The Effect of Public Subsidies on Innovation Investment Decision of Enterprises Based on Mean Variance Model Analysis

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Abstract—Based on the quadratic utility function and mean variance assumption, an innovation project investment decision model is established. In the model, innovative enterprise can choose to invest in innovative or non-innovative project. Enterprise manager is decision-maker, he determines the optimal level of effort and allocates his funds to innovative or non- innovative projects with the goal of maximizing the expected utility of investors, which is equivalent to maximizing the sharp ratio of the enterprise. It can be proved by the model that the amount of investment in innovative projects increases and the scope of investment can expand because of public subsidies. The investor will allocate more capital to innovative enterprises in that public subsidies increase the rate of return of innovative enterprises.

Keywords- innovation investment; public subsidy; investment decision model

1 Introduction

Technology-based small firms are a key source of innovation, job creation and productivity growth [1, 2]. On account of the public nature and externality of innovation activities, the best level of innovation activities can not be achieved by market mechanism only. This is also

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considered as a theoretical evidence for the government to provide policy support for enterprise innovation activities [3, 4, 5]. Scholars have failed to agree on the effect of public subsidies. Some scholars believe that public subsidies can stimulate enterprise innovation and promote enterprise investment in innovation. Public subsidies can improve the return on investment in innovative projects, reduce risks, and also convey good information to other investors to help companies attract more investment [6, 7, 8]. However, some scholars believe that public subsidies have crowding out effect on enterprises' innovation investment and reduce their investment in innovation activities [9, 10, 11]. Based on the mean variance hypothesis and the portfolio theory, the effect of ex ante public subsidies to enterprise innovation project on enterprise investment decision is studied in this paper.

The paper is organized in five sections. Section 1 defines the assumption. Section 2 provides a decision model for investors and enterprise managers. Section 3 provides a decision model of effort level for enterprise managers. Section 4 studies the effect of public subsidies on the assets allocation decision and effort level decision. Section 5 summarizes the conclusion.

2 Model assumption

The model assumes that there are two subjects, investors and innovative enterprises. Investors allocate their capital between risk-free assets and innovative enterprises. Risk-free interest rates (r_f) are constant throughout the investment period. The borrowing and lending of risk-free assets are unrestricted with the same interest rate for all the investors. Innovative enterprises can choose to invest in innovative or non innovative projects. The return of innovative projects is $E(r_v)$ and the standard deviation of return is σ_v. The return of non innovative projects is $E(r_E)$ and the standard deviation of returns is σ_E, $E(r_v) < E(r_E)$, $\sigma_v < \sigma_E$. The return of innovative or non innovative projects is not related.

Assume there are two time points (t=0,1). In the time of t=0, investors determine the proportion of investment to innovative enterprises according to their wealth and risk preference. In the time of t=1, all the projects are liquidated and the cash is distributed to investors.

In the time of t=0, investors determine the optimum ratio of risk-free assets and risk assets (investment to innovative enterprises) to maximize its utilities. Assume the utility function is (1):

$$U = W_1 - \frac{1}{2} A \cdot \sigma_{W_1}^2 \tag{1}$$

Here W_1 denotes the wealth in t=1, σ_W is standard deviation of W_1. Investor is risk aversion with coefficient A. In innovative enterprise, enterprise manager is the decision-maker. He determines the optimal level of effort and allocation of funds on innovative or non innovative projects with the target of maximizing the expected utility of investors.

3 The allocation decision of enterprise manager and investor

The wealth of investors is W_0 in t=0, the weight of investment to risk-free assets is w_f, the weight of investment to innovative enterprises is w_P, w_f + w_P=1. Defining the weight of innovative projects is w_v and weight of non innovative projects is w_e in the innovative enterprises, expected return of the innovative enterprises is (2):

$$E(r_P) = w_V \cdot E(r_V) + w_E \cdot E(r_E) \tag{2}$$

The variance of returns is (3):

$$\sigma_P^2 = w_V^2 \cdot \sigma_V^2 + w_E^2 \cdot \sigma_E^2 \tag{3}$$

To simplify the writing, defining (4)-(6):

$$R_P = E(r_P) - r_f \tag{4}$$

$$R_V = E(r_V) - r_f \tag{5}$$

$$R_E = E(r_E) - r_f \tag{6}$$

The decision problem of enterprise managers is to determine $x_1$and x_2 to maximize the Sharpe ratio of portfolio in innovative enterprises. The optimal weight of innovative projects and non innovative projects is (7)-(8):

$$w_V^* = \frac{R_V \cdot \sigma_E^2}{R_V \cdot \sigma_E^2 + R_E \cdot \sigma_V^2} \tag{7}$$

$$w_E^* = \frac{R_E \cdot \sigma_V^2}{R_V \cdot \sigma_E^2 + R_E \cdot \sigma_V^2} \tag{8}$$

