Ecological Dynamics and Stability Analysis of Predator-Prey Systems under the SEIR Infectious Disease Model

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Abstract. This paper investigates the stability of the Leslie-Gower predator-prey model under the influence of the SEIR infectious disease model. By incorporating the SEIR infectious disease model, the predator population is divided into four states: susceptible, exposed, infected, and removed. A predator-prey dynamic model is then established. Through dimensionless processing and discretization methods, the equilibrium points of the model and their stability are analyzed. The eigenvalues of the Jacobian matrix are computed to determine the stability of the equilibrium points, and numerical simulations are used to demonstrate the dynamic behavior of the system under different parameter conditions. The results indicate that the predation rate and disease transmission rate have significant effects on the stability of the system. Reducing these two parameters appropriately can stabilize the system.

Keywords: SEIR Infectious Disease Model, Predator-Prey Model, Stability, Numerical Simulation

1 Introduction

The predator-prey model is one of the classical models in ecology used to describe population interactions. In traditional Lotka-Volterra and Leslie-Gower models, the interaction between predators and prey is simplified to the growth of prey and the predation behavior of predators [1-3]. However, in real ecological systems, predator populations may be affected by infectious diseases, which significantly alter the dynamic characteristics of the population. To more accurately describe this complex ecological phenomenon, researchers have proposed various extended models to better reflect the interactions between predator and prey populations and the spread of infectious diseases in actual ecosystems [4,5].

In recent years, the SEIR model, as a classic infectious disease model, has been widely used to describe the spread of diseases in populations [6,7]. The SEIR model, by introducing the exposed (E) state, can more accurately describe the transmission process of infectious diseases with an incubation period [8]. This paper builds upon the traditional Leslie-Gower predator-prey model by incorporating the SEIR infectious disease model, dividing the predator population affected by the disease into four states: susceptible (y_S), exposed (y_E), infected (y_I), and removed (y_R), and develops a new predator-prey model.

CONF-CIAP 2025, January 17-23, Eskisehir, Turkey

DOI 10.4108/eai.17-1-2025.2355242

2 Model Construction

In the construction of the model, the following assumptions are made: the predation relationship between predators and prey follows the classical Leslie-Gower model [9,10]. Infectious diseases are transmitted within the predator population through contact, with the infection rate related to the frequency of contact between predator individuals. After being infected, the predation ability of the predator is significantly weakened. The specific model equations are as follows:

2.1 Dynamic Equation of the Prey Population

$$\frac{dx}{dt} = x\left(\frac{r_1}{1+ky} - \beta x - \frac{c_1(y_S+y_E+y_I)}{x+k_1}\right), \tag{1}$$

where x is the prey population size, and $y = y_S + y_E + y_I + y_R$ represents the total predator population. r_1 is the intrinsic growth rate of the prey, k is the impact coefficient of predators on the prey, β is the competition coefficient of the prey population, c_1 is the predation rate of predators on the prey, and k_1 is the protective coefficient of predators on the prey population.

2.2 SEIR Dynamic Equation of the Predator Population

Dynamic Equation of Susceptible Predators:

$$\frac{dy_S}{dt} = y_S\left(r_2 - \frac{c_2 y_S}{x+k_1}\right) - \alpha y_S y_I - d_1 y_S, \tag{2}$$

Dynamic Equation of Exposed Predators:

$$\frac{dy_E}{dt} = \alpha y_S y_I - \sigma y_E - d_2 y_E, \tag{3}$$

Dynamic Equation of Infected Predators:

$$\frac{dy_I}{dt} = \sigma y_E - e y_I - d_3 y_I, \tag{4}$$

Dynamic Equation of Removed Predators:

