions.

Keywords: expected cost rate; leased equipment; repair time threshold; minimal repair

1 Introduction

As technology advances and equipment becomes more complex, equipment maintenance becomes more and more specialized. In addition, due to the increasingly competitive market, companies are focusing more on their core competitive areas. Therefore, more and more enterprises and individuals prefer the form of equipment leasing or service outsourcing [1-2] to engage in production and operation activities, and completely transfer the daily maintenance of equipment to the leasing company [3]. As a result, the maintenance of leased equipment is increasingly receiving widespread attention from leasing companies and researchers.

In general, there are two main types of maintenance activities for leased equipment: corrective maintenance and preventive maintenance [4-6]. Corrective maintenance is the most commonly used form of equipment maintenance, which involves repairing equipment to operating condition after a failure has occurred. In practice, minimal repairs [7] are the most common method of corrective maintenance, which refers to repairing equipment to operating condition when it breaks down, but the failure rate of the equipment remains unchanged, also known as repairing as that just before failure. Preventive maintenance refers to upgrading the operating condition of leased equipment and reducing the likelihood of equipment failure.

Many scholars have done research on the maintenance optimization of leased equipment, which is currently focused on two aspects: (1) maintenance of new equipment. The literature

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[8-9] studied the preventive maintenance of leased equipment, in which the lessor would carry out minimal repairs to the equipment if it broke down during the lease period, by optimizing the preventive maintenance parameters, thus enabling the lessor to obtain a maintenance strategy with the minimum expected total cost. Pongpech and Murthy studied the case of fixed preventive maintenance intervals during the lease period L, in which the equipment preventive maintenance and a maintenance strategy of minimal repairs if the equipment breaks down during the preventive maintenance cycle [10]; Some authors used a combination of equal and variable cycles for the maintenance of leased equipment the literature [11-12]. (2) maintenance of used equipment. chattopadhyay and murthy studied a model of warranty cost for used equipment under free replacement or proportional warranty [13]; pongpech et al. studied a model of lease maintenance strategy considering equipment upgrade, but the unequal maintenance intervals in this model are not easy to operate in practical application [14]. Yeh et al. proposed two strategy models on the maintenance of used equipment [15]; Shafiee et al. studied an upgrade strategy model for used products with warranties [16]; Wang et al. studied a two-dimensional warranty problem for used products [17]; Park et al. proposed a model for a repair strategy that upgrades used products prior to sale and considers that when the repair time exceeds a given threshold, the dealer will issue a full refund to the consumer [18].

Little research has been done on the maintenance of leased equipment, and there is a lack of research on maintenance strategies that consider full or proportional refunds when the maintenance time exceeds a given threshold. The difference between this model and other existing studies is that it proposes two strategy models, full refund and proportional refund, to provide more options for the lessor to repair the equipment; it is solved using Newton's iterative algorithm and the optimal value of the lessor's expected total cost rate is obtained ECR(w*) and the corresponding optimal lease term w*; the case study shows that the pro-rata refund strategy is better than the full refund strategy.

The paper is structured as follows: Part I is an introduction; Part II is a description of the problem and the strategy; Part III is a mathematical model; Part IV gives the relevant theory and methods for model optimization; Part V is an analysis of the arithmetic cases and a study of the sensitivity of the parameters; and finally, a conclusion and outlook are given.

2 Description of the problem and strategy

The lessor leases the equipment to the lessee for use and charges the lessee a certain amount of rent. The parties sign a lease contract, which stipulates that: (1) if the equipment breaks down during the lease period, the lessor shall undertake minimal repairs to the equipment and the cost of minimal repairs shall be borne by the lessor. (2) If the equipment breaks down and affects the production and operation activities of the lessee, resulting in downtime losses, the lessee requires the lessor to pay the penalty costs for the breakdown. (3) When the minimal repair of the equipment takes longer than the pre-agreed repair time limit, the minimal repair is terminated and the lessee will receive a full refund or a pro-rata refund and the lease term is terminated. If the minimal repair time for a breakdown of the equipment does not exceed the pre-agreed repair time limit, the minimal repair will continue to be used until the end of the lease term.

As the costs of purchase, minimal repairs and refunds of the equipment are borne by the lessor.

