Impact of Solar Uncertainties on Distribution System Performance: A PEM-Based Probabilistic Load Flow Study

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Abstract. The increasing penetration of photovoltaic (PV) generation into distribution networks introduces significant variability and uncertainty, primarily due to fluctuating solar irradiance. These uncertainties directly affect system performance, particularly in terms of voltage stability, power losses, and overall reliability. Conventional deterministic load flow analysis cannot adequately capture such stochastic behaviour, making probabilistic approaches essential. This study presents a probabilistic load flow (PLF) framework based on the Point Estimate Method (PEM) to quantify the impact of solar uncertainties on distribution system performance. The proposed methodology is applied to the IEEE 34-bus radial distribution system, where solar generation is modelled as a stochastic variable with probability distributions derived from irradiance profiles. PEM is employed to efficiently approximate statistical moments of the system response. Performance metrics such as voltage deviation and system power losses, are evaluated under varying PV penetration levels. Simulation results reveal that solar uncertainties significantly influence the distribution of bus voltages and network losses, with higher penetration levels leading to increased probabilities of undervoltage and overvoltage conditions. The findings highlight the necessity of incorporating probabilistic analysis in planning and operation of PV-integrated distribution systems. The proposed PEM-based PLF framework provides utilities and planners with an efficient tool for assessing the risks associated with renewable energy variability, thereby supporting more resilient and reliable grid integration strategies.

Keywords: Probabilistic Load Flow (PLF), Point Estimate Method (PEM), Solar Uncertainty, Photovoltaic (PV) Integration, Distribution System Performance, Voltage Stability, Power Loss Analysis

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1 Introduction

The rapid growth of distributed photovoltaic (PV) generation is reshaping the planning and operation of modern distribution networks. While PV systems support decarbonization and have become increasingly cost-effective, their output remains inherently variable due to diurnal cycles, cloud movements, and seasonal weather patterns [1]. In radial distribution feeders—which typically exhibit high R/X ratios and limited voltage-regulation capability—such fluctuations can significantly affect bus voltages, branch currents, and feeder losses, especially when PV units are installed deep in the network [3], [15].

Traditional deterministic load-flow (LF) analyses assume fixed inputs and therefore fail to capture the probability of operational-limit violations under renewable-energy uncertainties [12], [13]. As PV penetration increases, utilities and planners require probabilistic tools capable of mapping uncertain inputs (e.g., solar irradiance, load variability) to the probability distributions of system states. These tools enable the evaluation of risk-centric performance indicators such as the likelihood of under/overvoltage, thermal overload, or increased losses [4], [7].

The probabilistic load flow (PLF) literature generally converges around three major methodological families. Monte Carlo Simulation (MCS) provides a benchmark for accuracy due to its broad sampling of input distributions, but its computational cost becomes prohibitive for large feeders or long-term analyses [4], [5], [6]. Analytical and moment-based approaches—including cumulant-based expansions and Gram–Charlier models—offer faster alternatives through moment propagation, although their accuracy may degrade under nonlinear or heavy-tailed uncertainties [7]–[10]. Between these two extremes, the Point Estimate Method (PEM) achieves a balance between accuracy and computational efficiency by evaluating the power-flow equations at a limited number of statistically derived concentration points [11], [12], [14], [15], [16].

A key component of PLF analysis is the accurate modeling of PV uncertainty. Solar irradiance is often represented using probability density functions (PDFs) such as the Beta or Weibull distributions, which effectively capture its bounded and asymmetric behavior in short-term and tropical conditions [17], [18], [19]. Selecting an appropriate PDF is crucial because the resulting risk metrics—such as voltage-violation probability—depend strongly on the underlying distribution.

Motivated by these challenges, this study develops a PEM-based PLF framework to quantify how solar-irradiance uncertainties influence distribution-system performance. The method integrates realistic irradiance probability models into an AC power-flow solver and evaluates statistical variations in voltage profiles and power losses. The robustness of the proposed approach is demonstrated using the IEEE 34-bus radial distribution feeder, a widely adopted benchmark for PV-integration studies.

