The Performance of Portfolio Optimization and Risk Control Strategies Based on Mean-Variance Model —A Study Under the Context of COVID-19

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Abstract— The study discussed the efficiency of portfolio selection methods based on the analysis of average return and standard deviation of return, from the perspective of long periods and specific time intervals. Several risk managing methods commonly utilized by the investors were discussed in the essay so as to assess their feasibility under unique economic setting. Specifically, the study applied selection strategies based on Markowitz's Portfolio Theory, and given that people have a proclivity for higher risk aversion due to the instability brought by the pandemic, the study compared two optimization goals for constructing portfolio—the minimal variance method and the maximum Sharpe ratio method. The analytical results based on the performance of the diversified portfolio within two decades revealed that generally, the selection strategies both had better performance after the emergence of COVID-19. However, specific constraints for investment capital would affect the result more negatively during the pandemic. The findings enable us to evaluate the present practice of traditional portfolio selection strategies and provide suggestions for investors on risk managing methods choosing under the disturbance of unexpected social events.

Keywords-portfolio selection; risk control; COVID-19

1 INTRODUCTION

The allocation and reallocation strategies for optimal portfolio have been the major consideration of investors, arousing the interests of many researchers as well. From the study done by Nagy and Obenberger [1], respondents were found to consider expected earnings, diversification needs, minimizing risk, and other variables to significantly influence stock purchase decisions. In addition, from previous research, mainly based on the Mean-Variance consideration proposed by Markowitz in 1952 [2], we have already developed commonly adopted methods for measuring the key factors people care about most when making investment decisions--the excess return and risk of the portfolio. Also, many scholars verified the efficiency of diversification, such as Sharpe [3], and the notion has been directing investing activities so far. Then, it is interesting to know whether the idea still holds. How would it be like if we apply such a method during the pandemic

ICIDC 2022, June 17-19, Qingdao, People's Republic of China

DOI 10.4108/eai.17-6-2022.2322674

setting at present? The paper would discuss the contemporary practice of the traditional portfolio construction concept proposed previously and its effectiveness, the main innovation of obtaining the effect brought by COVID-19 and the contingent adjustment of the selection strategy. Correspondingly, investors' risk preference would be the main factor that should be taken into consideration when devising the capital allocation strategy under specific societal conditions.

Referring to the current economic and social circumstances, largely affected by the emergence of COVID-19, there is a greater inclination for the investors to draw more emphasis on risk control. The speculation was mainly based on empirical studies in the 1970s. For instance, Cohn et al. [4] provided tentative evidence that risk-aversion would show a decrease when investor wealth increases. Furthermore, the study by Zhang and Wu [5] also supported the study, which demonstrated that fluctuation of labor income would significantly raise the level of people's risk aversion. Under the condition where individuals' income is generally unstable and decremental, the pursuit of minimal variance in the assets' return, indicating controllable price risk, would receive the acceptance from the investors, since it caters to the demand of risk-averse investors, even at the price of lower returns. Therefore, the minimal risk portfolio method would be the first strategy discussed in this paper.

The second strategy used in the paper is the maximum Sharpe ratio method. Based on the sequential study of Markowitz's Portfolio Theory conducted by Sharpe, portfolio performance could be measured by the reward-to-variability ratio. A portfolio with a maximum Sharpe ratio is also a feasible selection method widely accepted by investors since it would be considered efficient, which means the expected return cannot be increased without increasing its return variability. The variability cannot be reduced without return decreasing [6]. In this paper, for adaptation of the risk-aversion level, several constraints for risk control would be added to the selection procedure.

The study mainly focused on how the strategies would work out and how the performances in the long run and different sub-periods would vary. The results indicated that the two major kinds of portfolio selection strategies generally reached a better outcome after COVID-19 started to influence the stock market. Meanwhile, constraints for short sales would help investors maintain the fluctuations of both the level of return and risk within a certain range. They exert an even harder restrictive impact on the portfolio after the emergence of the pandemic. However, what is worth mentioning is that prohibiting short selling would lead to particularly unfavorable results in finding the maximum Sharpe ratio portfolio during this special period. This would be counterintuitive since many people expect a better outcome with intensive constraints under the unpredictable environment, hoping for loss reduction.

