The Dynamic Relation between Bitcoin Volatility and Stock Volatility: DCC-GARCH Approach

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Abstract—Bitcoin's performance during the COVID-19 pandemic has drawn a lot of attention, with many researchers wondering whether bitcoin can act as a hedge against the stock market, and how exactly the COVID-19 pandemic has changed bitcoin's connection to the world. This paper aims to investigate the dynamic conditional volatility correlation between bitcoin and stock markets before and after the COVID-19 outbreak. Bitcoin data and stock market data are collected from Coindesk and Invest, respectively. DCC-GARCH model is built to measure the dynamic correlation between bitcoin and stock markets. As a result, it shows the overall dynamic conditional correlations increase after the COVID-19 outbreak. Based on the dynamic correlations, this paper further conducts investment portfolio analysis for bitcoin and stock markets. The GARCH model is used to extract the volatilities of bitcoin and stock markets and calculate the dynamic optimal weight of bitcoin in the portfolio. Finally, this paper finds that the dynamic relation between bitcoin and the stock markets changes before and after the COVID-19 outbreak, and this change dramatically affects the optimal weight of bitcoin in the stock-bitcoin investment portfolio.

Keywords-component: bitcoin; stock market; DCC-GARCH; optimal portfolio

1 INTRODUCTION

In 2020, the COVID-19 pandemic leads to a tremendous worry to severe loss and the economic breakdown, and the significant financial turmoil associated with the pandemic would result in adjustments in optimal portfolio selections[1] (see Berkelaar et al., 2004[1]). Hence, it prompts investors to find a safe haven to hedge the risks, and bitcoin is considered as a markable shelter due to the weak correlation with traditional financial assets. However, some latest studies show that bitcoin is not a safe haven for the equity market during the COVID-19 pandemic, which is different from the previous researches (see Conlon, T.et al., 2020[2]). Salim Lahmiri and Stelios Bekiros (2020)[3] explore the evolution of the informational efficiency in 45 cryptocurrency markets and 16 international stock markets before and during the COVID-19 pandemic Cryptocurrencies show more instability and irregularity during the COVID-19 pandemic compared to international stock markets. Conlon, T., & McGee, R (2020)[4] also conduct empirical research and reckon that bitcoin does not act as a safe haven when considering the COVID-19 crisis in the short term. However, due to the time limit of this study, there is still a lack of long-term tracking of market reaction. The situation keeps changing after the short period of the pandemic, the stock and bitcoin markets began to rebounded strongly in the following

months as governments implemented monetary stimulus. The relations between stock market volatility and bitcoin fluctuations seem to change continuously.

Although several previous researchers have used the GARCH model to study bitcoin volatility, they do not discuss the dynamic relation between bitcoin volatility and stock market volatility, and that's probably why they get different conclusions for the relations during different times. Therefore, it is meaningful to figure out how the volatility relation between bitcoin and traditional assets changes before and after the COVID-19 outbreak. To study the dynamic volatility relation, this paper uses the DCC-GARCH model proposed by Engle (2002)[5]. This model provides a method to describe the time-varying relation between stock and bitcoin volatilities. Instead of using S&P 500 index and bitcoin price to get a static result, DCC-GARCH provides an option to calculate dynamic relation. To check the robustness of the results, we use a variety of stock markets to analyze whether the dynamic relations vary for different countries. Furthermore, according to the volatilities, we calculate optimal weights of different region's stock to construct a portfolio containing bitcoin that minimizes the risks.

The present paper is organized as follows: Section 1 provides the literature reviews relating to bitcoin volatility, and section 2 gives a brief data description. Furthermore, section 3 introduces the main model and methods. Thereinafter, section 4 presents the empirical results. Finally, section 5 gives the conclusion and summary.

The main findings can be summarized as follows: First, after COVID-19, the volatility relation is strengthened and keeps a positive relation most of the time. Second, the developed countries' stock markets have a more significant positive relationship after COVID-19, while developing countries' stock markets have a relatively week relationship except for Brazil. Brazil's stock market has a strong positive relationship with Bitcoin volatility. Third, we build an optimal portfolio by incorporating both bitcoin and stock markets and calculate the optimal weight of bitcoin in the portfolio.

