

Optimal Trade-in Return Policy Decision Based on Numerical Simulation

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Abstract. In order to retain existing customers and drive sales, firms introduce trade-in programs where participating consumers (referred to as trade-in consumers) have the option to return and request a refund for newly purchased products if they are dissatisfied. This study utilizes two mathematical analytical models and numerical simulation analysis to investigate the viability of implementing a trade-in return policy for trade-in consumers. Our findings indicate that when the hassle cost associated with consumer returns is low, implementing a trade-in return policy leads to higher profitability for the firm. Conversely, when the hassle cost is high, the firm benefits more from not implementing such a policy. Furthermore, the decision to implement a trade-in return policy is influenced by the product's durability, with higher durability increasing the firm's inclination to adopt such a policy. Lastly, implementing a trade-in return policy contributes to higher consumer surplus.

Keywords: Trade-in; Return policy; Simulation; Decision making

1 Introduction

Trade-in programs are widely implemented in various industries such as electronics, fashion apparel, and automotive, as they can help businesses retain existing customers and promote the sales of new products. For instance, Apple has implemented the "Apple Trade In" program, where consumers who possess a used iPhone and intend to purchase a new one can return their used device to Apple and receive a trade-in rebate that can be used towards their new purchase. Furthermore, with the development of e-commerce, consumer returns have become a prominent issue. For example, data shows that the return rate for offline products in physical stores is approximately 8%, while the return rate for online products is close to 25% [1]. Therefore, there is no doubt that whether or not a company provides a return policy for consumers is an extremely important question.

In recent years, scholars have conducted research on trade-in programs from various perspectives. For instance, Ray et al. analyzed and surveyed optimal product pricing and trade-in discounts [2]. Xiao and Zhou studied the optimal trade-in program when trade-in for cash, upgrade, and hybrid trade-in programs existed. [3]. Dong et al. explored within-brand and cross-brand trade-in options [4]. Zhu et al. explored optimal trade-in policies in a competitive environment [5]. Huang et al. studied the economics of the certified pre-owned (CPO) program in the context of trade-in [6]. In the field of consumer returns research, Chen and Chen investigated optimal channel selection and consumer return policies for retailers [7]. Yang and

Ji considered three innovative mechanisms related to effectively managing consumer returns for omnichannel retailers with both online channels and physical stores [8]. Wang and He studied the optimal decisions for modularity, price, and return policies in a dual-channel supply chain under mass customization [9]. Gans and De Giovanni [10] and Gans and De Giovanni [11] focused on consumer returns across different channels. He et al. explored omnichannel retail operations for refurbished consumer returns [12].

Based on the background mentioned above, the main research questions are as follows: (1) How does the implementation of a trade-in return policy by a firm affect the choice of replacement consumers considering joining the trade-in program? (2) Considering consumer returns, should a firm implement a trade-in return policy? How can the optimal product price and trade-in rebate be determined? (3) Which strategy is more beneficial for consumers? To address these questions, this paper considers a monopolistic firm that offers a trade-in program and can choose to implement a trade-in return policy for trade-in consumers. The paper mathematically models the scenarios of implementing or not implementing the policy and then conducts numerical simulation experiments to explore the issues.

2 Model setup

This paper primarily investigates whether a firm that offer a trade-in program should implement a return policy for trade-in consumers. Therefore, two scenarios are considered: (1) No trade-in return policy implementation (Model N), and (2) Implementation of trade-in return policy (Model R). The firm sells new products to consumers at a retail price of p and provides a trade-in program. The production cost of the new product is c , and the trade-in rebate is r . Consumers have the option to return the product based on their satisfaction. Our notation is summarized in Table 1. Before establishing our mathematical models, we consider the following scenario and make the necessary assumptions.

