Raw Material Inventory Optimization for Steel Enterprises Under Price Uncertainty

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Abstract. Due to the uncertainty of raw material prices, procurement and inventory costs account for a large proportion and are difficult to control in steel production. Aiming at the problem of raw material purchase and inventory in iron and steel enterprises with uncertain purchase price, in order to overcome the uncertainty of price and the conservatism of the solution, this paper establishes a robust optimization model for minimizing the total cost of purchase and inventory. Due to its inherent computational complexity, we introduced disturbance variables and employed duality theory to derive a tractable robust counterpart model. Finally, the model was applied to a real steel manufacturing enterprise to validate its effectiveness. The results indicate that, while the total cost of the robust model is slightly higher than that of the deterministic model for planning horizon of 3, 6, and 9, it becomes lower when the planning horizon is set to 12. For longer planning horizons, the robust model has a good effect on inventory and robustness under the condition of price fluctuations. Furthermore, we conducted an analysis of the impact of critical parameter thresholds on the total cost. The findings revealed that as the threshold value increases, the total cost also increases. Therefore, companies can select an appropriate threshold based on their specific circumstances to control costs.

Keywords: Robust optimization, Price uncertainty, Inventory management.

1 Introduction

Steel plays an important role in the construction of national economy and modern national defense. As a typical resource-consuming industry, raw material procurement and inventory cost is the main source of production cost of iron and steel products, and also the focus of many domestic iron and steel enterprises to reduce cost and increase efficiency.

Fig. 1. The process of steel production.

The main process of steel production is shown in **Fig. 1**. As an important raw material for the steel industry, iron ore has always been in the fluctuations, which has a great impact on the

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production cost of the steel industry, and it has brought difficulties to the management of raw material inventory. The more frequent the price of iron ore changes, the greater the impact on steel companies. Therefore, how to formulate scientific procurement and inventory decisions, reducing the total cost of enterprise procurement and inventory cost is essential for steel companies in the competitive market. In this context, it is necessary to study the management of raw material inventory under price uncertainty.

Varlas et al.[1] explored optimal inventory management strategies by comprehensively considering stochastic production, transportation, external demand, and sales loss. They introduced an analytical model based on Markov processes. Kawakami et al.[2] developed seasonal inventory management models for raw inventory of iron ore and coal for multiple suppliers and factories. Woo et al.[3] proposed a novel production-inventory control model for supply chain networks and formulated linear programming equations to solve this model. Zou et al.[4] developed a dynamic multi-objective optimization model to enhance coil resource utilization by optimizing total mismatch cost, residual inventory, and coil utilization, while accounting for various practical dynamic factors.

Huang et al.[5] established a spatiotemporal Markov model and utilized probabilistic chain adjustments to forecast inventory state changes, analyzing inventory variations during the multi-stage steel production process. Shujin et al.[6] integrated sales plans and historical data to construct a demand forecasting model and developed procurement and inventory optimization models with the objective of minimizing the total procurement and inventory costs.

Robust optimization model is an effective method to solve the problem containing uncertainty. The optimal solutions of the robust model are all feasible solutions of the model, that is, the robust optimization model has strong robustness.

Jackson et al.[7] addressed a single-warehouse, N-retailer, multi-period inventory allocation problem and obtained inventory strategies using robust optimization methods. Ruimin et al.[8] proposed a robust multi-objective mixed-integer nonlinear programming model considering cost parameters and demand uncertainty. Bertsimas and Thiele[9] first established a robust model to minimize procurement, inventory and shortage costs under the assumption of unknown demand distribution in the environment of a single enterprise, and then established a more general robust model under the network environment formed by multiple suppliers.

Based on the analysis of the current situation of raw material price and inventory in the steel industry, this paper proposes a robust optimization model which minimizes the total cost of raw material purchase and inventory when raw material price is uncertain. This model is effective in mitigating the volatility of market prices, and the obtained robust solution is less conservative compared to traditional robust methods.

The paper is organized as follows. In section 2, we build a robust model of raw material price uncertainty. In section 3, we apply this model to a real case and verify the effectiveness of the model. In section 4, we conclude this paper.

2 Robust Model of Raw Material Inventory

2.1 Problem Description

The paper investigates the development of procurement and inventory plans for a single steel enterprise and a single supplier under iron ore price uncertainty. For ease of model presentation, the assumptions underlying the model are presented below.

Relevant assumptions are as follows: 1) The price of raw materials will not change with different suppliers in the same ordering period; 2) All raw materials purchased in the current period arrive at the beginning of the period; 3) The funds and inventory capacity of steel enterprise are big enough.

2.2 Robust Model

The related notations in the model are shown in **Table 1**.

Table 1. Parameters description.

