Analyzing Numeracy of Middle School Students through the Lens of Cognitive Development Theory

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Abstract. This study aims to analyze the numeracy skills of middle school students with a multidimensional approach, including numeracy indicators and cognitive learning theories (Piaget, Bruner, and Vygotsky. Qualitative descriptive methods are used with data collection techniques such as numeracy tests, observations, interviews, and documentation. The results show that 24% of students are at the SPECIAL level (procedural understanding), 59% BASIC (literal understanding), 14% MODERATE (limited application), and 3% ADVANCED (partial reasoning). The analysis shows weaknesses in students' reasoning skills, where most students can only complete the initial steps without drawing logical conclusions on non-routine problems. Dominant error patterns include incorrect arithmetic operations, misconceptions of integers, and inability to conclude. Analysis based on Piaget^s Theory shows that most students are transitioning from the concrete operational stage to the formal operational stage, which limits their ability to think abstractly. Within the framework of Bruner^s Theory, many students at the enactive and iconic stages have difficulty switching to symbolic representations needed for more complex problems. Based on Vygotsky^s theory, it is suggested that students in the Zone of Proximal Development (ZPD) require more intensive scaffolding and social interaction to help them overcome difficulties in solving more difficult numeracy problems.

Keywords: Numeracy, Piaget; Bruner, Vygotsky, Cognitive Development Theory.

1 Introduction

In an increasingly mathematized world, the ability to navigate and make informed decisions depends on mastering numeracy skills across multiple domains [1-3]. Students' numeracy skills

have become a central issue in mathematics education studies across various countries. Numeracy is not simply defined as calculation skills but also reflects the ability to think logically, understand quantitative information, and apply mathematical concepts in different real-life situations [4-6]. Unfortunately, Indonesian students' PISA mathematics scores remain relatively low compared to those of other countries [7]. Based on the results of the 2022 Programme for International Student Assessment (PISA), Indonesia ranked lowest in mathematics competency with a score of 366, far below the OECD average score of 472 [8]. The PISA results indicate that most 15-year-old students in Indonesia have not yet reached the basic level of mathematics proficiency set by the OECD. These results suggest that middle school students in Indonesia frequently encounter challenges in developing numeracy skills, particularly when confronted with non-routine problems that require higher-order thinking [9][10]. It means that students are not yet able to use simple mathematical concepts and procedures in everyday contexts. These difficulties are not only related to procedural fluency but also to conceptual understanding and reasoning, which are significantly impacted by cognitive development processes [11][12].

A decline in numeracy skills often observed during the transition to middle school, where students are challenged by the demands of abstract thinking, such as algebra and mathematical modeling [13]. This situation highlights the importance of examining the cognitive factors that influence numeracy mastery during this critical period [14][15]. Previous studies have been conducted to analyze students' numeracy skills from diverse perspectives, ranging from problem-solving strategies [16], types of student errors [17], the influence of affective factors [18], to the effectiveness of particular learning approaches [19][20]. However, research that explicitly explores students' numeracy from the standpoint of cognitive learning theories remains limited. Yet, in fact, learning theories developed by Piaget [21], Bruner [22], and Vygotsky [23] provide a strong foundation for understanding how students construct mathematical understanding as they develop cognitively. More spesifically, in analyzing students' numeracy skills, cognitive learning theories also provide a meaningful framework for understanding the relationship between developmental stages and students' mathematical thinking.

Piaget emphasized that children construct knowledge through interactions with their environment, progressing through the sensorimotor (0–2 years), preoperational (2–7 years), concrete operational (7–12 years), and formal operational (12 years and above) stages [24]. However, the transition from the concrete operational to the formal operational stage is often challenging for middle school students, as their abstract thinking and logical reasoning skills are still developing [25][26]. This finding is consistent with research indicating that most students can perform basic procedures but struggle to draw logical conclusions from mathematical problems [27]. Complementing Piaget's perspective, Bruner emphasized the role of representation in learning through three stages: enactive (action-based), iconic (image-based), and symbolic (language-based and abstraction-based). Students' difficulty shifting from enactive and iconic representations to symbolic representations often hinders their ability to solve more complex numeracy problems [28]. Meanwhile, Vygotsky places a strong emphasis on how children learn through cultural values in society, through social interactions, and their ability to communicate with their peers [29]. The concept of the Zone of Proximal Development (ZPD) emphasizes that students can achieve higher levels of understanding with support (scaffolding) from teachers and peers [30]. It confirms that a constructivist learning environment, combined with appropriate support, can help students overcome limitations in reasoning and numeracy problem-solving [20].