2 Solar Irradiance Uncertainty and Data Representation

Solar resource characterization is essential for accurately modeling photovoltaic (PV) output in probabilistic power-flow studies. Solar energy is commonly expressed using two related metrics: solar insolation and solar irradiance. Solar insolation represents the total solar energy accumulated over a day on a unit surface (kWh/m²/day), and is widely used in global climate and renewable-energy datasets, including NASA's Surface Meteorology and Solar Energy (SSE) database [20]. In contrast, solar irradiance denotes the instantaneous power density of solar radiation (kW/m²) and is the primary input for PV output estimation and uncertainty modeling [17].

To integrate solar resource information into irradiance-based PV models, daily insolation values must be converted into equivalent average irradiance. In tropical regions such as Indonesia, this conversion commonly assumes an effective sunlight duration of approximately 12 hours per day, a practical simplification also employed in statistical solar-uncertainty analyses for tropical climates [19]. This conversion enables irradiance-based modeling while maintaining compatibility with global insolation datasets.

Solar irradiance is highly variable due to atmospheric scattering, cloud movement, and diurnal cycles. To capture this stochastic behavior, irradiance is treated as a random variable described by an appropriate probability density function (PDF). Several studies demonstrate that the Beta distribution effectively represents short-term irradiance or per-unit PV output because of its ability to model bounded and asymmetric distributions, especially under tropical and partly cloudy conditions [17], [18], [19]. For longer temporal aggregates such as monthly or annual averages, irradiance distributions tend to approximate a normal-like pattern due to averaging effects, consistent with the statistical behavior of solar-energy datasets reported in recent PV uncertainty analyses [18].

The upper physical limit of solar irradiance is governed by the solar constant, approximately 1.36 kW/m² at the top of the Earth's atmosphere while clear-sky irradiance at the surface typically reaches up to about 1.0 kW/m² [17], [20]. These physical boundaries are important constraints when formulating probabilistic models for PV generation.

In this study, solar resource data were obtained from the NASA SSE dataset [20] corresponding to the geographical coordinates of Universitas Negeri Medan, Indonesia (3.46° S, 98.76° E), for the period January–June 2025. The monthly average insolation extracted from NASA's dataset is presented in Table 1. Using the methodology described earlier, these insolation values were converted into average irradiance, as shown in Table 2. The resulting irradiance dataset was then used to compute the statistical moments such as mean (μ), variance (σ^2), and skewness (γ) required for the probabilistic load-flow analysis using the Point Estimate Method (PEM).

Table 1. Monthly average solar insolation (kWh/m²/day) for Universitas Negeri Medan based on NASA SSE dataset (January–June 2025)

January	February	March	April	May	June	Mean
4.5	5	5.5	5.7	5.5	5.2	5.23

The calculation of PV panel output power based on solar irradiance values requires the conversion of solar insolation (energy) data into solar irradiance. The resulting solar irradiance is assumed to be distributed over a 12-hour period of sunlight, using the following equation:

$$Solar\ Irradiance\ \left({}^{W}/_{m^2}\right) = \frac{Solar\ insolation\ ({}^{kWh}/_{m^2})}{Time\ duration} \quad (1)$$

The solar irradiance data for the period from January to June 2025 is presented in Table 2.

Table 2. Monthly average solar irradiance (kW/m²) for Universitas Negeri Medan converted from NASA SSE insolation data (January–June 2025)

January	February	March	April	May	June	Mean
0,375	0,416	0,458	0,475	0,458	0,433	0,436

3 Photovoltaic System (PV) Output Modeling

The electrical power produced by a photovoltaic (PV) module depends primarily on the incident solar irradiance, panel surface area, and the conversion efficiency of the solar cells. PV modules are typically characterized under Standard Test Conditions (STC), which assume an irradiance of 1 kW/m² and a cell temperature of 25°C. Under STC, the rated output of a PV module is expressed as the product of irradiance, active cell area, and module efficiency, as shown in (2). This formulation is widely used in PV performance assessment and uncertainty analysis, particularly when propagating irradiance variability into power-output variability [17].

$$PV_{R.O} = STC_{SI} \times A \times Eff \qquad (2)$$

Where

STC_{SI}= standard irradiance under STC (1 kW/m²),

A= module surface area (m²),

Eff= PV cell conversion efficiency,

$PV_{R.O}$= rated output power under STC.