2 DATA

Portfolio diversification is a strategy adopted by most investors since it lowers the unsystematic risk of the portfolio, given that not all asset categories, industries, or stocks move together. Consequently, holding various non-correlated assets can nearly eliminate unsystematic risk, which is induced by the specific corporation or industry. Based on this consideration, I selected 11 representative stocks, each given relatively high weight as a constituent of the S&P 500 from diverse industry segments, to simulate a common portfolio diversification strategy.

Simultaneously, an (S&P 500) equity index is included, and a 1-month Fed Funds rate is regarded as a proxy for risk-free rate in calculating excess returns.

As for the 11 securities selected, specifically they are NextEra Energy (NEE) from the Utility sector, ExxonMobil (XOM) from the Energy sector, Bank of America (BAC) from the Financial sector, Cisco (CSCO)from the telecom sector., Apple (AAPL) from Technology sector, Johnson & Johnson (JNJ) from Health Care sector, Protecter & Gamble (PG) from Consumer Staples sector, Amazon (AMZN) from Consumer Discretionary Sector, General Electric Company (GE) from Industrial sector, Ecolab (ECL) from Materials sector.

The stock prices for these stocks are shown in Figure 1 to Figure 12. In general, great discrepancies lay in the trends of the prices within the 20-year time. Still, most stocks, SPX included, showed similar changes in performance around February 2020, which demonstrated a plummet in stock price and a sudden increase in the frequency of trades.

Figure 1. Stock prices of S&P 500 (January 2001 to May 2021)

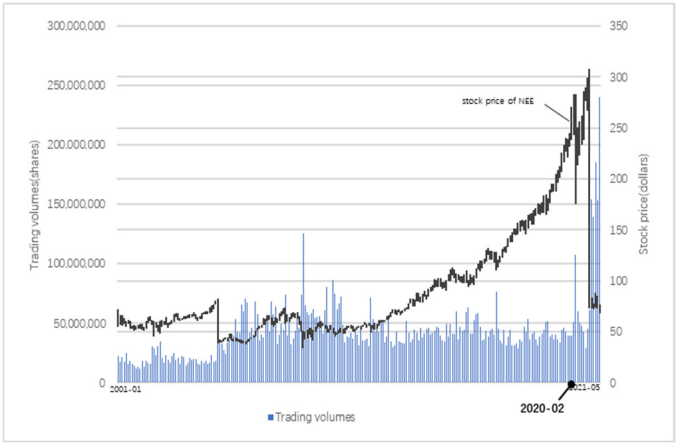

Figure 2. Stock prices of NEE (January 2001 to May 2021)

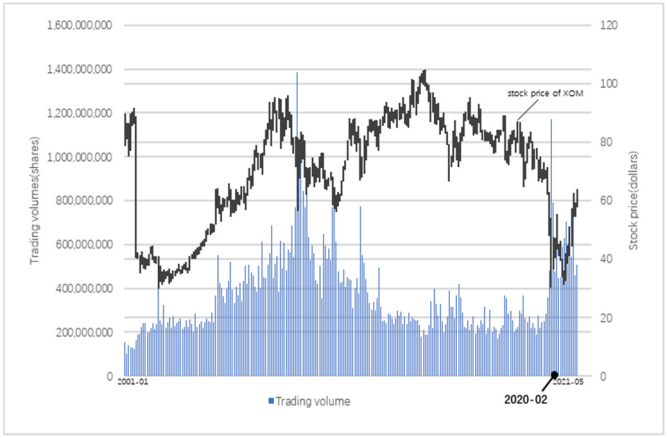

Figure 3. Stock prices of XOM (January 2001 to May 2021)