Based on the theoretical foundation, this study aims to analyze the numeracy skills of middle school students from a multidimensional perspective that integrates numeracy indicators (Special, Basic, Moderate, and Advanced) with cognitive learning theories by Piaget, Bruner, and Vygotsky. Using a qualitative descriptive method, this study explores students' numeracy skills, identifies error patterns, and interprets the difficulties they face within the framework of cognitive development theory.

2 Method

The type of research used was descriptive qualitative. Descriptive research aims to create systematic, factual, and accurate descriptions of the facts, nature, and relationships between the phenomena being studied [31]. The research design used phenomenology. The focus of phenomenology also focuses on direct, first-person experiences, not just limited to a single phenomenon. The subjects of this study were seventh-grade students at one of the junior high school in North Sumatera, Indonesia called SMP Negeri 14 Medan. Data collection techniques included observation, interviews, tests, and documentation. Data validity was tested using source triangulation and technique triangulation.

The indicators used to analyze students' numeracy literacy skills are: a) Understanding: using various numbers and basic mathematical symbols to solve practical problems in a real-life context; b) Application: analyzing questions and information presented in graphs, tables, charts, and other forms; and c) Reasoning: using interpretation of analysis results to predict and make decisions about solutions. The scores given for each indicator are presented in the following table.

Table 1 Scoring Guidelines for Each Numeracy Indicator

Score	Level	Criteria
0	**Special**	o The question was not completed or the paper was blank o The data was written incorrectly and was not used o The answer was incorrect
1	**Basic** Understanding	o Data is rewritten but not used o There is a correct strategy but it is not applied to the problem o Unable to achieve the correct result
2	**Moderate** Applying	o There is a strategy, it is implemented but the answer is wrong o The work is not good enough to achieve the answer o The answer is correct but there is no written calculation
3	**Advanced** Reasoning	Correct strategy correct calculation

To categorize the results of students' numeracy ability levels based on Nana Sudjana's theory of cognitive learning outcomes, the following table is used

Table 2 Cognitive Learning Outcome Indicators

Level	Description
Level 1 : Knowledge	Students are able to recall material without expecting the ability to use it.
Level 2 : Understanding	Students are able to understand problems based on memorized material but are unable to apply it.
Level 3 : Applying	Students are able to apply mathematical strategies and concepts.
Level 4 : Analysis	Students are able to decompose and identify parts of a problem and problem-solving strategies.
Level 5 : Synthesis	Students are able to integrate the results of their analysis in a structured manner to create a solution plan.
Level 6 : Evaluation	Students are able to consider the solutions they have created and evaluate the strategies used.

Table 3 Guidelines for Level of Numeracy based on Cognitive Learning Outcome Indicators

Numeracy Level	Cognitive Learning Outcome Level
Special	Knowledge
Basic	Understanding
Moderate	Application, Analysis
Advanced	Synthesis, Evaluation

Data analysis in the study used Miles and Huberman's interactive analysis technique [32]. The steps of the analysis technique are: 1) data collection, namely collecting all data obtained from observations, interviews, and documentation; 2) data condensation, namely selecting, focusing, simplifying, abstracting, and transforming data contained in field notes, so that accurate data is obtained; 3) data presentation in the form of brief descriptions, charts, relationships between categories, and the like; 4) verification of the findings obtained and then conclusions will be drawn [32].

Result

3.1 Levels of Numeracy Ability Based on Cognitive Learning Outcomes Theory

The following is a table showing the levels of Numeracy ability for each question item.

Table 4 Data on Student Numeracy Ability Level Scores in Solving Questions

Score	Level	Number					
		1	2	3	4a	4b	5
0	Special	2	4	7	7	6	8
1	Basic	12	15	18	16	9	4
2	Moderate	13	10	4	6	14	15
3	Advanced	2	0	0	0	0	1

The results of the table above show that in item 1 there are 12 (41%) students at the BASIC level (solving simple math or numeracy literacy problems) and 13 (45%) students at the MODERATE level. This means that most students are able to apply mathematics to solve numeracy problems in various contexts. For items 2, 3 and 4a; most students are at the BASIC level (solving simple math or numeracy problems); respectively 52%, 62% and 55%. For items 4b and 5; most students are at the MODERATE level (able to apply mathematics to solve numeracy problems in various contexts); respectively 48% and 52%.