For the lessor, it is a central concern to determine the optimal lease term w such that the equipment leased has the lowest desired total cost rate over the entire lease term. Therefore, this paper intends to take the lessor's perspective into account the purchase, minimal repair and refund of the equipment during the lease period, establish two strategic models, namely full refund repair and proportional refund repair, take the length of the lease period w of the equipment as the decision variable, and take the minimum value of the expected total cost rate ECR(τ) over the entire lease period as the optimization objective to optimally obtain the optimal expected total cost rate ECR(w*) and the corresponding value of the optimal lease period w*, so as to provide a decision basis for the lessor to lease the equipment.

3 Basic models

Symbolic notation:

$f(\cdot)$, $F(\cdot)$, $v_0(\cdot)$	Life density distribution, cumulative distribution and Failure rate functions for leased equipment
T, Y	Failure time, failure repair time
w	Length of equipment leased period
H (a, b)	Expected number of failures on the interval [a, b]
r_0	Maintenance time threshold
c_m, c_d	Minimal repair cost for equipment failures, failure penalty cost
N_{r_0}	Number of minimal repair failures on the interval [a, b] when the repair time threshold is r_0
t_f	The moment of the refund occurs during the lease term
g(y), $G(y)$	Density distribution of maintenance time, cumulative distribution
ECR	Expected cost rate
β, ϑ	Shape parameter, scale parameter of power-law distribution
κ, λ	Shape parameter, scale parameter of Weibull distribution
P_0	Purchase cost of equipment

Assuming that the expected total length of time for a refund to occur is $e(w)$, then it can be expressed as

$$e(w) = \mathrm{E}[T|0 \le T \le w, Y > r_0] = \frac{\int_0^w \overline{F}(t)^{\overline{G}(r_0)} \mathrm{d}t}{1 - [\overline{F}(w)]^{\overline{G}(r_0)}} \tag{1}$$

If the design life of the equipment is L years, the lessor's purchase cost is P_0. For calculation convenience, assume that the lessee's equipment lease cost is $\frac{w}{L}P_0$. The full refund (FR), proportional refund (PR) cost C(w) of the lessor during the lease period [0, w] is

$$C(w)=\begin{cases}\frac{w}{L}P_0, & FR,\\ \frac{w-e(w)}{L}P_0, & PR,\end{cases} \tag{2}$$

If the lease term of the equipment is w, the repair time threshold is r_0, and the minimal repair cost is c_m. On the lease time interval [0, w], the repair time of the products are all less than the given time threshold r_0, then the number of failures of the leased equipment is $N_{r_0}(0,w)$. If the minimal repair time of the equipment on the lease time interval [0, w] exceeds a given time threshold r_0 and occurs at the moment t_f, the number of failures of the leased equipment on the time interval [0, t_f] is $N_{r_0}(0,t_f)$. Let $\mathrm{H}(a,b)=\int_a^b v(u)du$ be the expected number of failures of the failure rate function $v(u)$ over the interval $[a,b]$. Since all faults occurring on the interval $[a,b]$ are repaired by minimal repairs prior to refund and none of the repair time exceed the repair time threshold r_0, then

$$\begin{aligned}E[N_{r_0}(a,b)] &= \sum\nolimits_{k=1}^{\infty} k\cdot P(N_{r_0}(a,b)=k)\\ &= \sum\nolimits_{k=1}^{\infty} k\cdot\frac{H(a,b)^k e^{-H(a,b)}}{k!}[G(r_0)]^k\\ &= H(a,b)\cdot G(r_0)\cdot[e^{-H(a,b)}]^{\overline{G}(r_0)}\end{aligned} \tag{3}$$

Using equation (3), it follows that

$$E[N_{r_0}(0,w)]=\int_0^w v(u)du\cdot G(r_0)\cdot[e^{-H(0,w)}]^{\overline{G}(r_0)} \tag{4}$$

Similarly, we obtain

$$E[N_{r_0}(0,t_f)]=\int_0^{e(w)} v(u)du\cdot G(r_0)\cdot[e^{-H(0,e(w))}]^{\overline{G}(r_0)} \tag{5}$$