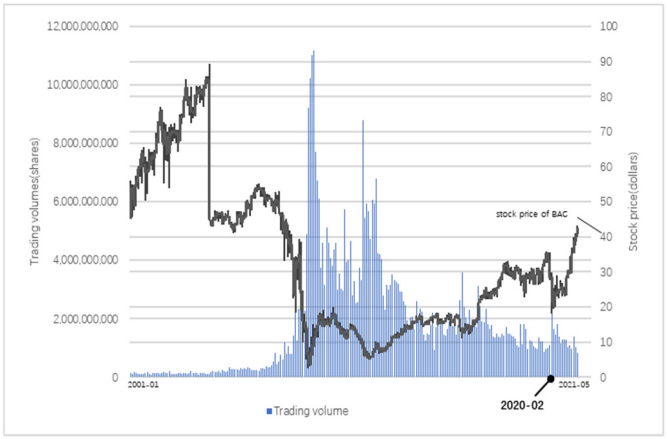

Figure 4. Stock prices of BAC (January 2001 to May 2021)

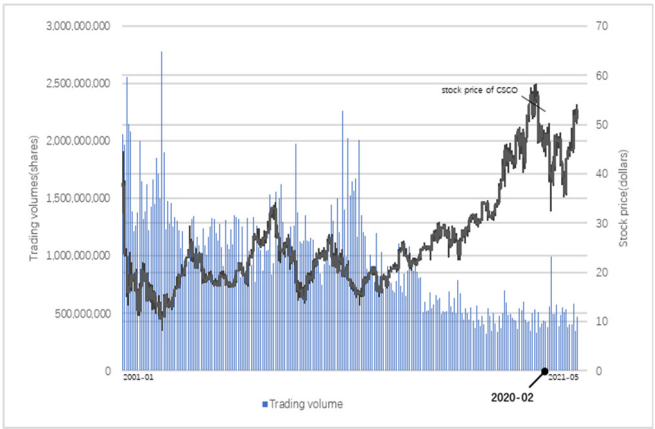

Figure 5. Stock prices of CSCO (January 2001 to May 2021)

Figure 6. Stock prices of AAPL (January 2001 to May 2021)

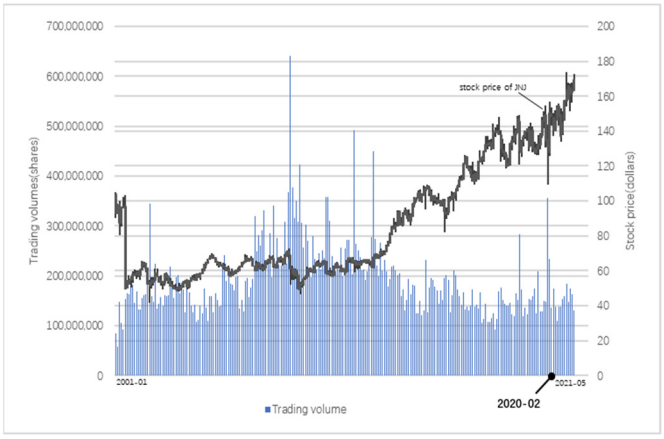

Figure 7. Stock prices of JNJ (January 2001 to May 2021)

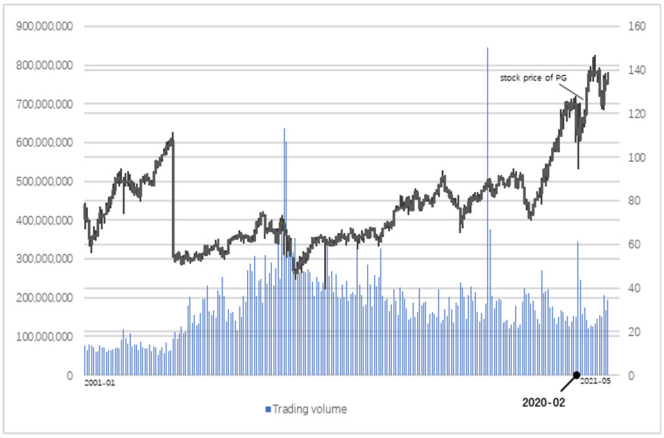

Figure 8. Stock prices of PG (January 2001 to May 2021)