According to constructivist learning theory, students at the basic numeracy level have two levels of understanding as proposed by Tighe and Wiggans. These two aspects of understanding are 'explanation' and 'interpretation'. It is evidenced by students' ability to solve simple numeracy problems. In this case, students at the basic level have a descriptive and translation mindset; students can describe problem statements and translate these statements into simple interpretations related to mathematical formulas and concepts. Furthermore, students at the advanced level have an 'application' level of understanding. In this case, students at this level have an 'application' mindset, where they can use their knowledge, skills, and understanding of problem statements to apply basic mathematical concepts in solving problems. This is shown by students' ability to apply mathematics to solve given numeracy problems. Based on the learning outcomes theory, the level of numeracy literacy ability of students at SMP Negeri 14 Medan is

Table 5 Levels of Students' Numeracy Ability Based on Learning Outcomes Theory

Total Score	Numeracy Level	Cognitive Learning Outcome Level	Total Students
0 - 4	Special	Knowledge	7
5 - 9	Basic	Understanding	17
10 - 13	Moderate	Applying Analysis	4
13 - 15	Advanced	Synthesis Evaluating	1

The results of the table above show that 24% of students are at the special level, 59% are at the basic level, 14% are at the proficient level and only 3% are at the advanced level. When viewed from student learning outcomes, the majority of students are at level 2, namely understanding (59%), and only a few students have the synthesis and evaluation level (3%). Furthermore, only 14% of students have application and analysis skills while the other 24% of students are at the knowledge level. These results indicate that most students are at the level of special cognitive processes and understanding, meaning students are able to understand mathematical facts, procedures and tools and have basic mathematical skills, basic computation, basic concepts and solve simple mathematical problems or numeracy literacy problems.

4 Discussion

4.1 Analysis of student answers based on numeracy ability indicators

The following is an analysis of the numeracy abilities of students at SMP Negeri 14 Medan:

4.1.1 Problem 1

Table 6 Students answer of problem 1

Score 1 (Understanding)	
 Fig 1. Student Answer with Score 1 on Problem 1	The student has written the information and questions given in the problem correctly and completely, which shows the student understands the problem well. The initial step in the student's solution is incorrect. The student multiplied all the marbles brought by the three children in the problem, but the result of the multiplication was not recorded. In addition, the student made another mistake, specifically subtracting 20 - 2 - 6, where the student introduced the numbers 2 and 6 themselves outside the information given in the problem. This indicates that the student's understanding, application, and reasoning regarding positive integers are still low, as the student misunderstood and applied the concept of positive integers, and was unable to reason to create a solution to the given problem.
Score 2 (Applying)	
 Fig 2. Student Answer with Score 2 on Problem 1	The student has written the information and questions given in the problem correctly and completely. This indicates that the student has a good understanding of the problem. The initial step in the solution provided by the student is correct, as the student adds up all the marbles brought by the three children in the problem, and the result of the addition is also correct. The student continues the solution to the next step to find the answer according to the question by sharing the results of the addition obtained with many children. The strategy used by the student is correct, and the result is also accurate. However, the student did not complete the solution to the final step, which would have allowed them to find the answer according to the question. This indicates that the student's understanding and application of positive integers are good, but the student's reasoning is still limited

because the solution to the problem is not perfect and complete.

Students have written the information in the problem correctly and completely. This indicates that students have a good understanding of the problem. The initial steps in the solution given by students are correct. Students can find the average number of marbles obtained by each child by adding all the marbles and dividing the total by the number of children. The results of the calculations obtained by students are also correct. Then, students continue the solution to the final step to determine the answer and correctly find out how many marbles Reno should give to Dodi and Anto. This indicates that students' understanding, application, and reasoning regarding positive integers is excellent, as they can comprehend and apply the concept of positive integers and reason to provide accurate and complete solutions to the given problems.