$$\frac{dy_R}{dt} = e y_I - d_4 y_R, \tag{5}$$

where y_S, y_E, y_I, and y_R represent the number of susceptible, exposed, infected, and removed predators, respectively. r_2 is the growth rate of susceptible predators, c_2 is the predation rate of predators on prey, α is the infection rate for susceptible predators to become exposed predators, σ is the rate at which exposed predators become infected predators, e is the recovery or removal rate of infected predators, and d_1, d_2, d_3, d_4 are the natural death rates of susceptible, exposed, infected, and removed predators, respectively. The system of differential equations is obtained by solving equations (1), (2), (3), (4), and (5):

$$
\begin{cases}
\frac{dx}{dt} = x\left(\frac{r_1}{1+ky} - \beta x - \frac{c_1(y_S+y_E+y_I)}{x+k_1}\right), \\
\frac{dy_S}{dt} = y_S\left(r_2 - \frac{c_2 y_S}{x+k_1}\right) - \alpha y_S y_I - d_1 y_S, \\
\frac{dy_E}{dt} = \alpha y_S y_I - \sigma y_E - d_2 y_E, \\
\frac{dy_I}{dt} = \sigma y_E - e y_I - d_3 y_I, \\
\frac{dy_R}{dt} = e y_I - d_4 y_R,
\end{cases}
\tag{6}
$$

Let the initial values of each subpopulation at the start of the model, i.e., at $t = 0$, be given as $\mathrm{S}(0)$, $\mathrm{E}(0)$, $\mathrm{I}(0)$, $\mathrm{R}(0)$ for susceptible, exposed, infected, and removed predators, respectively, and $\mathrm{X}(0)$ for the prey population size. These values will serve as the initial conditions for the system of differential equations.

3 Model Processing

3.1 Variable Substitution and Dimensionless Transformation

In order to simplify the analysis of the model, the original predator-prey model undergoes a dimensionless transformation. The following variable substitutions are considered:

$$
T = r_1 t, \quad u = \frac{\beta}{r_1}x, \quad v_S = \frac{c_1\beta}{r_1^2}y_S, \quad v_E = \frac{c_1\beta}{r_1^2}y_E, \quad v_I = \frac{c_1\beta}{r_1^2}y_I, \quad v_R = \frac{c_1\beta}{r_1^2}y_R.
$$

At the same time, new dimensionless parameters are introduced:

$$
s = \frac{\beta k_1}{r_1}, \quad \gamma = \frac{r_2}{r_1}, \quad f = \frac{c_2}{c_1}, \quad d = \frac{k r_1^2}{c_1 \beta}, \quad \alpha' = \frac{\alpha c_1}{r_1}, \quad \sigma' = \frac{\sigma}{r_1}, \quad e' = \frac{e}{r_1}, \quad p_i = \frac{d_i}{r_1},
$$

Based on the above variable substitutions, the original predator-prey model can be rewritten in dimensionless form as:

$$
\begin{cases}
\frac{du}{dT} = u\left(\frac{1}{1+d(v_S+v_E+v_I+v_R)} - u - \frac{v_S+v_E+v_I}{u+s}\right), \\
\frac{dv_S}{dT} = v_S\left(\gamma - \frac{f v_S}{u+s}\right) - \alpha' v_S v_I - p_1 v_S, \\
\frac{dv_E}{dT} = \alpha' v_S v_I - \sigma' v_E - p_2 v_E, \\
\frac{dv_I}{dT} = \sigma' v_E - e' v_I - p_3 v_I, \\
\frac{dv_R}{dT} = e' v_I - p_4 v_R.
\end{cases}
\tag{7}
$$