To estimate the electrical power output under actual field conditions, the STC irradiance value is replaced with the real-time or average solar irradiance measured at the study location. Accordingly, the PV output under non-STC operating conditions is modeled using (3):

$$PV_{out} = Solar_{irr} \times A \times Eff \qquad (3)$$

where PV_{out} is the PV array output for an irradiance level $Solar_{irr}$(kW/m²).

The active area A of the PV array may also be expressed as:

$$A = \frac{PV_{R.O}}{Eff \times STC_{SI}} \qquad (4)$$

where $PV_{R.O}$ is the rated capacity of the PV module or array. Substituting (4) into (3) yields a simplified linear expression that relates PV output directly to solar irradiance:

$$PV_{out} = n \times P_R \times \frac{Solar_{irr}}{STC_{SI}} \qquad (5)$$

where n denotes the number of PV modules in the array. This formulation is consistent with PV performance modeling approaches that treat PV output as a linear transformation of irradiance for small perturbations around STC conditions [17], [18].

Equation (5) shows that PV power output is linearly proportional to irradiance. Thus, if solar irradiance is modeled as a random variable with known statistical moments, the PV output will also inherit those moments through linear transformation. For a linear model $Y = kX$, the corresponding output statistics follow:

$$\mu_Y = k\mu_X \qquad (6)$$

$$\sigma_Y = k\sigma_X \qquad (7)$$

$$\gamma_Y = k\gamma_X \qquad (8)$$

These moment-preservation properties make linear PV modeling fully compatible with moment-based probabilistic techniques such as the Point Estimate Method (PEM), which requires only the mean, variance, and skewness of input variables [11], [12], [14]. Once the irradiance statistics are determined (Section 4), they can be consistently propagated to obtain the corresponding probabilistic PV output used later in the load-flow analysis.

4 Input Modeling with Beta Distribution

Solar irradiance is a bounded random variable whose values lie between zero and a physical maximum determined by atmospheric conditions. Due to this bounded and often asymmetric behavior, the Beta probability density function (PDF) is widely regarded as an appropriate model for short-term irradiance and PV power variability, particularly in tropical regions with highly dynamic cloud patterns [17], [18], [19].

$$\mu_X = \frac{\alpha}{\alpha + \beta} \qquad (9)$$

$$\sigma_X^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \qquad (10)$$

$$\gamma_X = \frac{2(\beta - \alpha)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}} \qquad (11)$$

These moments fully characterize the Beta distribution and are directly usable by Hong's 2-point PEM formulation, which relies on the mean, variance, and skewness to derive statistically equivalent concentration points for probabilistic load-flow evaluations [12], [14], [16].

In this study, the Beta distribution is used to model the uncertainty of solar irradiance derived from the NASA SSE dataset (Section 2). The empirical moments computed from the monthly irradiance data are matched with equations (9), (10), (11) to obtain the Beta parameters $\alpha$and β. Once the Beta PDF is established, the probabilistic nature of irradiance can be propagated into PV output through the linear transformation framework described in Section 3.

Thus, the Beta distribution serves as the probabilistic foundation for irradiance modeling, while the linear PV model ensures that these statistical properties are accurately transmitted into the PV generation model used in the PEM-based probabilistic load flow. The combined use of equations (9) - (11) and (6) - (8) guarantees mathematical consistency and realistic modeling of uncertainty across both irradiance and PV output domains.