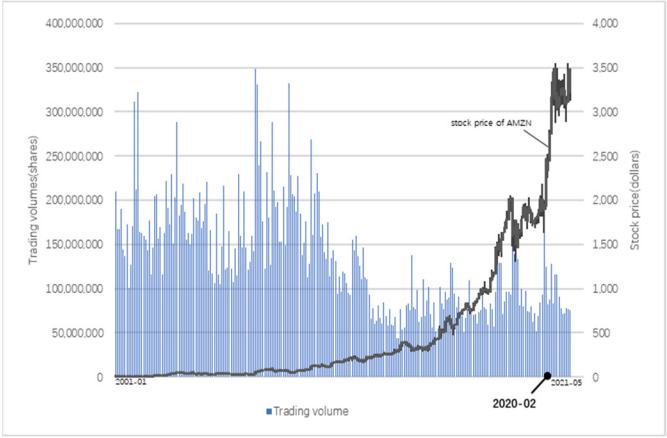

Figure 9. Stock prices of AMZN (January 2001 to May 2021)

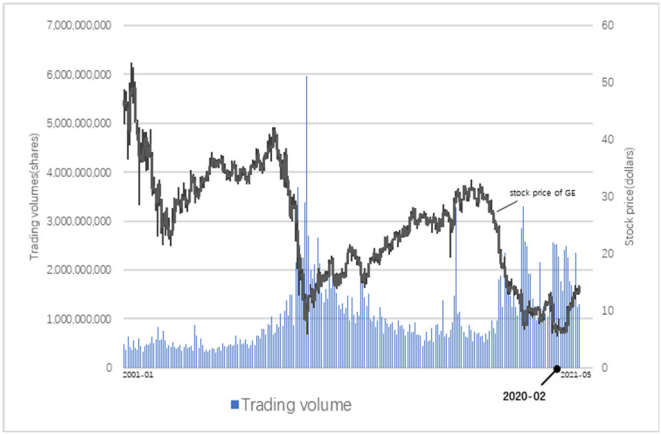

Figure 10. Stock prices of GE (January 2001 to May 2021)

Figure 11. Stock prices of ECL (January 2001 to May 2021)

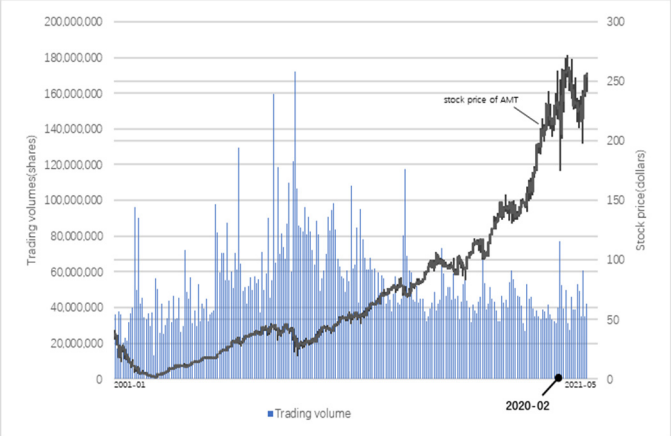

Figure 12. Stock prices of AMT (January 2001 to May 2021)

Source: Wind

Regarding the macro-social factor, the disturbance of the unexpected epidemic--COVID-19 should also be considered since its global economic impact was hard to overlook. Whether the unexpected shock influenced the mechanism of the investing strategies mentioned, amd how they were affected were the main problems the study tried to reveal. Consequently, in addition to the general analysis of the entire period from 2011 to 2021, the research would also discuss the problem under different external societal conditions by splitting the period into two intervals (before and after the emergence of COVID-19). It is important to highlight that a sudden global market crash began in February 2020, indicating that the stock market reflected the economic impact brought by COVID-19 from that time. In addition, based on the observations of the stock price of the selected stocks, sector index, and global index, abnormal fluctuations during the same period led to the confirmation. Therefore, the 20 years' time interval was divided contingently, with one covering the time before February 2020 and the other after it. The breakpoint has been identified in Figure 1.

3 METHOD

The variables I used in the paper are shown in Table 1.