3.2 Discretization of the Model

To discretize the dimensionless model, the forward Euler method is applied, converting the continuous-time model into a discrete-time model. Let the time step be ΔT, with the time point $T_n = n\Delta T$. The discretized model equations are as follows:

$$\begin{cases} u_{n+1} = u_n + \Delta T \cdot u_n \left(\frac{1}{1+d(v_{S,n}+v_{E,n}+v_{I,n}+v_{R,n})} - u_n - \frac{v_{S,n}+v_{E,n}+v_{I,n}}{u_n+s} \right), \\ v_{S,n+1} = v_{S,n} + \Delta T \cdot \left(v_{S,n} \left(\gamma - \frac{f v_{S,n}}{u_n+s} \right) - \alpha' \ v_{S,n} v_{I,n} - p_1 v_{S,n} \right), \\ v_{E,n+1} = v_{E,n} + \Delta T \cdot \left(\alpha' \ v_{S,n} v_{I,n} - \sigma' \ v_{E,n} - p_2 v_{E,n} \right), \\ v_{I,n+1} = v_{I,n} + \Delta T \cdot \left(\sigma' \ v_{E,n} - e' \ v_{I,n} - p_3 v_{I,n} \right), \\ v_{R,n+1} = v_{R,n} + \Delta T \cdot \left(e' \ v_{I,n} - p_4 v_{R,n} \right). \end{cases} \tag{8}$$

4 Existence of Equilibrium Points

To determine all equilibrium points of the model, the following system of equations needs to be solved:

$$\begin{cases} u = u\exp\left(\Delta T \cdot \left(\frac{1}{1+d(v_S+v_E+v_I+v_R)} - u - \frac{v_S+v_E+v_I}{u+s} \right) \right), \\ v_S = v_S\exp\left(\Delta T \cdot \left(\gamma - \frac{f v_S}{u+s} - \alpha' \ v_I - p_1 \right) \right), \\ v_E = v_E\exp\left(\Delta T \cdot \left(\alpha' \ v_S v_I - \sigma' \ - p_2 \right) \right), \\ v_I = v_I\exp\left(\Delta T \cdot \left(\sigma' \ v_E - e' \ - p_3 \right) \right), \\ v_R = v_R\exp\left(\Delta T \cdot \left(e' \ v_I - p_4 \right) \right). \end{cases} \tag{9}$$

The constants of this model yield the equilibrium points: $E_0(0,0,0,0,0)$, $E_1(u_1,0,0,0,0)$, and $E_2(u_2, v_{S,1}, v_{E,1}, v_{I,1}, v_{R,1})$, where:

$$v_{S,1} = \frac{(\gamma - p_1)(u_2+s)}{f}, \quad v_{E,1} = \frac{\alpha' \ v_{S,1} v_{I,1}}{\sigma' \ + p_2}, \quad v_{I,1} = \frac{\sigma' \ v_{E,1}}{e' \ + p_3}, \quad v_{R,1} = \frac{e' \ v_{I,1}}{p_4},$$

And u_2 is the positive real root of the following quadratic equation:

$$\phi_2 u^2 + \phi_1 u + \phi_0 = 0,$$

where:

$$\phi_2 = df(\gamma - p),$$
$$\phi_1 = f(\gamma - p) + d(\gamma - p)(v_{S,1})^2 + f^2,$$
$$\phi_0 = ds(\gamma - p)(v_{S,1})^2 + f(\gamma - p)s - f^2$$

For the zero equilibrium point $E_0(0,0,0,0,0)$ and the axial equilibrium point $E_1(u_1,0,0,0,0)$, their existence is self-evident. The following theorem demonstrates the existence of the equilibrium point $E_2(u_2, v_{S,1}, v_{E,1}, v_{I,1}, v_{R,1})$.

Theorem

If $v_{S,1} > 0$, and

$$ds(\gamma - p)(v_{S,1})^2 + f(\gamma - p)s < f^2,$$

then the model has a unique positive equilibrium point $E_2(u_2, v_{S,1}, v_{E,1}, v_{I,1}, v_{R,1})$, satisfying:

$\phi_2 > 0 \, \phi_2 > 0 \;\; \phi_2 > 0,\; \phi_1 > 0,\; \phi_0 < 0 \;\; \phi_1 > 0 \;\; \phi_1 > 0 \;\; \phi_0 < 0 \;\; \phi_0 < 0$