Table 1. Notation

Symbol	Definition
v	Consumers' valuation of a new product, $v \sim U(0,1)$
δ	Product durability, $0 < \delta < 1$
α	Product satisfaction rate, $0 < \alpha < 1$
t	Consumers' hassle cost of returning the product
s	Unit salvage value of the used product
c_n/c_r	Unit production cost/return cost of the new product
M	Proportion of new consumers, $0 < M < 1$
U_j^i	Consumers' utility
D_j^i	Demand for products
R_j^i	Return quantity for products
π^i	Profit for the firm
i	$i \in N, R$, where N and R represent no trade-in return policy and trade-in return policy, respectively

Symbol	Definition
j	$j \in n, r$, where n and r represent new consumers and replacement consumers, respectively
Decision variables	
p^i	Retail price of the new product
r^i	Trade-in rebate

Assumption 1. Without loss of generality, we assume that the total market is normalized to 1, and there are two types of consumers in the market: new consumers (who do not possess any used products) and replacement consumers (who already have a used product), accounting for proportions M and $1 - M$, respectively.

Assumption 2. We assume consumers' valuation for the new product is v and $v \sim U(0,1)$. We use δ to represent the value discount rate for used products, which also indicates the product's durability. Therefore, the consumers' valuation of the used product is δv .

Assumption 3. If a consumer is satisfied with the newly purchased products, he/she will not return it. We assume the product satisfaction rate is $\alpha (0 < \alpha < 1)$. If the consumer is dissatisfied with the product, similar to Fan et al. [13], the value of the product to the consumer is assumed to be 0. And if the consumer chooses to return the product, he/she will incur a hassle cost t , and the firm will incur a return cost c_r . For new consumers, the firm implements a full refund policy. For replacement consumers who participate in the trade-in program, when the firm does not implement a trade-in return policy (Model N), they are unable to choose to return the product. However, when the firm implements a trade-in return policy (Model R), if a replacement consumer is dissatisfied with the product, he/she also enjoys a full refund policy. The firm will refund the actual amount paid by the consumer $p - r$ and the trade-in rebate r . The trade-in rebate can be used by the consumer for the next purchase. Moreover, we assume that $t < p$ to ensure that consumers choose to return the product when they are dissatisfied (if the firm offers a return policy).

Assumption 4. After the firm collects a used product, it obtains its salvage value s . And when a trade-in consumer returns the newly purchased product, the firm does not refund the used product itself.

Based on the assumptions mentioned above, the consumer behavior and product demand can be described as follows. For new consumers purchasing a new product, when they are dissatisfied with their purchase, they choose to return the product, resulting in a utility $U_n^i = \alpha(v - p^i) - t(1 - \alpha)$, $i \in \{N, R\}$. Only when $U_n^i \geq 0$, new consumers will purchase the new product. Therefore, the demand of the new product for new consumers is $D_n^i = M \left(1 - \frac{\alpha p^i + (1-\alpha)t}{\alpha}\right)$, and the return quantity for new consumers is $R_n^i = (1 - \alpha)D_n^i$. When the firm does not implement a trade-in return policy, for replacement consumers participating in the trade-in program, they are unable to return the dissatisfactory product. Therefore, their utility derived from participating in the trade-in program is $U_r^N = \alpha(v - p^N + r^N - \delta v) + (1 - \alpha)(0 - p^N + r^N - \delta v)$. Only when $U_r^N \geq 0$, replacement consumers will choose to participate in the trade-in program. Thus, the trade-in demand for replacement consumers is $D_r^N = (1 - M) \left(1 - \frac{p^N - r^N}{\alpha - \delta}\right)$. When the firm implements a trade-in return policy, for replacement consumers, if they are dissatisfied with the product they obtained through the trade-in, they can return it. The firm will not refund the used product itself, but it will refund the trade-in rebate. Consequently, the