TABLE 1. KEY VARIABLES

Variable Names	Definitions
wi	The weight of stock i in the portfolio, like w_{NEE} was the weight given to stock NEE. The sum of the weights would be 1.
r_f	Return of risk-free assets, which is approximated by the 1-month Fed Funds rate.
ri	The annual average excess return of stock i
Cov(R_i,R_j)	The covariance of the returns of stock i and stock j

Sharpe ratio	$\frac{r(\vec{w})}{\sigma(\vec{w})}$
σ	The standard deviation of the portfolio returns.

Next, the portfolios formed by the 11stocks and the SPX would be investigated.

In the study, two models were utilized as methods for portfolio constitution setting. For the majority of investors, being risk-averse is their shared characteristic. When measuring whether the portfolio could live up to their expectation, the main consideration would be the portfolio's risk level. As for the measurement of risk, I selected the standard deviation of the portfolio as its representative in accordance with the widely utilized Markowits Model. As a result, the portfolio with minimal variance or standard deviation was supposed to be what they desire. This was generally the *Minimal Variance Portfolio Method base*, where the portfolio selection goal was indicated as formula (1) and (2).

$$\sigma(\vec{w}) = \sum_{i=1}^{12} \sum_{j=1}^{12} Cov(R_i, R_j). \quad (1)$$

$$\sigma(\vec{w}) \rightarrow \min_{\vec{w}} \quad .(2)$$

However, this might not be the most adopted choice for the investors since the return of the investments is also what they attached great significance to when allocating their money for different securities. Based on the comprehensive weighing of the two factors, finding the diversified portfolio with the greatest returns at the lowest level of risk, which meant the one with the highest Sharpe ratio, would be the common pursuit. And whatever the economic condition is and how it would change, the demand is not likely to change much since aggrandizing their earnings is the shared and ultimate goal that would not change. Therefore, the Maximum Sharpe Ratio Portfolio Method would give a simulation of the favorable portfolio which could be calculated by formula (3), (4) and (5):

$$r(\vec{w}) = (w_{SPX} * r_{SPX} + w_{NEE} * r_{NEE} + w_{XOM} * r_{NEE+} w_{BAC} * r_{BAC} + w_{CSCO} * r_{CSCO} + w_{AAPL} * r_{AAPL} + w_{JNJ} * r_{JNJ+} w_{PG} * r_{PG} + w_{AMZN} * r_{AMZN} + w_{GE} * r_{GE} + w_{ECL} * r_{ECL} + w_{AMT} * r_{AMT}) - r_f \quad . (3)$$

$$\sigma(\vec{w}) = \sum_{i=1}^{12} \sum_{j=1}^{12} Cov(R_i, R_j) \quad (4)$$

$$\frac{r(\vec{w})}{\sigma(\vec{w})} \rightarrow \max_{\vec{w}} \quad .(5)$$

Nevertheless, based on certain regulations restricting the trades and capital allocation of specific types of accounts and funds, the portfolio structure is likely to differ greatly and influence the performance of the portfolio. Meanwhile, different level of risk aversion would be another significant factor that could affect the allocation of capital and the optimization result as well.

To be specific, in this paper, the performance of the portfolio under 4 constraints, 3 with additional restrictions, would be analyzed. As for the restrictions, including the restrictive effect on margin accounts by Regulation T, the short positions prohibiting, which was a typical limitation on trades of open-ended mutual funds, and the arbitrary "box" constraints on weights required by the clients are mainly focused and analyzed.

Constraint 1: No constraint. The condition without constraint would depict the greatest degree our optimization could reach under the two models.

Constraint 2: Allowing margin securities to purchase within 50 percent of the total purchase price. This was also what FINRA had requested on trades with margin accounts from when the regulation T was put forward. As for calculations, the simulation would be simplified as $\sum_{i=1}^{12} | wi | \leqq 2$.

Constraint 3: $| wi | \leqq 1$ A type of "box" constraint set by the investors confines short selling within a certain degree regarding the level of their risk aversion.

Constraint 4: No permission on short selling. It is considered a strict limitation with a strong effect since the investment could only be made by owned equity. Meanwhile, the constraint is compulsory when investing in open-ended mutual funds where short selling is under prohibition. Requiring $wi \geq 0$, for $\forall i$ would be the simulative setting.