Proof

The roots of the quadratic equation satisfy the root-coefficient relationship:

$$m + n = -\frac{\phi_1}{\phi_2}, \quad mn = \frac{\phi_0}{\phi_2}.$$

Since $r_2 > E$, it follows that $\gamma > p$, thus:

$$\phi_1 = dsf(\gamma - p) + d(\gamma - p)\left(v_{S,1}\right)^2 + f^2 > 0, \quad \phi_2 = df(\gamma - p) > 0.$$

If $\phi_0 > 0$, then $m + n < 0$, and $mn < 0$. The equation will have two negative real roots only when $\Delta = \phi_1^2 - 4\phi_2\phi_0 > 0$; if $\Delta < 0$, the equation has no real roots. In this case, model (9) has no positive equilibrium points.

If $\phi_0 < 0$, then $m > 0$, and $n > 0$. The equation has one positive real root and one negative real root only when $\Delta = \phi_1^2 - 4\phi_2\phi_0 > 0$. In this case, model (9) has a unique positive equilibrium point $E_2\left(u_2, v_{S,1}, v_{E,1}, v_{I,1}, v_{R,1}\right)$.

5 Stability of the Equilibrium Points

5.1 Calculation of the Jacobian Matrix

To analyze the stability of the equilibrium points, the Jacobian matrix at the corresponding points is calculated, and its eigenvalues are solved to analyze stability [11-13].

This section focuses on the stability analysis of the equilibrium point $E_2\left(u_2, v_{S,1}, v_{E,1}, v_{I,1}, v_{R,1}\right)$, which represents the coexistence of all predator and prey populations. The corresponding Jacobian matrix J is the linear approximation of the model near this point. The elements of the matrix are the partial derivatives of each equation in the model with respect to all variables:

$$J = \begin{bmatrix} \frac{\partial u_n}{\partial u_{n+1}} & \frac{\partial u_n}{\partial v_{S,n+1}} & \frac{\partial u_n}{\partial v_{E,n+1}} & \frac{\partial u_n}{\partial v_{I,n+1}} & \frac{\partial u_n}{\partial v_{R,n+1}} \\ \frac{\partial v_{S,n}}{\partial u_{n+1}} & \frac{\partial v_{S,n}}{\partial v_{S,n+1}} & \frac{\partial v_{S,n}}{\partial v_{E,n+1}} & \frac{\partial v_{S,n}}{\partial v_{I,n+1}} & \frac{\partial v_{S,n}}{\partial v_{R,n+1}} \\ \frac{\partial v_{E,n}}{\partial u_{n+1}} & \frac{\partial v_{E,n}}{\partial v_{S,n+1}} & \frac{\partial v_{E,n}}{\partial v_{E,n+1}} & \frac{\partial v_{E,n}}{\partial v_{I,n+1}} & \frac{\partial v_{E,n}}{\partial v_{R,n+1}} \\ \frac{\partial v_{I,n}}{\partial u_{n+1}} & \frac{\partial v_{I,n}}{\partial v_{S,n+1}} & \frac{\partial v_{I,n}}{\partial v_{E,n+1}} & \frac{\partial v_{I,n}}{\partial v_{I,n+1}} & \frac{\partial v_{I,n}}{\partial v_{R,n+1}} \\ \frac{\partial v_{R,n}}{\partial u_{n+1}} & \frac{\partial v_{R,n}}{\partial v_{S,n+1}} & \frac{\partial v_{R,n}}{\partial v_{E,n+1}} & \frac{\partial v_{R,n}}{\partial v_{I,n+1}} & \frac{\partial v_{R,n}}{\partial v_{R,n+1}} \end{bmatrix}$$