Parameter	Parameters description
$\tilde{P}_t$	unit variable price of iron ore in period *t* in the planning horizon
h	inventory holding cost per unit of iron ore
ss	safety stock
D_t	demand in period *t* in the planning horizon
T	collection of the planning horizon
Z_t	shortage in period *t*
s	unit shortage cost
I_t	inventory available at the beginning of period *t*
Q_t	inventory ordered in period *t*
U	the range of unit variable price

For iron and steel enterprises, the total cost of procurement and inventory is affected by the uncertainty of raw material prices. The following mathematical model is established for the inventory problem of price uncertainty:

$$\min \max_{\tilde{P}_t \in U, t\in T} \sum_{t\in T} \left(hI_{t+1} + sZ_t + \tilde{P}_t Q_t\right)$$

$$\text{s.t.} \quad I_{t+1} = I_t + Q_t - D_t \qquad t \in T \tag{1}$$

$$I_t \geq ss \qquad t \in T \tag{2}$$

$$Z_t = D_t - I_t - Q_t \qquad t \in T \tag{3}$$

$$I_t, Q_t, Z_t \geq 0 \qquad t \in T \tag{4}$$

where hI_{t+1} in the objective function refers to the inventory cost, that is, the related expenses incurred when raw materials are stored in the material yard; sZ_t refers to the out-of-stock cost, that is, the loss due to the interruption of the supply of raw materials; $\tilde{P}_t Q_t$ refers to the purchasing cost, that is, the cost to purchase raw materials; Constraint (1) represents the inventory balance constraint, which means that in each period of the planning horizon, the beginning inventory of raw materials plus the ordering stock of the current period minus the demand of the current period is the beginning inventory of the next period; Constraint (2) represents the safety stock constraint, which means that in order to prevent supply shortage in each period of the planning period, a certain safety stock will be set, and the stock of raw materials shall not be lower than the safety stock; Constraint (3) represents the shortage stock, which means that the sum of inventory and purchase quantity at the beginning of the current period is less than the demand of the current period.

The above model is nonlinear programming and difficult to solve, so we transform the model. The transformed model is established by introducing variable *C*. It can be seen that the above model and the transformed model are equivalent.

$$\min \quad C$$
$$\text{s.t.} \quad \max_{\tilde{P}_t \in U, t \in T} \sum_{t \in T} \left(hI_{t+1} + sZ_t + \tilde{P}_t Q_t \right) \leq C \qquad t \in T \tag{5}$$
Constraints (1) - (4).

The transformed model is generally difficult to solve directly, so it needs to be transformed into a convex optimization problem by mathematical optimization theory, that is, a robust equivalence problem.

We can see that only the unit cost of raw materials is uncertain, and other parameters are deterministic parameters in (5), so it can be rewritten as:

$$\sum_{t \in T} (hI_{t+1} + sZ_t) + \max_{\tilde{P}_t \in U, t \in T} \sum_{t \in T} \tilde{P}_t Q_t \leq C \tag{6}$$

Using historical data, the interval, median value, mean value and variance of the unit price of raw materials in each period can be obtained, so as to provide effective data information of the unit price for this model. Let the range of the uncertain set be as follows:

$$U = \left\{ P_t \middle| P_t \in \left[\overline{P}_t - \widehat{P}_t, \overline{P}_t + \widehat{P}_t \right], t \in T \right\} \tag{7}$$

Where $\overline{P}_t$ is the average unit price during the *t*th period, $\widehat{P}_t$ is the fluctuation range of unit price during the *t*th period, here is the median value of unit price during the *t*th period. Let the disturbance variable $\xi_t = \left(\tilde{P}_t - \overline{P}_t \right) / \widehat{P}_t$ be a random variable with [-1,1] symmetrical distribution, then $\tilde{P}_t = \overline{P}_t + \widehat{P}_t \xi_t$, the uncertain disturbance can be expressed as:

$$\begin{aligned} \max \quad & \sum_{t=1}^{T} \widehat{P}_t \, Q'_t \\ \text{s.t.} \quad & -Q_t \leq Q'_t \leq Q_t \quad t \in T \\ & Q'_t \geq 0 \qquad\qquad\quad t \in T \end{aligned} \tag{8}$$

where $Q'_t = Q_t \xi_t$.