4 RESULT

Firstly, the study would focus on analyzing the portfolio performance based on the Minimal variance method. As is shown in Table 2 and Table 3, I found fewer differences in the least risk level under different constraints. However, the prohibition of short selling exerted a relatively great influence on the weights set. However, Table 4 and Table 5 show that during the period from February 2020 to May 2021, not merely the allocation of different stocks differed enormously from the former period, but also the constraints' impact on creating a portfolio with the lowest risk was more easily observed. The latter minimal risk portfolio inclined elevating the weights of SPX, NEE, XOM, CSCO, PG, GE, and withdrawing capital from the others. Also, there were fewer discrepancies on the level of risk when short selling was forbidden, indicating that though it posed a greater restriction on risk control, the level of risk could be maintained even under such unpredictable conditions. Moreover, what is worth mentioning is that the actual restrictive power of Regulation T seemed to be more conspicuous since the advent of COVID-19, given that the portfolio structure and the lowest variance attainable under the constraint differed greatly from that under "free" condition. We may conclude that allocating less capital for a short position would be a better way for risk controlling in the unstable external environment.

TABLE 2. WEIGHTS OF INDIVIDUAL STOCKS IN THE MINIMAL VARIANCE PORTFOLIO

Minimal Variance Portfolio (January 2001 to May 2021)				
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***
w_{SPX}	0.348	0.348	0.348	0.100
w_{NEE}	0.305	0.305	0.305	0.321

w_{XOM}	0.058	0.058	0.058	0.097
w_{BAC}	-0.049	-0.049	-0.049	0.000
w_{CSCO}	-0.053	-0.053	-0.053	0.000
w_{AAPL}	-0.001	-0.001	-0.001	0.006
w_{JN}	0.135	0.135	0.135	0.156
w_{PG}	0.208	0.208	0.208	0.223
w_{AMZN}	-0.023	-0.023	-0.023	0.000
w_{GE}	-0.016	-0.016	-0.016	0.000
w_{ECL}	0.074	0.074	0.074	0.080
w_{AMT}	0.014	0.014	0.014	0.017

TABLE 3. KEY RESULTS FROM MINIMAL VARIANCE PORTFOLIO

Minimal Variance Portfolio (January 2001 to May 2021)				
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***
Portfolio return	0.088	0.088	0.088	0.103
Standard deviation	0.096	0.096	0.096	0.098
Sharpe Ratio	0.916	0.916	0.916	1.051

TABLE 4. WEIGHTS OF INDIVIDUAL STOCKS IN THE MINIMAL VARIANCE PORTFOLIO IN 2 SUB-PERIODS

	Minimal Variance Portfolio (January 2001 to February 2020)				**Minimal Variance Portfolio (February 2020 to May 2021)**			
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***	***Constraint 1***	***Constraint 2***	***Constraint 3***	***Constraint 4***
w_{SPX}	0.378	0.378	0.378	0.070	1.270	-0.036	1.000	0.000
w_{NEE}	0.278	0.278	0.278	0.293	0.300	0.216	0.306	0.333
w_{XOM}	0.080	0.080	0.080	0.134	0.320	0.000	0.333	0.000
w_{BAC}	-0.039	-0.039	-0.039	0.000	-0.830	-0.113	-0.777	0.000
w_{CSCO}	-0.065	-0.065	-0.065	0.000	0.016	-0.057	-0.010	0.000
w_{AAPL}	0.006	0.006	0.006	0.015	-0.255	-0.161	-0.193	0.000
w_{JN}	0.124	0.124	0.124	0.155	0.065	0.312	0.087	0.171
w_{PG}	0.202	0.202	0.202	0.205	0.521	0.002	0.551	0.104
w_{AMZN}	-0.016	-0.016	-0.016	0.000	-0.319	0.147	-0.327	0.000
w_{GE}	-0.062	-0.062	-0.062	0.000	0.087	0.203	0.102	0.140
w_{ECL}	0.104	0.104	0.104	0.114	-0.159	-0.132	-0.119	0.000
w_{AMT}	0.012	0.012	0.012	0.013	-0.017	0.622	0.047	0.252