2 DATA

2.1 Data description

(1) Bitcoin price

Bitcoin price data series with a daily frequency is obtained from the CoinDesk website (coindesk.com). The sample spans from 3/12/2019 to 3/12/2021, covering the year before the COVID-19 breakout and the year after that. Moreover, we calculate the returns by comparing the close price difference among days.

(2) Stock index

We collect data of stock closing price covering 6 countries with a daily frequency. Specifically, stock prices are represented by the stock price indices from Investing (Investing.com). The countries we choose include 3 main developed countries (the United States, the United Kingdom, and Japan) and 3 developing countries (China, India, and Brazil).

2.2 Summary statistics

For each price series including Bitcoin price and 6 countries' stock index performance, we make price and index daily data summary and description as Table 1 shows. Moreover, the return data are represented by the daily return of bitcoin and daily change of stock index, which are shown in Table 2.

TABLE 1. DAILY BITCOIN PRICE AND STOCK INDEX

stats	Bitcoin	US	China	Japan	UK	India	Brazil
mean	12873	3174	3278	6578	1918	1336	1842
min	3846	2237	2692	4965	1428	876.1	1036
max	57637	3939	3874	8344	2211	1735	2429
std	10415	341.5	221.4	661.2	205	158.4	361
skewness	2.52	0.44	0.23	0.73	-0.25	0.28	-0.37
kurtosis	8.77	2.67	2.52	3.31	1.58	3.78	1.93

We can see that the Bitcoin price series is more volatile due to the higher standard deviation, skewness, and kurtosis. In this term, Bitcoin is riskier comparing with the stock market.

TABLE 2. DAILY BITCOIN AND STOCK INDEX RETURN

stats	Bitcoin	US	China	Japan	UK	India	Brazil
mean	0.003	7E-04	-0	3E-04	-0	0.001	-0
min	-0.37	-0.11	-0.04	-0.06	-0.11	-0.13	-0.18
max	0.149	0.086	0.071	0.074	0.089	0.088	0.164
std	0.039	0.015	0.013	0.012	0.014	0.015	0.027
skewness	-1.14	0.264	0.019	-0.19	-0.89	-0.75	-0.2
kurtosis	16.38	15.12	5.84	10.61	13.39	15.89	14.42

As Table2 showed, Bitcoin has the highest expected return as it has the highest mean of daily return, while the stock daily return means in China, UK and Brazil are negative. Similarly, the highest daily return volatility belongs to Bitcoin. It is noted that Bitcoin owes the highest expected return while bears the most risks.

3 METHODOLOGY

3.1 DCC-GARCH model

We apply the DCC-GARCH model proposed by Engle (2002)[5] to study the volatility relation between Bitcoin and segmented stock markets. One of the major advantages of this model is the detection of the dynamic conditional correlations over time, which make it possible to detect the dynamic Bitcoin fluctuation in response to stock market shock as more investors know about Bitcoin. Furthermore, the dynamic conditional correlations measure is appropriate to investigate

possible contagion effects due to herding behavior in emerging financial markets during crisis periods [see Corsetti et al.(2005)[6], Boyer et al.(2006)[7], and Syllignakis and Kourtas(2011)[8]]. Another advantage of the DCC-GARCH model is that this model estimates correlation coefficients of the standardized residuals and so accounts for heteroscedasticity directly (see Chiang et al., 2007[9]). DCC-GARCH can continuously adjust the correlation for the time-varying volatility without any bias from volatility. Therefore, it provides a superior measure for correlation (see Cho and Parhizgari, 2008[10])

The application of the DCC-GARCH model includes two steps: the first step is to estimate the univariate GARCH model, and the second step is the estimation of the dynamic conditional correlations.

The multivariate DCC-GARCH model is defined as follows:

$$H_t = \mu_t + H_t^{1/2}\varepsilon_t \quad [5] \tag{1}$$

$$\begin{cases} H_t = D_t R_t D_t \\ R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \quad [5] \\ D_t = diag(\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, \ldots, \sqrt{h_{NN,t}}) \end{cases} \tag{2}$$

Where $X^t = (X_{1t}, X_{2t}, \ldots, X_{Nt})$ is the vector of the past observations, H_t is the multivariate conditional variance, $\mu_t = (\mu_{1t}, \mu_{2t}, \ldots, \mu_{Nt})$ is the vector of conditional returns. $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt})$ is the vector of the standardized residuals, R_t is a N*N symmetric dynamic correlations matrix and D_t is a diagonal matrix of conditional standard deviations of return series (see Celik et al., 2012[11]).