5 Solar PV Representation in AC Load Flow

In AC power-flow studies, grid-connected photovoltaic (PV) units are modeled as active-power injections at their respective buses. Because PV systems typically operate as PQ buses, their contribution to the network can be expressed as a negative load:

$$P_{inj,k} = -PV_{out,k} \qquad (12)$$

The modeling approach can be illustrated in Figure 1 where the PV unit at Bus 2 injects 2 kW of active power into the network and it is treated as negative load, thereby reducing the net load at the bus according to equation (12), the effective load seen by Bus 2 becomes $Load_{bus2} = Load_1 - PV_{out}$. This modeling approach treats PV generation as an active-power injection (negative load), which is standard in AC load-flow analysis for inverter-based distributed generation.

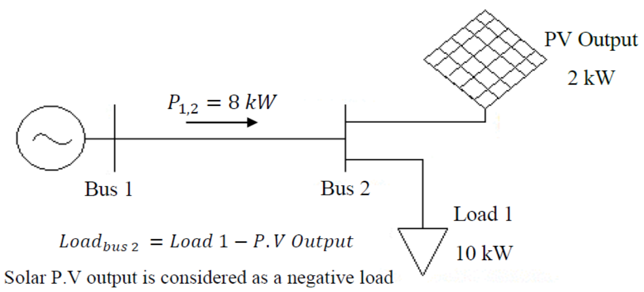

Figure 1. PV Model in Load Flow Study

6 Point Estimate Probabilistic Load Flow Method (PEM)

The Point Estimate Method (PEM) is a moment-based probabilistic technique that evaluates the impact of uncertain inputs on system performance using a limited number of deterministic load-flow evaluations. Instead of relying on thousands of samples as in Monte Carlo Simulation (MCS), PEM reconstructs the output statistics from concentration points derived from the mean, variance, and skewness of the input distributions, achieving high computational efficiency while preserving accuracy for nonlinear systems [11], [12], [14].

Let the AC power-flow model be expressed generically as:

$$\mathrm{y} = F(\mathrm{x}) \qquad (13)$$

where x contains uncertain inputs (PV output derived from irradiance), and y represents the resulting state variables such as voltages and feeder losses. PEM approximates the distribution of y by evaluating Fat a set of statistically defined concentration points.

6.1. *2m* Hongs Scheme ($\boldsymbol{K} = \mathbf{2}$)

This study adopts Hong's two-point estimate method (2PEM), a widely applied technique in probabilistic load flow due to its favorable balance between computational effort and approximation accuracy [12], [14], [16]. For an uncertain variable X, two concentration points are generated around the mean as:

$$\xi_{1,2} = \mu_X \pm \lambda\sigma_X \qquad (14)$$

where μ_X, σ_X, and $\gamma_X$are the mean, standard deviation, and skewness of X, respectively, and λ is a skewnes

The weights associated with the two concentration points are:

$$w_1 = \frac{1}{2}\left(1 + \frac{\gamma_X}{\sqrt{\gamma_X^2 + 4}}\right) \qquad (15)$$

$$w_2 = 1 - w_1 \qquad (16)$$

These weights allow 2PEM to capture asymmetry in the input distribution that an important feature for solar irradiance modeled using skewed Beta distributions.

6.2 PEM-Based Probabilistic Load Flow Procedure

The 2PEM probabilistic load-flow process used in this study consists of the following steps:\

1. Model uncertain PV inputs:
   Solar irradiance uncertainty is modeled via a Beta distribution (Section 4), and PV output is derived as a linear transformation (Section 3).
2. Compute statistical moments:
   The moments $(\mu_X, \sigma_X, \gamma_X)$ for irradiance are obtained using the Beta PDF.
3. Generate concentration points:
   Apply (14) to generate $\xi_{1,2}$ and their corresponding weights using equations (15) and (16).
4. Map irradiance to PV injections:
   Each concentration point $X = \xi_i$ is converted into PV output using the linear model as described in equation (6) – (8) and injected at its bus as negative load using equation (12).
5. Run deterministic load flows:
   Solve the power flow for each concentration point while other variables are fixed at their mean values.
6. Reconstruct probabilistic outputs:
   The moments of network voltages, branch flows, and total losses are obtained by combining deterministic outputs using the concentration weights.