The sharp ratio of portfolio in innovative enterprise is (9):

$$S_P = \frac{\left(R_V^2 \cdot \sigma_E^2 + R_E^2 \cdot \sigma_V^2\right)^{\frac{1}{2}}}{\sigma_E \sigma_V} = \left[\left(\frac{R_V}{\sigma_V}\right)^2 + \left(\frac{R_E}{\sigma_E}\right)^2\right]^{\frac{1}{2}} \tag{9}$$

The optimal weight of risk assets in portfolio of investor is (10):

$$w_P^* = \frac{1}{A \cdot W_0} \cdot \frac{R_V \cdot \sigma_E^2 + R_E \cdot \sigma_V^2}{\sigma_V^2 \cdot \sigma_E^2} \tag{10}$$

4 The decision of effort level for enterprise manager

Assuming the effort level of enterprise manager has some impacts on the return and risk of innovative projects, but has no effects on the return and risk of non innovative projects. Symbol $f_r(\eta)$ denotes the part influenced by the effort of enterprise manager, η denotes the level of effort, $\eta > 0$. The effect of effort level on the return of innovative projects is not single-reduction. There is a point $\eta = \eta_0$, $f_r'(\eta_0) = 0$. When $\eta < \eta_0$, $f_r'(\eta) > 0$, the effect of effort level is positive. When $\eta > \eta_0$, $f_r'(\eta) < 0$, the effect of effort level is negative. With the increasing of effort level, the effect on return is marginal diminishing and the effect on costs is marginal increasing, $f_r''(\eta) < 0$. Assume $f_r(\eta)$ is second-order continuous differentiable concave function, when $\eta \to \infty$, $f_r(\eta) = -\infty$.

Symbol $f_\sigma(\eta)$ denotes the part influenced by the effort of enterprise manager. The risk of innovative projects will decrease if enterprise manager increases the level of effort, $f_\sigma'(\eta) < 0$, but the effect is marginal diminishing, $f_\sigma''(\eta) > 0$. Assume $f_r(\eta)$ is second-order differentiable geometric convex function. If $\eta \to \infty$, then $f_r(\eta) = 0$.

So the optimization problem of enterprise manager is to maximize the shape ratio. The first-order condition for solving this optimization problem is (11):

$$S_V' = \frac{f_r'(\eta) \cdot \left[\sigma_{V0} + f_\sigma(\eta)\right] - f_\sigma'(\eta) \cdot \left[R_{V0} + f_r(\eta)\right]}{\left[\sigma_{V0} + f_\sigma(\eta)\right]^2} = 0 \tag{11}$$

Assume when $\eta = \eta^*$, $S_V' = 0$, obviously, when $\eta = \eta_0$, $S_V' > 0$ .If S_V' is single-reduction when $\eta > \eta_0$, then $\eta^* > \eta_0$. The next will prove that S_V' is single-reduction when $\eta > \eta_0$.

Defining the function (12):

$$g_\sigma(\eta) = \frac{1}{\sigma_{V0} + f_\sigma(\eta)} \tag{12}$$

According to the properties of geometric convex functions, $g_\sigma'(\eta) > 0$, $g_\sigma''(\eta) < 0$.

Rewriting that (13):

$$S_V = g_\sigma(\eta)\left[R_{V0} + f_r(\eta)\right] \tag{13}$$

The first order derivative is (14):

$$S_V' = g_\sigma'(\eta)\left[R_{V0} + f_r(\eta)\right] + g_\sigma(\eta) \cdot f_r'(\eta) \tag{14}$$

The second order derivative is (15):

$$S_V'' = g_\sigma''(\eta)\left[R_{V0} + f_r(\eta)\right] + 2g_\sigma'(\eta) \cdot f_r'(\eta) + g_\sigma(\eta) \cdot f_r''(\eta) \tag{15}$$

For that $g_\sigma''(\eta) < 0$, $f_r''(\eta) < 0$, $g_\sigma'(\eta) > 0$, when $\eta > \eta_0$, $f_r'(\eta) < 0$, so $S_V'' < 0$, S_V' is single-reduction.