$\rho_{ij,t}$ is determined by the geometrically weighted average of standardized residuals (The farther away from period T, the weight decreases geometrically).

$$\rho_{ij,t} = \frac{\sum_{s=1}^{t-1} \lambda^s \varepsilon_{i,t-s} \varepsilon_{j,t-s}}{\sqrt{(\sum_{s=1}^{t-1} \lambda^s \varepsilon_{i,t-s}^2)(\sum_{s=1}^{t-1} \lambda^s \varepsilon_{j,t-s}^2)}} \quad [11] \tag{3}$$

Where $\varepsilon_{i,t-s}$ is normalized disturbance term (the variance is normalized to 1), and λ^s is geometric weight. Written in matrix form, its dynamics are determined by the following two equations:

$$R_t = (diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}) \tag{4}$$

$$Q_t = (1 - \lambda_1 - \lambda_2)R + \lambda_1 \varepsilon_{t-1}\varepsilon_{t-1}' + \lambda_2 Q_{t-1} \quad [11] \tag{5}$$

3.2 Optimal portfolio

In the second part, to minimize the risk of investment, we construct a portfolio containing bitcoin and the stock markets. Investors can utilize the optimal portfolio to invest in bitcoin to hedge against the stock index portfolios. According to Akhtaruzzaman et al., 2020[12], the optimal weight of bitcoin in a portfolio of the country stock index portfolio i/Bitcoin at time t, denoted by $w^{(i/Bitcoin)}$.

$$w_t^{i/Bitcoin} = \frac{h_t^i - h_t^{i/Bitcoin}}{h_t^i - 2h_t^{i/Bitcoin} + h_t^{Bitcoin}} \ [12] \tag{6}$$

Where h_t^i, $h_t^{Bitcoin}$ and $h_t^{i/Bitcoin}$ are the conditional volatility of the country i's stock market portfolio, bitcoin, and covariance at time t, respectively.

4 EMPIRICAL RESULTS

4.1 The static correlation during the pre-and the post-COVID19 year

Table 3 and Table 4 show the static correlation between Bitcoin and some countries' stock index pre-and-post-COVID19. Comparing the correlations between pre-and-post the COVID 19 a significant increase of volatility correlation between Bitcoin and the stock index can be found. In the pre-COVID19 period, the correlation between bitcoin and the stock market is ambiguous, in which bitcoin has a negative correlation with the stock in the US, India, and Brazil while a positive correlation with the stock in China, Japan, and the UK. However, the correlation has changed after COVID19, all the correlations become positive in the year after the COVID19 break. Moreover, in the two most mature financial international markets, the US and UK, the correlations become high, at 0.2112 and 0.2866. Nevertheless, the volatility correlation between China stock market and bitcoin shows the smallest change after COVID19, increasing slightly from 0.0202 to 0.0799.

TABLE 3. THE VOLATILITY CORRELATION BEFORE COVID-19

	Bitcoin	US	China	Japan	UK	India	Brazil
Bitcoin	1.000						
US	-0.005	1.000					
China	0.020	0.226	1.000				
Japan	0.002	0.123	0.435	1.000			
UK	0.028	0.540	0.491	0.291	1.000		
India	-0.020	0.005	0.351	0.296	0.405	1.000	
Brazil	-0.045	0.554	0.276	0.089	0.531	0.120	1.000

TABLE 4. THE VOLATILITY CORRELATION AFTER COVID-19

	Bitcoin	US	China	Japan	UK	India	Brazil
Bitcoin	1.000						
US	0.211	1.000					
China	0.080	0.081	1.000				
Japan	0.148	-0.012	0.392	1.000			
UK	0.287	0.523	0.393	0.205	1.000		
India	0.065	0.227	0.520	0.186	0.469	1.000	
Brazil	0.236	0.508	0.321	0.057	0.570	0.448	1.000