It is known that the lessee will penalize the lessor when the equipment breaks down for minimal repair at a penalty cost of c_d. From equations (1), (2), (4) and (5), the lessor's expected total cost rate is

$$ECR(w)=\begin{cases}\dfrac{P_0+((c_m+c_d)\cdot E[N_{r_0}(0,t_f]+\frac{w}{L}P_0)\cdot(1-\overline{F}(w)^{\overline{G}(r_0)})+(c_m+c_d)\cdot E[N_{r_0}(0,w]\cdot\overline{F}(w)^{\overline{G}(r_0)}}{\int_0^w\overline{F}(w)^{\overline{G}(r_0)}dt}, & FR\\ \dfrac{P_0+((c_m+c_d)\cdot E[N_{r_0}(0,t_f]+\frac{w-e(w)}{L}P_0)\cdot(1-\overline{F}(w)^{\overline{G}(r_0)})+(c_m+c_d)\cdot E[N_{r_0}(0,w]\cdot\overline{F}(w)^{\overline{G}(r_0)}}{\int_0^w\overline{F}(w)^{\overline{G}(r_0)}dt}, & PR\end{cases}\tag{6}$$

4 Model Optimization

Equation (6) above is an expression for the lessor's expected total cost rate, which depends not only on the failure rate function of the product, but also on the length of the product's lease term w. Deriving equation (6) with respect to the variable w, it follows that

$$\frac{d}{dw}ECR(w)=0\tag{7}$$

As the expected total cost rate function ECR(w) is non-linear, an analytical solution with respect to the variable w cannot be obtained. Newton's iterative method is widely used as a computational tool to find the optimal solution of the function. Therefore Newton's iterative method will be applied to find the approximate solution w^* of the expected total cost rate function ECR(w), so as to further derive the optimal value of the expected total cost rate ECR(w^*).

5 Example Analysis

Assuming that a lessor acquires a particular model of car for rental, it is of interest to the lessor to determine the optimal lease period w such that the equipment being leased has the lowest expected total cost rate over the entire lease period. Assume that the failure rate function of the product is

$$v(t)=\frac{\beta}{\theta}(\frac{t}{\theta})^{\beta-1},\beta>1,\theta>0\ .\tag{8}$$

The repair time of the product follows a two-parameter Weibull distribution:

$$g(y)=\frac{\kappa}{\lambda}(\frac{y}{\lambda})^{\kappa-1}e^{-(\frac{y}{\lambda})^{\kappa}},y\geq 0,\kappa,\lambda>0\ .\tag{9}$$

Assuming the values of the parameters are P_0=20, L=10, κ=2, λ=0.05, r_0=1/12, c_m=2 and c_d=2. When θ=2, β=2, Using Newton's iterative method, we can obtain that in the full refund scenario w^*=4.9868, $ECR(w^*)$=8.3765. That is the length of the optimal lease term w^* is 4.9868 and its corresponding optimal value of the lessor's expected total cost rate ECR(w^*) is 8.3765. Similarly, it is possible to obtain in the case of a pro-rata refund w^*=4.9868, $ECR(w^*)$=6.3765. That is, the length of the optimal lease term w* is 4.9868 and its corresponding optimal value of the dealer's expected total cost rate ECR(w*) is 6.3765. A comparison shows that the optimal lease period is the same under both strategies, with all other parameters held constant, but the expected total cost rate is lower under the pro-rata refund strategy than under the full refund strategy, as shown in Figure 1.

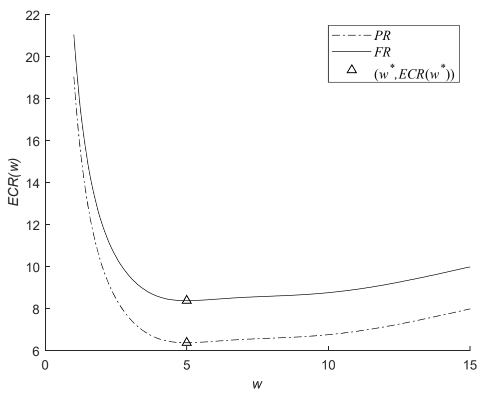

Fig. 1 Expected total cost rate ECR(w) curves under both strategies when the parameters are taken to the same value

5.1 Variation in the lessor's expected optimal value under the two strategies

Table 1and 2 show the corresponding values of the optimal lease term w^* and the optimal expected total cost rate ECR(w^*) when the other parameters are held constant and the parameters β, c_d, c_m and P_0 are taken to different values respectively. From Table 1, it can be concluded that as the values of parameters c_d, P_0 and c_m increase, the value of the optimal expected total cost rate ECR(w^*) increases. However, the value of the optimal lease period w^* decreases as the values of the parameters c_d and c_m increase, and the value of the optimal lease period w^* increases as the value of the parameter P_0 increases. From Table 1, we can also conclude that the value of the optimal expected total cost rate ECR(w^*) increases with increasing values of the parameter β when the values of the parameter β are taken as 2 and 3 respectively. However, the value of the optimal lease term w^* decreases as the value of the parameter β increases. Similarly, a similar conclusion can be drawn from Table 2.