If the expected return rate changes from $E(r_{V0}) + f_r(\eta)$ to $E(\hat{r}_{V0}) + f_r(\eta)$ because of some factors, and defining is (16):

$$\hat{R}_{V0} = E(\hat{r}_{V0}) - r_f \text{ , } \hat{R}_{V0} > R_{V0} \tag{16}$$

The first order derivative of the sharp ratio is (17)

$$\hat{S}_V' = \frac{f_r'(\eta)\cdot\left[\sigma_{V0}+f_\sigma(\eta)\right]-f_\sigma'(\eta)\cdot\left[\hat{R}_{V0}+f_r(\eta)\right]}{\left[\sigma_{V0}+f_\sigma(\eta)\right]^2}=0 \quad (17)$$

Because of: $f_r'(\eta^*)<0$, $f_\sigma'(\eta^*)<0$, $\hat{R}_{V0}>R_{V0}$

so when $\eta=\eta^*, \hat{S}_V'>0$. According before, S_V' is single- reduction when $\eta>\eta_0$, hence there is η^{**}, $\eta^{**}>\eta^*$, to satisfy $\hat{S}'(\eta^{**})=0$.

5 The effect of public subsidies on the investment decision

It is assumed that all innovation project investments can receive a certain proportion of public subsidies from the government, the proportion is R_S. So the expected return rate will increase R_S because of public subsidies for an investment to innovative projects. We assume that non innovative projects cannot receive public subsidies.

Under innovative projects market equilibrium, average return on investment of all innovative projects is $E(r_{V0})$, and $R_{V0}=E(r_{V0})-r_f$. Average standard deviation is σ_{V0}. They are not affected by the effort of enterprise managers.

If the investment of innovative projects increases because of public subsidies, $E(r_{V0})$ will decrease and σ_{V0} will not change. The amount of innovative projects invested is defined as Q_V under innovative projects market equilibrium. $Q_V = f_D(R_{V0})$. The first order derivative is $f_D'(R_{V0})$, $f_D'(R_{V0})<0$.

The investment allocated to innovation projects is (18):

$$Q_V = f_S(R_{V0}) = W_0\cdot w_P^*\cdot w_V^* = \frac{1}{A}\cdot\frac{R_{V0}+f_r(\eta^*)}{\left[\sigma_{V0}+f_\sigma(\eta^*)\right]^2} \quad (18)$$

Assuming that there are $R_{V0,1}$ and $R_{V0,2}$, $R_{V0,2}>R_{V0,1}$, the optimal effort level of enterprise manager is η_1^* when $R_{V0}=R_{V0,1}$, and the optimal effort level is η_2^* when

$R_{V0} = R_{V0,2}$. According before, $\eta_2^* > \eta_1^*$, $f_r(\eta_2^*) < f_r(\eta_1^*)$, $f_\sigma(\eta_2^*) < f_\sigma(\eta_1^*)$, defining that (19)-(21):

$$f_S(R_{V0,1}) = \frac{1}{A} \cdot \frac{R_{V0,1} + f_r(\eta_1^*)}{\left[\sigma_0 + f_\sigma(\eta_1^*)\right]^2} \tag{19}$$

$$f(R_{V0,2}, \eta_1^*) = \frac{1}{A} \cdot \frac{R_{V0,2} + f_r(\eta_1^*)}{\left[\sigma_0 + f_\sigma(\eta_1^*)\right]^2} \tag{20}$$

$$f_S(R_{V0,2}) = \frac{1}{A} \cdot \frac{R_{V0,2} + f_r(\eta_2^*)}{\left[\sigma_0 + f_\sigma(\eta_2^*)\right]^2} \tag{21}$$

Because of $R_{V0,2} > R_{V0,1}$, $f(R_{V0,2}, \eta_1^*) > f_S(R_{V0,1})$. When $R_{V0} = R_{V0,2}$, the optimal effort level of enterprise manager should maximize the Sharpe ratio, so that we can obtain (22):

$$\frac{R_{V0,2} + f_r(\eta_2^*)}{\left[\sigma_0 + f_\sigma(\eta_2^*)\right]^2} > \frac{R_{V0,2} + f_r(\eta_1^*)}{\left[\sigma_0 + f_\sigma(\eta_1^*)\right]^2} \tag{22}$$

Under the innovative projects market equilibrium with public subsidies and without public subsidies, the investments to innovative projects are (23):

$$f_D(\hat{R}_{V0}) = f_S(\hat{R}_{V0} + R_S) = \frac{1}{A} \cdot \frac{R_{V0} + R_S + f_r(\eta^{**})}{\left[\sigma_0 + f_\sigma(\eta^{**})\right]^2} \tag{23}$$