There are extreme cases in the uncertain set U, but in the actual steel raw material market, such cases rarely occur, so the disturbance variable ξ_t takes the worst case rarely. According to the above formula, the objective function of the minimization problem solved larger, and the solution result is more conservative. In order to adjust the robustness of the conservative level of the solution in the model, the constraint parameter Γ is introduced to describe the variation range of the uncertain parameter, and each period has a constraint condition of the constraint parameter Γ:

$$U(\Gamma_t) = \{ P_t | P_t \in \left[\overline{P}_t - \xi_t \widehat{P}_t, \overline{P}_t + \xi_t \widehat{P}_t \right], \ \sum_{k=1}^{t} \xi_k \leq \Gamma_t, 0 \leq \xi_k \leq 1 \} \tag{9}$$
$$t \in T \ \ k \in \{1, 2, \cdots, t\}$$

Therefore, the uncertain disturbance without considering the order can be expressed as:

$$\begin{aligned} \max \ & \sum_{t=1}^{T} \widehat{P}_t \, \xi_t \\ \text{s.t.} \ & \textstyle\sum_{k=1}^{t} \xi_k \le \Gamma_t \qquad t \in T \\ & 0 \le \xi_t \le 1 \qquad t \in T \end{aligned} \tag{10}$$

This model is a linear programming model. According to the strong duality in the dual theory of linear programming, that is, if the original problem (dual problem) has a definite optimal solution, then the dual problem (original problem) also has a definite optimal solution. Therefore, this model can be expressed as the following formula after being converted by dual theory, and the objective function value of the optimal solution is the same:

$$\begin{aligned} \min \ & C \\ \text{s.t.} \ & \textstyle\sum_{t\in T}(hI_{t+1} + sZ_t) + \sum_{t\in T} Q_t(\bar{P}_t + \Gamma_t y_t + m_t) \le C \\ & \textstyle\sum_{k=1}^{t} y_k + m_t \ge \widehat{P}_t \qquad t \in T \ \ k \in \{1,2,\cdots,t\} \\ & \text{Constraints(1)-(3)} \\ & y_t, m_t, I_t, Q_t, Z_t \ge 0 \qquad t \in T \end{aligned} \tag{11}$$

So far, this section converts the robust optimization model into a deterministic linear programming model without uncertain parameters.

3 Case Study

3.1 Historical Price Analysis of Iron Ore

According to the actual price data of 62% grade iron ore in Beijing Iron Ore Trading Center from 2018 to 2021, we analyze and calculate the price data. By illustrating the unbiasedness and consistency, we can think that it is reasonable and feasible to replace the overall expectation and variance with the sample mean and variance. Therefore, the mean and variance of monthly iron ore prices can be obtained by analyzing the collected iron ore prices. The price ranges, mean and variance of monthly iron ore prices as shown in **Table 2**.

Table 2. Price ranges and the mean and variance of iron ore prices.

Month	Price (yuan/ton)	Statistics (μ|σ^2)	Month	Price (yuan/ton)	Statistics (μ|σ^2)
1	[521,1159]	739|60640.4	7	[453,1528]	914|122882.8
2	[521,1193]	740|52711.2	8	[475,1275]	825|56367
3	[445,1191]	729|63614.6	9	[490,1091]	777|29731.7
4	[440,1305]	747|78906.7	10	[532,926]	757|19186.4
5	[450,1644]	836|139479.5	11	[520,905]	688|14319.4
6	[445,1528]	885|143682.0	12	[540,1159]	756|35152.0

As shown in **Table 2**, we can see that the mean of prices is not only large, but also the change in the mean is large. In addition, the range and variance of prices are large, indicating that the change of prices is not stable.

3.2 Application of Robust Models

According to the iron ore purchasing and inventory data of steel enterprise B and the planned annual demand in 2022, we apply the model in Section 2 to solve the problem. The parameters in the model are selected as follows:

(1) A month is a period, the planning period $T = 3,6,9,12$;

(2) Steel enterprise needs 1.5 times more iron ore than the planned amount of blast furnace pig iron;

(3) At present, some large domestic iron and steel enterprises take the safety inventory of raw materials for about five days as the optimal inventory structure. Therefore, in this model, according to the average iron ore demand in the second half of 2022, the value of safety inventory ss is 216000 tons;

(4) The site where enterprise B stores iron ore is its own storage yard, and the cost of its own storage yard in the production plant is about 0.6(yuan/(ton·day)). Therefore, the monthly unit inventory cost is h=18(yuan/(ton·day));

(5) In the production of iron and steel enterprises, raw materials are not allowed to be out of stock, so we take the value of unit out of stock cost s to infinity.

According to the above data, CPLEX optimization software is used to solve the problem with actual data, and the obtained objective function values of the deterministic model and robust model are shown in **Table 3**.

Table 3. Total cost.