Table 1 Optimal values taken under the FR strategy when the parameters β, c_d, P_0 and c_m are taken to different values respectively

c_d	P_0	β=2						β=3					
		c_m=3		c_m=5		c_m=7		c_m=3		c_m=5		c_m=7	
		w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR
1.0	15	4.5312	7.0349	4.0150	8.4269	3.6846	9.7270	4.0359	8.0878	4.0000	9.7013	2.2070	11.0064
	20	4.9868	8.3765	4.3699	9.8593	4.0150	11.2359	4.0645	9.7053	4.0250	11.3221	4.0000	12.9351
	25	5.4133	9.6705	4.6865	11.2332	4.2860	12.6768	4.0888	11.3207	4.0463	12.9409	4.0193	14.5562
2.0	15	4.2348	7.7456	3.8375	9.0860	3.5495	10.3522	4.0157	8.8951	3.9879	10.5069	2.0897	11.4782
	20	4.6249	9.1351	4.1745	10.5578	3.8791	11.8975	4.0422	10.5144	4.0111	12.1289	3.9904	13.7407
	25	4.9868	10.4707	4.4629	11.9664	4.1401	13.3686	4.0645	12.1316	4.0315	13.7489	4.0090	15.3627
4.0	15	3.8375	9.0860	3.5459	10.3522	3.2861	10.5603	3.9879	10.5069	2.0897	11.4782	1.9173	12.3179
	20	4.1745	10.5578	3.8791	11.8975	3.6487	13.1796	4.0111	12.1289	3.9904	13.7407	2.1750	14.8377
	25	4.4629	11.9664	4.1400	13.3686	3.9046	14.7079	4.0315	13.7489	4.0090	15.3627	3.9925	16.9745

Table 2 Optimal values taken under the PR strategy when the parameters β, c_d, P_0 and c_m are taken to different values respectively

c_d	P_0	β=2						β=3					
		c_m=3		c_m=5		c_m=7		c_m=3		c_m=5		c_m=7	
		w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR	w^*	ECR
1.0	15	4.5312	5.5349	4.0150	6.9269	3.6846	8.2270	4.0359	6.5878	4.0000	8.2013	2.2070	9.5063
	20	4.9868	6.3765	4.3699	7.8593	4.0150	9.2359	4.0645	7.7053	4.0250	9.3221	4.0000	10.9351
	25	5.4133	7.1705	4.6865	8.7331	4.2860	10.1768	4.0888	8.8207	4.0463	10.4409	4.0193	12.0562
2.0	15	4.2348	6.2456	3.8375	7.5860	3.5495	8.8522	4.0157	7.3951	3.9879	9.0069	2.0897	9.9782
	20	4.6249	7.1351	4.1745	8.5578	3.8791	9.8975	4.0422	8.5144	4.0111	10.1289	3.9904	11.7407
	25	4.9868	7.9707	4.4629	9.4664	4.1401	10.8686	4.0645	9.6317	4.0315	11.2489	4.0090	12.8627
4.0	15	3.8375	7.5860	3.5459	8.8522	3.2861	10.0603	3.9879	9.0069	2.0897	9.9782	1.9173	10.8179
	20	4.1745	8.5578	3.8791	9.8975	3.6487	11.1796	4.0111	10.1289	3.9904	11.7407	2.1750	12.8377
	25	4.4629	9.4664	4.1400	10.8686	3.9046	12.2079	4.0315	11.2489	4.0090	12.8627	3.9925	14.4745

6 Conclusions

In this paper, we propose two strategy models, a full refund repair strategy and a proportional refund repair strategy, from the lessor's perspective, to address the maintenance problem of leased equipment. The results of the numerical study show that the optimal expected total cost rate under the proportional refund (PR) strategy is lower than that under the full refund (FR) strategy compared to the two refund strategies. The findings of this paper can provide a basis for lessors to make decisions on the optimal product maintenance plan. How to design the optimal two-dimensional maintenance plan based on the two strategy models is a direction for further research.

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