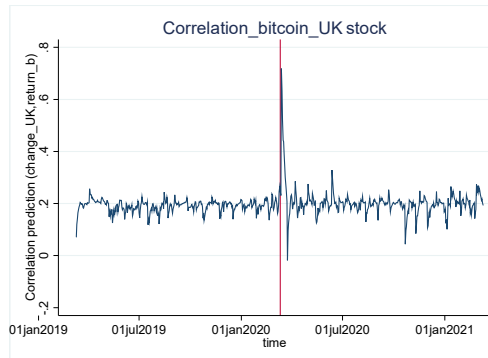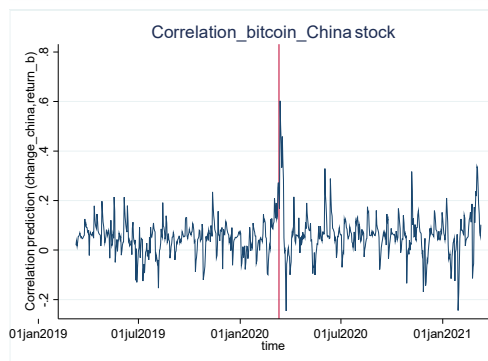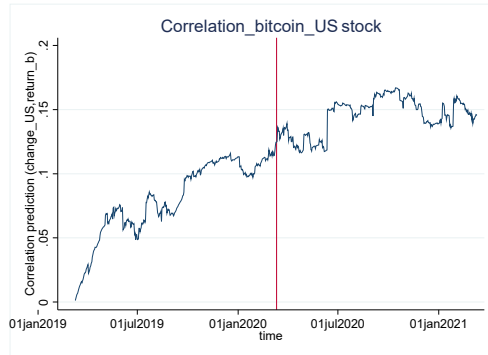## 4.2 The dynamic conditional correlation during the pre-and the post-COVID19 years

We compute the dynamic correlations between Bitcoin and the stock index of 6 countries by the DCC-GARCH model. Hence, we present 6 countries' DCCs in Figure 3, including 3 developed countries: the US, UK, and Japan, and 3 developing countries: China, India, and Brazil.
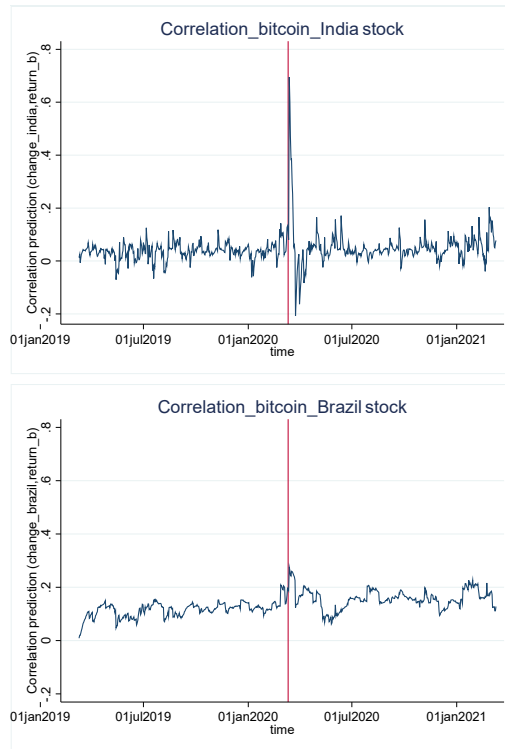
As the figure1 shows, the red line represents the announcement of the COVID19 break. On March 11, 2020, World Health Organization declared the COVID19 has constructed the global pandemic. Since then, the world realized the unprecedented virus would damage the economy and normal life. Thus, we choose this time point as a dividing line to compare what happened before and after COVID19. Additionally, one year was selected as an observation period, and the results were compared one year before and one year after the outbreak.

We can see that when the COVID19 was defined as a global pandemic, the volatility correlation increased dramatically, indicating that bitcoin and the stock markets experienced a shock simultaneously. Subsequently, the dynamic correlation fell to a negative value quickly. With the advance of time, DCCs recovered to a higher level, and the overall level after the outbreak was higher than the overall level before the outbreak.

For a short period, bitcoin could indeed act as a safe haven against stock market risk. Considering the price series, we can infer that bitcoin slowed and stopped the dramatic decrease before the stock market. That seems to mean that while bitcoin wasn't seen as a safe haven during the initial panic, people calmed down one month after the outbreak, making it somewhat of a safe haven against the stock market. By the time the panic had died down and stocks had rebounded, the dynamic volatility correlation between bitcoin and stocks had returned to relatively normal. At this point, Bitcoin gradually lost its role as a hedge against the stock market.