Planning horizon T	Total cost of deterministic models(yuan)	Total cost of robust models(yuan)
3	2.6709×10^{9}	2.8458×10^{9}
6	5.89×10^{9}	6.1479×10^{9}
9	8.7105×10^{9}	9.3139×10^{9}
12	1.5121×10^{10}	1.2194×10^{10}

The robust optimization model seeks the optimal solution in the worst case. Compared with the solution result of the deterministic inventory model, the objective value of the robust optimization inventory model is larger when the planning period $T = 3,6,9$, while the objective function value of the robust model is smaller when $T = 12$, which is because the deterministic model modeling is sometimes inaccurate. The robust optimization model established by statistical analysis is more in line with the reality. The robust optimization model can make the total cost of procurement and inventory change less when the price of raw materials fluctuates, avoid the more serious loss caused by the uncertainty of raw material price fluctuations, so as to enhance the anti-risk ability of raw material procurement and inventory of iron and steel enterprises.

3.3 Impact of Budgets of Uncertainty

Taking the planning period as an example, which is set to 6, this section observes the impact of budgets of uncertainty on total costs of robust models by changing Γ based on actual data. The changes in total costs are shown in **Fig. 2**.

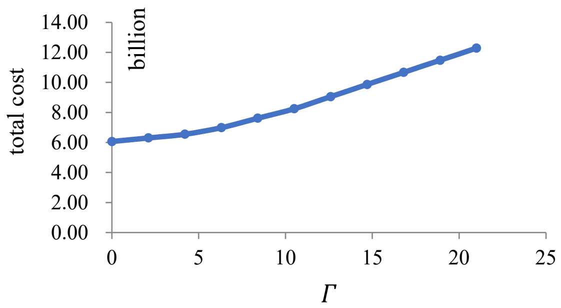

Fig. 2. The changes in total costs of robust model.

As shown in **Fig. 2**, we can observe that the total cost of robust models increases as the budgets of uncertainty increases. This is because a larger budget of uncertainty results in a more conservative strategy, which leads to higher costs under the uncertainty of iron ore prices. In practice, companies can choose appropriate budgets of uncertainty to mitigate price uncertainty while avoiding excessive conservatism.

4 Conclusion and Future Work

In this paper, we propose a robust optimization model under price uncertainty to minimize the total cost of purchase and inventory. To prevent overly conservative solutions, the model imposed constraints on disturbance variables at each period and derived a solvable robust counterpart model. Based on an analysis of historical pricing data, the model was applied to a steel enterprise to validate its effectiveness. The results indicate that for longer planning horizons, the robust model exhibits superior robustness and lower total costs compared to the deterministic model. Moreover, we analyzed the impact of budgets of uncertainty variations on the total cost under the robust model. The results indicate that the total cost increases with the increase of the budgets of uncertainty. In practice, companies can select an appropriate threshold to control costs according to their specific requirements.

Future work will consider more constraints, such as storage capacity constraints and funding constraints. In addition, consideration will be given to the impact of purchasing changes by steel companies on other companies in the relevant supply chain.

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References

[1] Varlas, G., Vidalis, M., Koukoumialos, S., Diamantidis, A.: Optimal inventory control policies of a two-stage push–pull production inventory system with lost sales under stochastic production, transportation, and external demand. Top 29(3), 799-832(2021).

[2] Kawakami, K., Kobayashi, h., Nakata, K.: Seasonal Inventory Management Model for Raw Materials in Steel Industry. INFORMS Journal on Applied Analytics 51(4), 312-324(2021).
[3] Woo, Y. B., Moon, I., Kim, B. S.: Production-Inventory control model for a supply chain network with economic production rates under no shortages allowed. Computers & Industrial Engineering 160, 107558(2021).
[4] Zou, F., Guo, Q., Yen, G. G.: Dynamic evolutionary multiobjective optimization for open-order coil allocation in the steel industry. Applied Soft Computing 146, 110661(2023).
[5] Huang, J., Meng, Y., Liu, F., Liu, C., Li, H.: Modeling and predicting inventory variation for multistage steel production processes based on a new spatio-temporal Markov model. Computers & Industrial Engineering 164, 107854(2022).
[6] Shujin, J., Jian, Y., Jing, W., Bin, D.: Demand prediction and purchase optimization decision model for alloys in steel making. Baosteel Technical Research 16(4), 33-39(2022).
[7] Jackson, P. L., Muckstadt, J. A., Li, Y.: Multiperiod stock allocation via robust optimization. Management Science 65(2), 794-818(2019).
[8] Ruimin, M. A., Lifei, Y. A. O., Maozhu, J. I. N., Peiyu, R. E. N., Zhihan, L. V.: Robust environmental closed-loop supply chain design under uncertainty. Chaos, Solitons and Fractals 89(15), 195-202(2016).
[9] Bertsimas, D., Thiele, A.: A Robust Optimization Approach to Inventory Theory. Operations Research 54(1), 150-168(2006).