Let $v_E + v_I + v_R + v_S$ be denoted as v_A;; let $v_E + v_I + v_S$ be denoted as v_P. The values of the elements in the first row of the matrix are as follows:

$$j_{11} = \frac{-u(-v_P + (s+u)^2) + (s+u)^2}{(s+u)^2} \exp\left(\frac{s - u(s+u)(dv_A + 1) + u - (dv_A + 1)v_P}{(s+u)(dv_A + 1)}\right)$$

$$j_{12} = -u\left(\frac{d}{(dv_A + 1)^2} + \frac{1}{s+u}\right)\exp\left(-u + \frac{1}{dv_A + 1} - \frac{v_P}{s+u}\right)$$

$$j_{13} = -u\left(\frac{d}{(dv_A + 1)^2} + \frac{1}{s+u}\right)\exp\left(-u + \frac{1}{dv_A + 1} - \frac{v_P}{s+u}\right)$$

$$j_{14} = -u\left(\frac{d}{(dv_A + 1)^2} + \frac{1}{s+u}\right)\exp\left(-u + \frac{1}{dv_A + 1} - \frac{v_P}{s+u}\right)$$

$$j_{15} = -du\exp\left(\frac{s - u(s+u)(dv_A + 1) + u - (dv_A + 1)v_P}{(s+u)(dv_A + 1)}\right)\frac{1}{(dv_A + 1)^2}$$

The remaining elements can be solved in a similar way and are not listed here.

5.2 Eigenvalue Calculation and Stability Analysis

The relevant parameters for the equations are assigned reasonable values. The eigenvalues of the above Jacobian matrix are solved using Python software. The five obtained eigenvalues are:

$$\lambda_1 = 1.0000, \lambda_2 = 0.0899, \lambda_3 = 1.0000, \lambda_4 = 0.9999, \lambda_5 = 0.0000$$

$\lambda_1 = \lambda_3 = 1.0000$ indicates that the system is neutrally stable along these directions, but not asymptotically stable, which may lead to periodic behavior or sustained oscillations.

$\lambda_2 = 0.0899$, which is less than 1, shows that this direction is asymptotically stable.

$\lambda_4 = 0.9999$ suggests that the system is close to neutral stability along this direction, and theoretically, periodic behavior or boundary stability may occur.

$\lambda_5 = 0.0000$ means there is no change along this direction, indicating that the system is at rest in this direction.

The above analysis shows that the system has not fully reached asymptotic stability, but is instead in a state of boundary stability. Periodic fluctuations or sustained oscillations may occur along certain directions.

6 Image Analysis

6.1 Time Series Plot of the System

By plotting the system's time series, the changes in the predator or prey populations during the evolutionary process can be shown, allowing observation of whether periodic behaviors or other dynamic features exist.

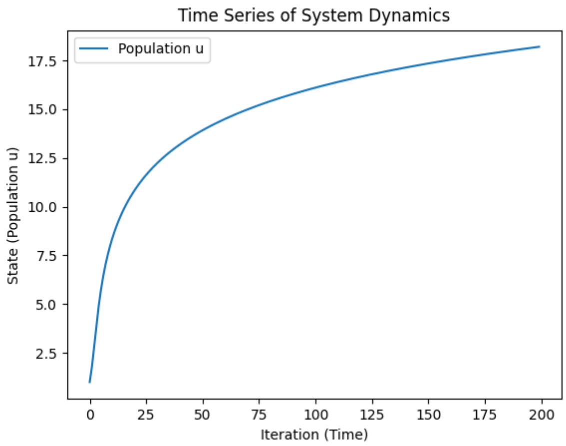

Fig. 1. Time Series Plot of the System

Figure 1 shows the trajectory of the system state u as it changes with the increasing number of iterations over time. The system state grows rapidly in the early stages and then tends to stabilize. This may be related to the rapid reproduction of predators and the swift spread of infectious diseases.

Additionally, there is no apparent periodic oscillation or chaotic phenomenon in the plot, indicating that the system's dynamic behavior is monotonic and tends toward a stable equilibrium point. This is consistent with the theoretical analysis results, indicating the system's asymptotic stability.