utility obtained by replacement consumers through the trade-in program is $U_r^R = \alpha(v - p^R + r^R - \delta v) + (1 - \alpha)(r^R - \delta v - t)$. Only when $U_r^R \geq 0$, replacement consumers will choose to participate in the trade-in program. Hence, in this case, the trade-in demand for replacement consumers is $D_r^R = (1 - M)\left(1 - \frac{\alpha p^R - r^R + (1 - \alpha)t}{\alpha - \delta}\right)$, and the return quantity for replacement consumers is $R_r^R = (1 - \alpha)D_r^R$. Moreover, trade-in programs are not only crucial for the firm but also closely related to consumers. Therefore, consumer surplus is a key area to explore when examining trade-in practices. Specifically, consumer surplus consists of two components: consumer surplus from new consumers and consumer surplus from repeat consumers. Under Model N and Model R, the consumer surplus can be derived as follows, respectively:

$$CS^N = M \int_{\frac{\alpha p^N + (1 - \alpha)t}{\alpha}}^1 U_n^N dv + (1 - M) \int_{\frac{p^N - r^N}{\alpha - \delta}}^1 U_r^N dv \quad (1)$$

$$CS^R = M \int_{\frac{\alpha p^R + (1 - \alpha)t}{\alpha}}^1 U_n^R dv + (1 - M) \int_{\frac{\alpha p^R + (1 - \alpha)t - r^R}{\alpha - \delta}}^1 U_r^R dv \quad (2)$$

Next, we will establish mathematical analysis models for both Model N and Model R, wherein the firm simultaneously determines the price of the new product p^i and trade-in rebate r^i to maximize its profit π^i , $i \in \{N, R\}$.

2.1 No trade-in return policy implementation (Model N)

When a trade-in return policy is not implemented, the profit function of the firm can be represented as follows:

$$\max \pi^N(p^N, r^N) = (p^N - c_n)(D_n^N - R_n^N) - c_r R_n^N + (p^N - c_n - r^N + s)D_r^N \quad (3)$$

By solving the model, the firm's optimal decisions are shown in Theorem 1.

Theorem 1. *When the firm does not implement a trade-in return policy, the optimal price of the new product and trade-in rebate are as follows: $p^{N*} = \frac{\alpha(1+c_n)+(1-\alpha)(c_r-t)}{2\alpha}$, $r^{N*} = \frac{\alpha(1-\alpha+\delta+s)+(1-\alpha)(c_r-t)}{2\alpha}$.*

The optimal product demand and firm's profit can be obtained from Theorem 1, as shown in Table 2.

Table 2. Optimal product demand and firm's profit under Model N and Model R

No trade-in return policy (Model N)	Trade-in return policy (Model R)
$D_n^{N*} = \frac{M(\alpha(1-c_n)-(1-\alpha)(c_r+t))}{2\alpha}$	$D_n^{R*} = \frac{M(\alpha(1-c_n)-(1-\alpha)(c_r+t))}{2\alpha}$
$D_r^{N*} = \frac{(1-M)(s+\alpha-\delta-c_n)}{2(\alpha-\delta)}$	$D_r^{R*} = \frac{(1-M)(\alpha(1-c_n)-(1-\alpha)(c_r+t)+s-\delta)}{2(\alpha-\delta)}$
$\pi^{N*} = \frac{(1-M)\alpha(c_r-s-\alpha+\delta)^2 + M(\alpha-\delta)(\alpha(1-c_n)-(1-\alpha)(c_r+t))^2}{4\alpha(\alpha-\delta)}$	$\pi^{R*} = \frac{(1-M)\alpha(\alpha(1-c_n)-(1-\alpha)(c_r+t)-\delta+s)^2 + M(\alpha-\delta)(\alpha(1-c_n)-(1-\alpha)(c_r+t))^2}{4\alpha(\alpha-\delta)}$

2.2 Implementation of trade-in return policy (Model R)

When a trade-in return policy is implemented, the profit function of the firm can be represented as follows:

$$\max \pi^R(p^R, r^R) = (p^R - c_n)(D_n^R - R_n^R + D_r^R - R_r^R) - c_r(R_n^R + R_r^R) + (s - r^R)D_r^R \quad (4)$$

By solving the model, the firm's optimal decisions are shown in Theorem 2.