However, if we take a slightly longer view, bitcoin is not a very good haven. It is shown that the overall level of dynamic correlations increases significantly in the year after COVID19 in comparison with the year before the COVID19. This means that the correlation between stock market volatility and Bitcoin volatility in most countries became higher after the outbreak, which means that the stock market is increasingly linked to Bitcoin and bitcoin is not suitable as a hedge asset for a long period after the outbreak.

Correlation_bitcoin_US stock

Correlation_bitcoin_China stock

Correlation_bitcoin_UK stock

Correlation_bitcoin_Japan stock
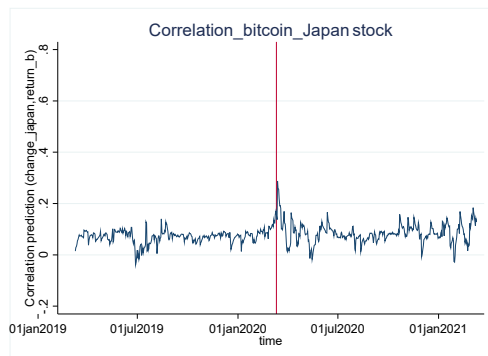
Fig. 1. DCC-GARCH relationships

As Figure2 shows, the first red vertical line represents March 23, 2018, and the second red vertical line represents March 11, 2020, the announcement day of COVID 19. If we look at DCCs in the US four years before the outbreak and one year after the outbreak, we can see that in addition to the sharp increase after the COVID19 outbreak, DCCs also rose to a high level between 2018 and 2019. Looking back at the major events of this period, we find that on March 23, 2018 (Beijing time), US President Donald Trump officially signed a memorandum of Understanding on trade with China at the White House, officially announcing the start of the trade war between China and the US. DCCs between Bitcoin and the US stock market showed similar growth during the crisis as they did during the COVID-19 pandemic, as the trade war between China and the US sparked fears about the economic outlook and stock markets began to fall in panic. This seems to presage an increase in DCCs between bitcoin and the stock market during the crisis, so bitcoin is not a good hedge against the stock market.

Fig. 2. DCC-GARCH relationships in the US

4.3 Optimal weight of bitcoin in portfolio

Results showed in Figure3, illustrate that the optimal weights of Bitcoin vary as time changes. Moreover, it shows, to maintain the expected return and minimize risks, we should increase the weights of Bitcoin in the early crisis period, approximately from April 2020 to July 2020. As the graph presented, during this pandemic period, the optimal weights of Bitcoin require a significant increase, implying that Bitcoin has a certain hedging effect in this short period.

Comparing the results of pre-COVID19 with the post-COVID19, the share of bitcoin required to be allocated after the outbreak is larger than that before the outbreak. However, as the epidemic progresses, the optimal proportion of overall bitcoin gradually declines. And the hedge effect is disappearing. After July 2020, the required level of bitcoin in the portfolio decreased to a low level. And one year after the outbreak, the weight of Bitcoin required is close to the level before the outbreak.