7 Results and Discussion
7.1 System Data

The Point Estimate Method (PEM) with Hong's characteristic is applied to the IEEE 34 distribution system to assess the probabilistic performance under the integration of solar photovoltaic (PV) generation. The system is modeled with uncertain variables, particularly solar irradiance, which affects the output of PV panels. By using the probabilistic load flow (PLF) framework, we estimate the impact of solar uncertainties on feeder losses, bus voltages, and branch loading risks.

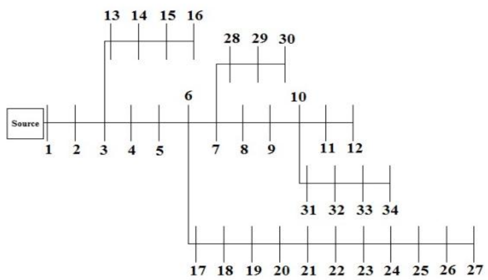

Figure 2. The test system of IEEE 34 bus

For the load, the data used represents the peak load condition occurring at 2:00 p.m. It should be noted that the load profile is defined in probabilistic terms and derived from the available historical load data. The data in Table 3 corresponds to the load at each bus, which is relatively used or taken as a reference when performing power flow analysis or other studies on the IEEE

34-bus distribution system without incorporating uncertainty conditions (i.e., deterministic analysis). This dataset represents the load that consumes the highest amount of electrical energy within a 24-hour period, commonly referred to as the peak load condition. Total load in peak condition is 4.64 MW and 2.87 MVar.

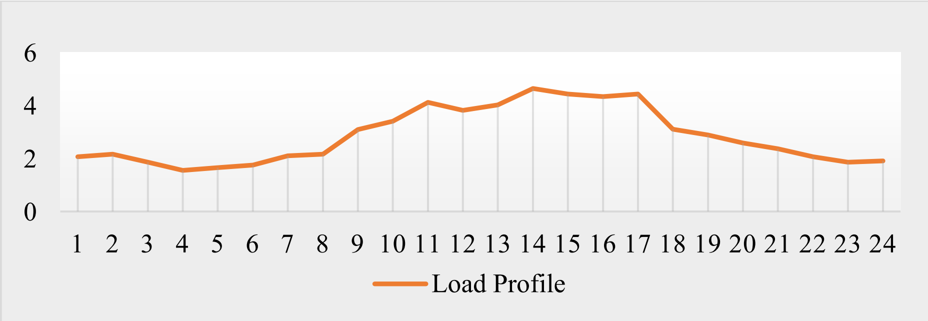

Figure 3. Statistical 24-hour load data of the system

7.1 Results of Probabilistic Performance

In this study, six PV-DG units are deployed at buses 23, 24, 25, 26, 27, and 33 of the IEEE 34-bus feeder to assess the impact of spatial PV placement on the probabilistic voltage and loss profiles. The impact of solar irradiance uncertainty on bus voltages was analyzed by comparing the voltage profiles under deterministic conditions versus probabilistic conditions. The results revealed that as the level of PV penetration increased, the voltage at certain buses exhibited larger deviations from the nominal values. In particular, buses closer to the PV generation units showed increased volatility in voltage levels due to fluctuations in solar irradiance. The average voltage at the buses was within acceptable limits, but the variation in voltage increased with higher solar irradiance uncertainty. The standard deviation of voltage increased, indicating higher uncertainty in voltage profiles under varying solar conditions. The probabilistic analysis also provided risk metrics, showing a higher probability of undervoltage and overvoltage conditions at specific buses as PV penetration increased.