TABLE 5. KEY RESULTS FROM MINIMAL VARIANCE PORTFOLIO IN 2 SUB-PERIODS

	Minimal Variance Portfolio (January 2001 to February 2020)				**Minimal Variance Portfolio ((February 2020 to May 2021)**			
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***	***Constraint 1***	***Constraint 2***	***Constraint 3***	***Constraint 4***
Portfolio return	0.093	0.093	0.093	0.105	-0.098	0.072	-0.101	0.086
Standard deviation	0.092	0.092	0.092	0.095	0.040	0.087	0.041	0.093
Sharpe Ratio	1.016	1.016	1.016	1.112	-2.428	0.826	-2.470	0.928

Next, similar analytic process would be conducted on the portfolio with maximum Sharpe ratio, which was another focus of the study. As is shown in Table 6 and Table 7, investors investing with margin accounts or in open-ended mutual funds were not likely to be affected by the contingent restrictions on owning portfolios with the greatest return-deviation ratio since the optimal portfolios did not differ much on Sharpe ratios in the long run.

Comparing the data before and after the outbreak of COVID-19, several interesting changes were explicitly shown. Firstly, Table 8 demonstrates that weights adding on AMZN, GE, AMT was the major conversion observable under different limitations. Moreover, the integral ascendance of the Sharpe ratio level was obvious, especially when no constraints were set. However, the limitations seemed more effective since the optimal portfolios' Sharpe ratios with constraints showed a moderate increase compared to those without.

But what indicated by Table 9 as well was that the constraints of allowing up to 50 percent of the purchase price of margin securities showcased its impact more evidently after February 2020. The maximum Shape ratio of portfolio without short-selling even decreased during the same period, which meant they might not be effective investing strategies in certain environment. Comparably, the certain setting for "box" constraint could reach better outcomes both before and after the health event.

TABLE 6. WEIGHTS OF INDIVIDUAL STOCKS IN THE MAXIMUM SHARPE RATIO PORTFOLIO

Maximum Sharpe Ratio Portfolio (January 2001 to May 2021)				
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***
w_{SPX}	-1.723	-0.414	-1.000	0.000
w_{NEE}	0.470	0.393	0.424	0.384
w_{XOM}	0.230	0.040	0.141	0.000
w_{BAC}	0.183	0.038	0.113	0.000
w_{CSCO}	-0.003	-0.050	-0.047	0.000
w_{AAPL}	0.389	0.217	0.289	0.177
w_{JN}	0.313	0.159	0.230	0.056
w_{PG}	0.376	0.234	0.311	0.134
w_{AMZN}	0.205	0.093	0.146	0.058
w_{GE}	-0.036	-0.035	-0.058	0.000
w_{ECL}	0.476	0.263	0.360	0.158
w_{AMT}	0.120	0.062	0.091	0.033

TABLE 7. KEY RESULTS FROM THE MAXIMUM SHARPE RATIO PORTFOLIO

Maximum Sharpe Ratio Portfolio (January 2001 to May 2021)				
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***
Portfolio return	0.358	0.224	0.285	0.180
Standard deviation	0.193	0.131	0.156	0.124
Sharpe Ratio	1.851	1.712	1.824	1.456