US

UK

Japan

Brazil
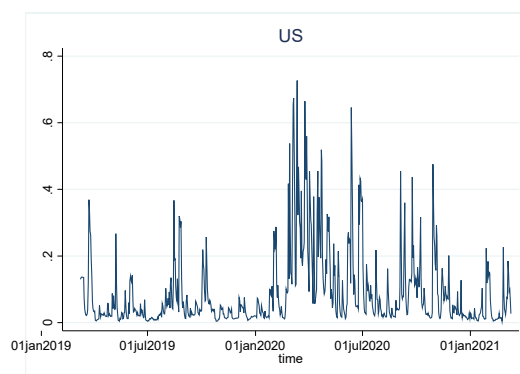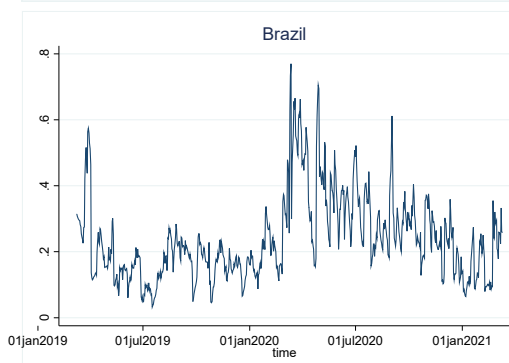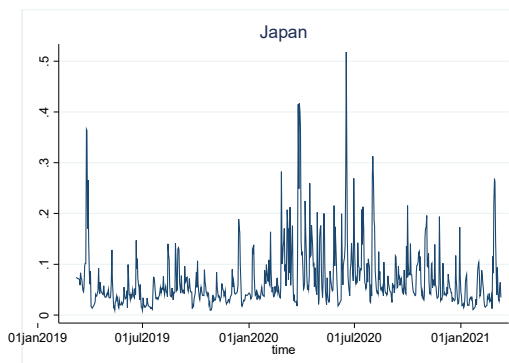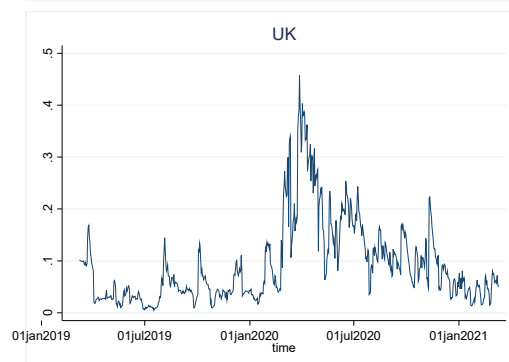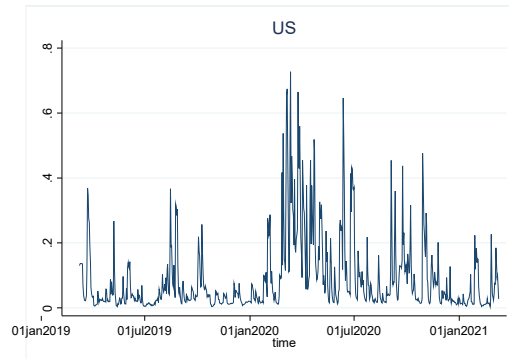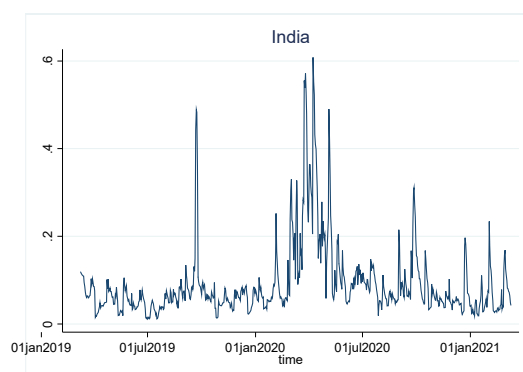
Fig. 3. Optimal weights of Bitcoin in the Stock-Bitcoin portfolios

5 CONCLUSION

This paper tries to test the difference in the conditional volatility correlation between periods before and after COVID-19 by employing the DCC-GARCH model. We examine the contagion effect of COVID-19 in three emerging stock markets, three developed stock markets, and the bitcoin market for the period before and after COVID-19, with a sample period from March 11, 2019, to March 11, 2021. We find that the conditional volatility correlation has a significant change after the COVID-19 outbreak. Generally, over a one-year sample period, overall DCC levels were higher in the year after the outbreak than that before the outbreak. More specifically, DCC rose rapidly at the beginning of the outbreak when the stock market and the bitcoin market had a contagion effect. However, bitcoin stopped falling soon, which led to the rapid decline of DCC. In the following time, DCC in each country increased slowly. In summary, there is a short period when bitcoin can act as a safe haven against the stock market. But overall, bitcoin and the stock market have become more closely linked since the outbreak, and bitcoin is not a good hedge against the stock market. Then we calculated the optimal allocation between bitcoin and the stock market in the stock-bitcoin portfolios, and the results support our earlier view. To increase the expected return of the portfolio after the COVID 19 outbreak, we can increase our bitcoin allocation from April to July 2020. However, after these periods, the optimal allocation of bitcoin is not high, which implying bitcoin is not a good hedge against the stock market.

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