Without public subsidies, the investments to innovative projects are: (24)

$$f_D(R_{V0}) = f_S(R_{V0}) = \frac{1}{A} \cdot \frac{R_{V0} + f_r(\eta^*)}{\left[\sigma_0 + f_\sigma(\eta^*)\right]^2} \tag{24}$$

Defining a function (25):

$$H(x) = f_D(x) - f_S(x) \tag{25}$$

because of $f_D(x)$ is single-reduction and $f_S(x)$ is single-increasing, $H(x)$ is single-reduction. $H(R_{V0}) = 0$. Defining a function: (26)

$$G(x) = f_D(x) - f_S(x + R_S) \tag{26}$$

Because of that:

$$\begin{aligned} G(\hat{R}_{V0}) &= f_D(\hat{R}_{V0}) - f_S(\hat{R}_{V0} + R_S) \\ &= H(\hat{R}_{V0}) - \left\lfloor f_S(\hat{R}_{V0} + R_S) - f_S(\hat{R}_{V0}) \right\rfloor \\ &= 0 \end{aligned}$$

$f_S(x)$ is single-increasing.

$$f_S(\hat{R}_{V0} + R_S) - f_S(\hat{R}_{V0}) > 0$$

$$H(\hat{R}_{V0}) > 0$$

$$f_D(\hat{R}_{V0}) = f_S(\hat{R}_{V0} + R_S) > f_D(R_{V0}) = f_S(R_{V0})$$

This demonstrates that the amount of investment increases and the scope of investment can be expended because of public subsidies. The average quality of innovative projects will decline.

In addition:

$$\begin{aligned} G(\hat{R}_{V0}) &= f_D(\hat{R}_{V0}) - f_S(\hat{R}_{V0} + R_S) \\ &= f_D(\hat{R}_{V0}) - f_D(\hat{R}_{V0} + R_S) + H(\hat{R}_{V0} + R_S) \\ &= 0 \end{aligned}$$

$f_D(x)$ is single-reduction, $f_D(\hat{R}_{V0}) - f_D(\hat{R}_{V0} + R_S) > 0$, so $H(\hat{R}_{V0} + R_S) < 0$, $\hat{R}_{V0} + R_S > R_{V0}$, $\eta^{**} > \eta^{*}$. The expected return of innovative enterprises will increase and efforts level of enterprise manager will increase.

Since non innovative projects cannot receive public subsidies from the government, the market equilibrium of non innovative projects is not affected by public subsidies, consequently the amount of investments to non innovative projects will not change.

6 An Example

It is assumed that the return and risk data of the two types of projects are shown in Table 1.

Table 1 Calculation example data

no public subsidy		there are public subsidies	
R_V	15%	R_V	20%
R_E	10%	R_E	10%
σ_V	25%	σ_V	25%
σ_E	20%	σ_E	20%

It can be calculated that the sharp ratio is 0.781 without public subsidies and 0.943 with government subsidies. The fund allocation of the two types of projects is shown in Figure 1. The solid line indicates the fund allocation without public subsidies, and the dotted line indicates the fund allocation with public subsidies.

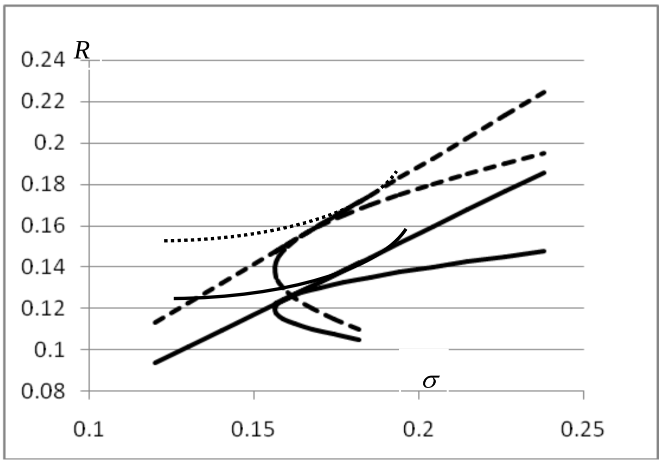

Figure 1. Schematic diagram of fund allocation.

7 Conclusion

On the mean-variance assumption, the effect of ex ante public subsidies to innovative projects on capital allocation of innovation enterprises and investors under market equilibrium has been studied. The study demonstrates that the amount of investments in innovation projects increases and the scope of investment can expand because of public subsidies. Although the average quality of innovative projects diminishes for this, the expected return of innovative enterprise will increase because of public subsidies, therefore innovative enterprise manager would enhance his endeavor. Investor will allocate more capital to innovative enterprise in that the return-risk ratio of innovative enterprise increases because of public subsidies.

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