Theorem 2. *When the firm implements a trade-in return policy, the optimal price of the new product and trade-in rebate are as follows: $p^{R*} = \frac{\alpha(1+c_n)+(1-\alpha)(c_r-t)}{2\alpha}$, $r^{R*} = \frac{s+\delta}{2}$.*

The optimal product demand and firm's profit can be obtained from Theorem 2, as shown in Table 2.

3 Simulation analysis

In this section, we conduct numerical simulations on the optimal product quantity, firm's profit, the difference in firm profit between the two scenarios, and consumer surplus. We analyze the impact of various important parameters on these outcomes. The parameter values selected based on the assumptions mentioned earlier are presented in Table 3.

Table 3. Parameter values

	M	c_r	s	α	c_n	δ	t
Figure 1, 2(a)	0.6	0.1	0.45	0.7	0.4	0.6	[0, 0.6]
Figure 2(b), 3(a), 4(a)	0.6	0.1	0.45	0.7	0.4	[0, 0.7]	0.1, 0.15, 0.2
Figure 3(b)	0.6	0.1	0.45	0.7, 0.8, 0.9	[0.1, 0.6]	0.6	0.1
Figure 4(b)	0.6	0.1	0.45	[0.6, 1]	0.4	0.6	0.1

3.1 Comparison of optimal product quantities

Figure 1 demonstrates the impact of consumer hassle cost of returns on optimal product quantities for replacement consumers in two scenarios. The following conclusions can be drawn: (1) When the consumer hassle cost is low, implementing a trade-in return policy in Model R significantly encourages replacement consumers to participate in the trade-in program. Despite the possibility of returns, the actual quantity of new products sold to replacement consumers through the trade-in program remains higher compared to when the trade-in return policy is not implemented. In Model N, the optimal product demand for replacement consumers is lower than the optimal product demand for replacement consumers minus the return quantity in Model R. (2) For a moderate hassle cost, implementing a trade-in return policy in Model R still stimulates more replacement consumers to participate in the trade-in program. However, the increased hassle cost reduces the effectiveness of the trade-in return policy, resulting in a lower quantity of new products sold to replacement consumers compared to when the trade-in return policy is not implemented. In Model N, the optimal product demand for replacement consumers surpasses the actual quantity of new products sold to replacement consumers in Model R, but it remains lower than the optimal product demand for replacement consumers in Model R. (3)

Under a high hassle cost, the incentive for replacement consumers to participate in the trade-in program diminishes, rendering the trade-in return policy less effective. In this case, the optimal product demand for replacement consumers in Model N exceeds that in Model R.

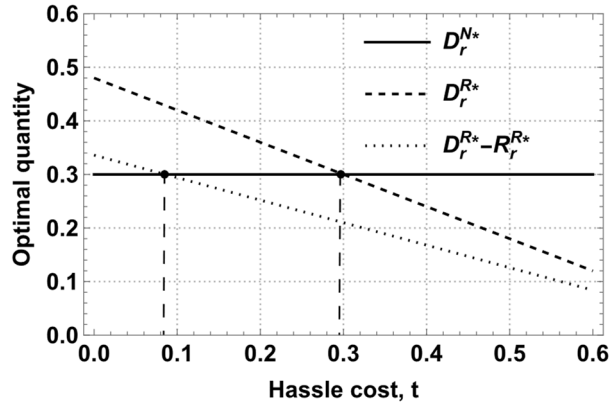


Fig.1. Comparison of optimal product quantities under Model N and Model R.