For different parameter settings, especially with different combinations of the predation rate c_I and disease transmission rate α, numerical simulations show that the system's dynamic characteristics change significantly. As the predation rate increases, the predator population grows more rapidly in the initial stages. However, in the long term, the predator population tends to stabilize, while the prey population gradually decreases [14,15]. This is because the high predation pressure on the prey leads to a reduction in the prey population. However, when the prey population decreases to a certain threshold, the growth of the predator population slows, and the system approaches stability. When the disease transmission rate is high, the dynamic behavior of the predator population becomes unstable. This is due to the accelerated infection of predators, which increases their mortality rate, and the prey population, in turn, shows an increasing trend, leading the system to become overall unstable [16,17].

These numerical simulation results align with the characteristic value calculations from the theoretical analysis, further verifying the system's dynamic behavior under different parameter conditions. The parameters c_I and α are critical control factors for the system's stability, determining the long-term stability of the predator-prey-disease system.

6.2 Bifurcation Diagram

To better observe the system's long-term behavior and potential bifurcation phenomena, this study plots a bifurcation diagram, changing a specific parameter and observing the system's dynamics at different parameter values [18]. The following diagram shows the state changes of the system when the predation rate is altered.

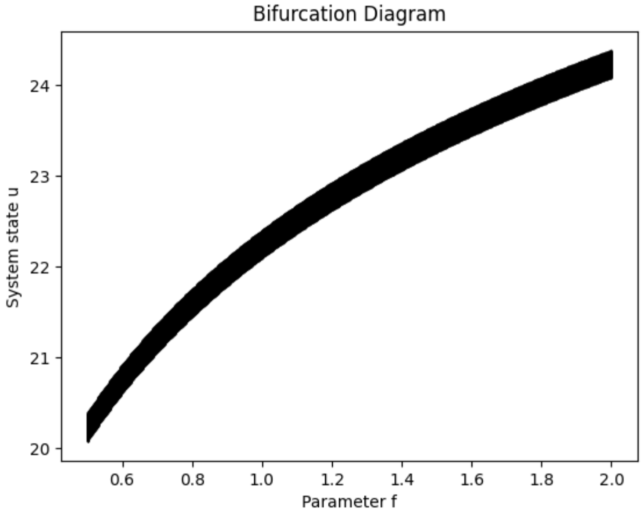

Fig. 2. Bifurcation Diagram of the System

From Figure 2, it can be seen that the system state u shows a monotonically increasing trend as the predation rate f increases, with a smooth curve and no significant bifurcation. This suggests that, within this parameter range, the system is relatively stable, without any periodic oscillation or chaotic behavior.

This may also indicate that the range of variation for the parameter f is not large enough to cause the system to transition from a stable state to other dynamic states. The smooth trend of the system state u with changes in the predation rate f suggests that the impact of this parameter on the system's overall state is gradual and does not immediately trigger unstable dynamic behavior.

7 Conclusion Summary

This study investigates the dynamic behavior and stability of a predator-prey system under the influence of the SEIR model. Through dimensional analysis and numerical simulations, the model's equilibrium points were analyzed, and system stability under different parameter conditions was explored using the characteristic values of the Jacobian matrix.

Numerical simulation results show that the predation rate c_l and disease transmission rate α are the key parameters influencing the system's dynamic behavior. At higher predation rates, the predator population grows more rapidly, but simultaneously, the prey population decreases quickly, leading the system to become unstable. Higher disease transmission rates also exacerbate the infection of predators, increasing system instability. Reducing the predation rate and disease transmission rate can effectively slow the system's fluctuations and make the system asymptotically stable.

Moreover, through simulations with different parameter combinations, this study found that the predator population's exposure-to-infection conversion rate σ and the predator's recovery or removal rate e also have significant effects on the system's dynamic behavior. A higher conversion rate accelerates the infection process of predators, causing dramatic population fluctuations, while increasing the recovery rate of predators helps the system approach equilibrium.

Declaration

This project is supported by the 2024 Undergraduate Research and Development Program of Ocean University of China.

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