TABLE 8. WEIGHTS OF INDIVIDUAL STOCKS IN THE MAXIMUM SHARPE RATIO PORTFOLIO IN SUB-PERIODS

	Maximum Sharpe Ratio Portfolio (January 2001 to February 2020)				**Maximum Sharpe Ratio Portfolio ((February 2020 to May 2021)**			
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***	***Constraint 1***	***Constraint 2***	***Constraint 3***	***Constraint 4***
w_{SPX}	-1.539	-0.328	-1.539	0.000	25.533	0.000	1.000	0.000
w_{NEE}	0.488	0.409	0.488	0.405	-13.145	0.052	-0.085	0.000
w_{XOM}	0.228	0.031	0.228	0.000	-50.215	-0.234	-0.923	0.000
w_{BAC}	0.171	0.038	0.171	0.000	43.244	0.000	0.455	0.000
w_{CSCO}	-0.006	-0.063	-0.006	0.000	1.194	0.000	-0.021	0.000
w_{AAPL}	0.346	0.201	0.346	0.164	-22.057	0.000	-0.621	0.001
w_{JN}	0.307	0.152	0.307	0.038	30.449	0.325	0.835	0.000
w_{PG}	0.353	0.242	0.353	0.136	-67.630	-0.116	-1.000	0.000
w_{AMZN}	0.189	0.090	0.189	0.055	47.234	0.393	0.955	0.646
w_{GE}	-0.125	-0.110	-0.125	0.000	10.409	0.321	0.409	0.239
w_{ECL}	0.475	0.277	0.475	0.170	-17.307	-0.151	-0.486	0.000
w_{AMT}	0.112	0.061	0.112	0.031	13.292	0.409	0.483	0.114

TABLE 9. KEY RESULTS FROM THE MAXIMUM SHARPE RATIO PORTFOLIO IN SUB-PERIODS

	Maximum Sharpe Ratio Portfolio (January 2001 to February 2020				**Maximum Sharpe Ratio Portfolio ((February 2020 to May 2021)**			
	Constraint 1	***Constraint 2***	***Constraint 3***	***Constraint 4***	***Constraint 1***	***Constraint 2***	***Constraint 3***	***Constraint 4***
Portfolio return	0.362	0.232	0.362	0.181	20.493	0.255	0.442	0.327
Standard deviation	0.181	0.125	0.181	0.118	5.126	0.132	0.145	0.227
Sharpe Ratio	2.004	1.861	2.004	1.527	3.998	1.933	3.048	1.440

From the observation of samples covering an approximately 20-year period, from January 2001 to May 2021, it could be concluded that a short-selling ban would impose great restrictions both in finding portfolio with minimal standard deviation and maximum Sharpe ratio. Simultaneously, the "up to 50% margin founding" constraint was more influential when the optimization goal was to maximize Sharpe ratio.

Significant indication could be acquired through comparison of the results. Firstly, the general level of the least standard deviation was even lower after the emergence of COVID-19, which was out of expectation. Still, it may also be induced by the deficiency of data. Secondly, though with greater and more erratic changes, we could acquire the most favorable portfolio performance under two models when no constraints were set. Meanwhile, the "box" constraint was considered the least restrictive constraint, bringing the most similar result to that under the condition without constraints, indicating that it may be the best choice when profitability and risk-controlling demands coexist. Moreover, it is worth mentioning that short selling forbidden exerted greater adverse effect on the portfolio, given that when the general level of attainable greatest Sharpe ratio ascended after February 2020, the maximum ratio under the "box" constraint was even lower than that of before.

5 CONCLUSION

This paper mainly discussed the performance of two basic portfolio selection methods with additional constraints on leverage, by analyzing average portfolio return and the standard deviation of it. The main conclusion is that, in general, the optimization method, either to minimize variance or to maximize Sharpe ratio of the portfolio, rendered an even better outcome after the advent of COVID-19. Though the portfolio definitely generated the best results without constraints, several constraints exerted on short selling did show favorable effects on maintaining the risk and return of the portfolio on a certain level. As a result, the findings showed that investors should make the contingent adjustment to the constraints on short sales. However, some defects were found that when calculating for the portfolio with the lowest standard deviation, though the results under the "box" constraint provided solutions with particularly low risk, it was not likely to make sense in real practice, since its adverse influence on the portfolio returns was particularly unfavorable. The same result could be acquired when conducting the same calculation procedures under no constraints.

The study provids us with a new interpretation of the widely applied methods for risk control. Although restrictions on leverage could help alleviate the price risk of the portfolio, their effectiveness would be weakened under specific 'black swan' events, and may even do more harm to the overall return of the portfolio. However, the statistical outcome could be affected by the limited data, since the time interval from the emergence of COVID-19 to the present is relatively short. Consequently, it is considered significant and to reconduct the research with prolonged time intervals during the outbreak of pandemic.

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