3.2 Comparison of the firm's optimal profit

Figure 2 illustrates how consumer hassle cost of returns and product durability impact the optimal profit for two scenarios. The following observations can be made: (1) When the consumer hassle cost is low, implementing a trade-in return policy enables the firm to generate higher profits. Conversely, when the consumer hassle cost is high, not implementing the trade-in return policy leads to greater profitability. This is because a lower consumer hassle cost stimulates the demand for trade-in among replacement consumers in Model R, making it advantageous for the firm to implement the trade-in return policy and provide superior after-sales service. However, if the consumer hassle cost is high, the demand for trade-in among replacement consumers decreases in Model R, and the firm cannot attract their participation through improved after-sales service. Hence, not implementing the trade-in return policy becomes more beneficial. (2) As the consumer hassle cost increases, the profits for both scenarios decline. Therefore, firms should aim to simplify the return process, offer convenient return options, and provide support to minimize the consumer hassle cost of returns. (3) As product durability increases, the firm's profit exhibits an initial decline followed by an increase. This is attributed to the following factors: On one hand, higher product durability encourages replacement consumers to continue using pre-owned products rather than participating in the trade-in program, thereby reducing the firm's profitability (negative impact). On the other hand, improved product durability also attracts a larger customer base, leading to increased profits from the sale of new products (positive impact). Consequently, when product durability is relatively low, the negative impact outweighs the positive impact, resulting in an overall decrease in the firm's profit. Conversely, when product durability is higher, the positive impact becomes dominant, contributing to an overall increase in the firm's profit.

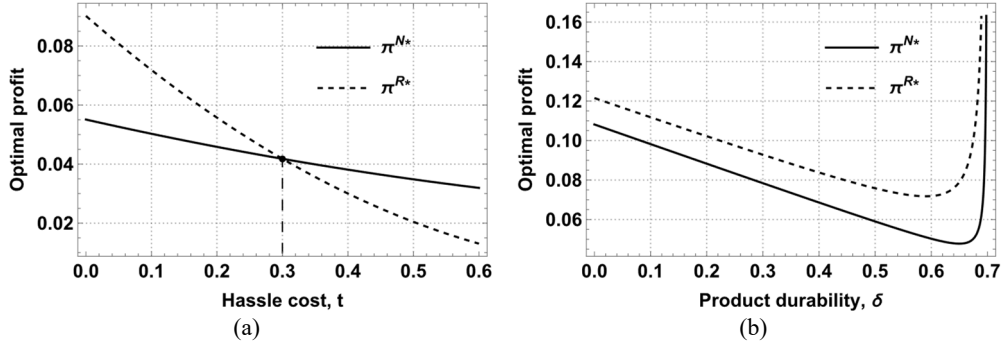


Fig.2. Comparison of the firm's optimal profit under Model N and Model R.

3.3 Analysis of the difference in the firm's profit between Model N and Model R

Figure 3 illustrates the impact of product durability, consumer hassle cost of returns, product production cost, and product satisfaction rate on the difference in the firm's profit between Model N and Model R ($\pi^{R*} - \pi^{N*}$). The following observations can be made: (1) Increasing product durability leads to a higher difference in the firm's profit. This suggests a greater inclination for the firm to implement a trade-in return policy in order to encourage consumer participation. (2) Higher consumer hassle cost of returns results in a reduced difference in the firm's profit. Therefore, when implementing a trade-in return policy, the firm should consider providing convenient return options to minimize the cost and inconvenience for consumers. (3) The difference in the firm's profit initially increases and then decreases as production costs rise. Excessively high or low production costs are unfavorable for firms implementing a trade-in return policy. (4) As the product satisfaction rate increases, the impact of production costs on the difference in the firm's profit diminishes. This is because higher product satisfaction reduces the demand for the return policy, thereby minimizing its influence on the firm's profit.