Table 3. Probabilistic voltage profile of system

No of Bus	Voltage (μ)	Voltage (σ)	Angle (μ)	Angle (σ)
1	1	0	0	0
2	0,996	0,001	0,04	0,053
3	0,992	0,002	0,074	0,1
4	0,987	0,004	0,167	0,167
5	0,982	0,005	0,234	0,225
6	0,978	0,006	0,292	0,278
7	0,975	0,007	0,36	0,323
8	0,974	0,007	0,396	0,352

9	0,972	0,008	0,44	0,391
10	0,971	0,009	0,46	0,411
11	0,971	0,009	0,46	0,419
12	0,97	0,009	0,46	0,424
13	0,992	0,002	0,085	0,104
14	0,991	0,002	0,085	0,11
15	0,991	0,002	0,082	0,114
16	0,991	0,002	0,082	0,114
17	0,975	0,007	0,348	0,325
18	0,972	0,008	0,394	0,368
19	0,969	0,009	0,455	0,418
20	0,967	0,01	0,504	0,46
21	0,964	0,011	0,547	0,499
22	0,962	0,012	0,605	0,547
23	0,96	0,012	0,653	0,588
24	0,958	0,013	0,698	0,63
25	0,957	0,014	0,72	0,652
26	0,957	0,014	0,729	0,66
27	0,957	0,014	0,722	0,665
28	0,975	0,007	0,366	0,328
29	0,975	0,007	0,37	0,332
30	0,975	0,007	0,372	0,334
31	0,971	0,009	0,466	0,418
32	0,97	0,009	0,472	0,424
33	0,97	0,009	0,475	0,428
34	0,97	0,009	0,473	0,43

Total losses were also assessed in probabilistic scenario and the result also compared with the deterministic scenario. The total feeder losses increased with higher levels of PV generation, especially in scenarios where solar irradiance fluctuated significantly. Probabilistic load flow highlighted that under high irradiance uncertainty, the variability in feeder losses could lead to more frequent operational inefficiencies and increased losses, particularly in the feeder sections near PV installations. Mean Losses: The mean losses were relatively low in the base case but showed a noticeable increase when solar variability was factored in. Standard Deviation of Losses: The standard deviation of losses was higher in the probabilistic case, indicating greater variability in power losses due to fluctuating solar generation.

Table 4. Probabilistic power losses

Losses	Mean	STD
MW	0,147	0,07
Mvar	0,043	0,02

The deterministic load flow analysis assumes fixed input values and does not account for the variability in PV generation. In contrast, the probabilistic analysis, which incorporates the uncertainty in solar irradiance, reveals a more realistic picture of the distribution system's behavior under varying solar conditions. The deterministic analysis tends to underestimate the variability and risk in voltage levels, feeder losses, and branch loading, making probabilistic load flow a crucial tool for more accurate and risk-aware system planning and operation.

7 Conclusion

In conclusion, the integration of solar generation into distribution systems introduces significant uncertainties that must be accounted for in load flow analysis. The PEM-based PLF method, as demonstrated on the IEEE 34-bus system, provides an efficient and effective way to quantify the impact of these uncertainties on system performance. The results highlight the importance of incorporating probabilistic methods in distribution system planning and operation, particularly as PV penetration increases. This approach not only provides a more realistic assessment of system behavior but also helps in identifying areas of potential risk that may not be apparent in deterministic analyses. The findings emphasize the need for utilities and planners to use probabilistic tools to support more reliable and resilient grid integration strategies for renewable energy sources.

Compared to Monte Carlo simulations (MCS) and other probabilistic load flow methods, PEM offers several advantages. While MCS is considered the benchmark for accuracy, it comes with a high computational cost, especially when dealing with large systems or long time periods. PEM, on the other hand, requires fewer evaluations, making it much more computationally efficient while still providing reliable results. This makes PEM particularly suitable for real-time analysis and large-scale systems where the computational burden of methods like MCS would be prohibitive. By using fewer evaluations, PEM can maintain a balance between accuracy and computational efficiency, making it a practical choice for modern distribution system analysis.

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