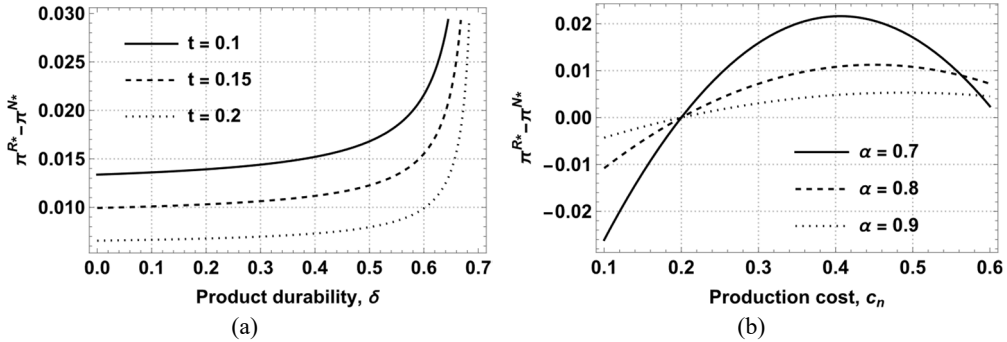


Fig.3. Comparison of the difference in the firm's optimal profit between Model N and Model R.

3.4 Comparison of consumer surplus

Figure 4 depicts the influence of product durability and product satisfaction rate on consumer surplus under two scenarios. The findings indicate that implementing a trade-in return policy leads to higher consumer surplus. With increasing product durability, consumer surplus initially

decreases and then increases. Similarly, as the product satisfaction rate rises, consumer surplus initially declines and subsequently rises under Model N, while it consistently decreases under Model R.

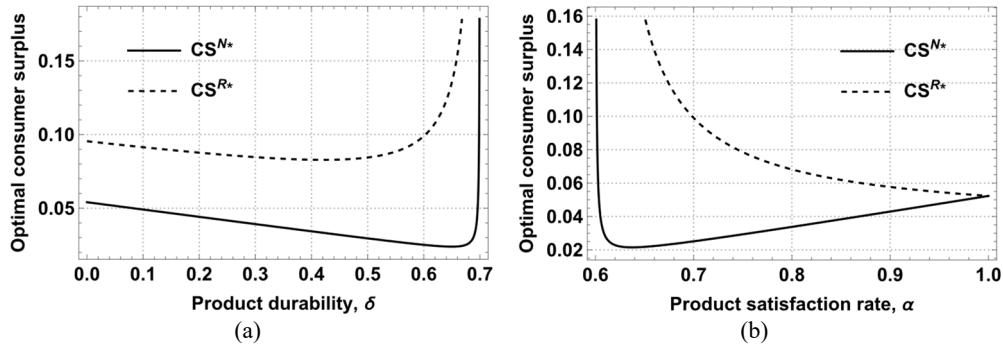


Fig.4. Comparison of consumer surplus under Model N and Model R.

4 Conclusion

To retain existing consumers and drive new product sales, firms are increasingly proactive in implementing trade-in programs. Motivated by the popularity of such programs and the practical significance of consumer returns, this study primarily investigates the motivations behind firms adopting trade-in return policies for replacement consumers who possess used products. By employing optimization and simulation analysis, the study draws the following key conclusions and managerial insights: (1) When the consumer hassle cost is low, implementing a trade-in return policy leads to higher firm profits. Conversely, when the consumer hassle cost is high, the firm benefits more from not implementing such a policy. Moreover, as consumer hassle costs increase, profits decline in both scenarios. Thus, firms should streamline the return process, offer convenient return options, and support consumers to minimize hassle costs associated with returns. (2) Higher product durability encourages firms to implement trade-in return policies, thereby incentivizing consumers to trade in their used products. (3) Implementing a trade-in return policy results in higher consumer surplus compared to not implementing one.

Given the limitations of this study, future research can expand in two areas. Firstly, real-life replacement consumers may possess used products of varying quality levels. Hence, future studies could consider the heterogeneous quality of used products within trade-in return policies. Secondly, this study solely focuses on the trade-in return policy of a monopolistic firm, thus future research can explore competitive scenarios as well.

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