Score 3 (Reasoning)

Fig 3. Student Answer with Score 3 on Problem 1

4.1.2 Problem 2

Table 7 Students answer of problem 2

Score 1 (Understanding)

Fig 4. Student Answer with Score 1 on Problem 2

The student has written the information and questions given in the problem correctly and completely. This indicates that the student has a good understanding of the situation. The solution presented by the student is in the form of a number line. The number line drawn by the student is incorrect because it does not show the correct number line. This can be seen from the fact that the number to the left of the number 0 is positive, and the student wrote the number 0 twice on one number line. This indicates that the student lacks an understanding of and is mistaken about the concept of the number line. Therefore, the student is

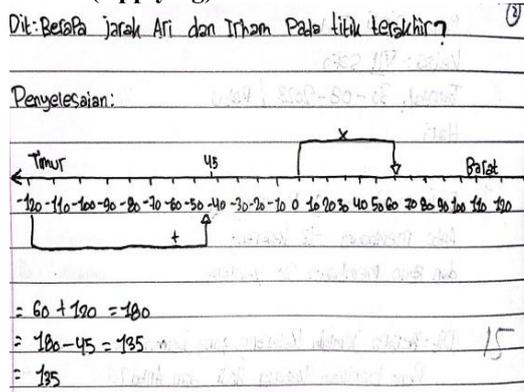unable to solve and answer the problem. This indicates that the student's understanding, application, and reasoning regarding the concept of integers remain very low.

Score 2 (Applying)

Fig 5. Student Answer with 2 Score in Problem 2

Students can accurately and completely record the information and questions provided in the problem. This suggests that they have a good understanding of the problem. The initial steps of the solution presented by students are in the form of a number line. The number line drawn by students is universally correct, but there are several errors. Specifically, students are incorrect in determining the direction of west and east on the number line, and students also make mistakes in calculating Irham's distance from 120 to 45. The correct distance for Irham on the number line should be 120 to 75 because 120 - 45 = 75. However, the final result of the solution carried out by students is correct because it still uses a negative sign as a representation of 'reversing direction' in the problem, so students add up the initial distance traveled by the two children and then subtract it from the reverse distance traveled by Irham. This indicates that students' understanding and application of the concept of numbers are good, as evidenced by their ability to solve and answer problems, despite some errors still remaining.

4.1.3 Problem 3

Table 8 Students answer of problem 3

Score 1 (Understanding)	Students have not been able to write the information and questions given in the problem accurately, which indicates that they have not fully understood the situation. The solution steps

Fig 6. Student Answer with a Score of 1 on Problem 3

Score 2 (Applying)

presented by students contain many errors and mistakes, namely student errors in converting the weight of each material from kilograms to grams, students convert the weight units of materials in the problem by converting them to whole numbers, but students ignore the weight of the material informed in the problem in the form of fractions, so the conversion results are wrong. In addition, there are errors in converting whole numbers into fractions. The strategies and solution results presented by students are wrong. This indicates that students' understanding, application, and reasoning regarding the concept of fractions remain low, as they are unable to solve and answer problems accurately and completely.

Students can accurately write the information and questions given in the problem, which indicates that they understand the situation well. The solution step presented by the student is to add up all the measurements of the ingredients. This strategy is correct; however, the student misunderstood the additional information from the problem, resulting in an incorrect value for one of the ingredients, specifically the measurement of flour. However, the student has completed the solution thoroughly, despite some errors remaining. This indicates that students' understanding and application of the concept of fractions are good, but their reasoning about the concept of fractions remains low.

Fig 7. Student Answer with a Score of 2 on Problem 3

4.1.4 Problem 4

Table 9 Students answer of problem 4

Score 1 (Understanding)	
 Fig 8. Student Answer with Score 1 on Problem 4	Students have not written the information and questions given in the problem entirely and correctly, which indicates that students do not understand the situation well. The strategy in the solution steps presented by students is incorrect, as students multiply the number of children at each table. The strategy and solution results presented by students are inaccurate because they do not align with the question asked in the problem. This indicates that students' understanding, application, and reasoning regarding the concept of fractions are still very limited, as evidenced by their inability to solve and correctly answer problems.

Score 2 (Applying)

Fig 9. Student Answer with a Score of 2 on Problem 4

Students can accurately write the information and questions given in the problem, which indicates that they understand the situation well. The solution steps presented by students are correct in determining the number of children at Table B after Bima joins, and students can also choose the table that has the same number of children as Table B after Bima joins. However, students have not completed the solution completely because students do not write how many parts of the pudding each Bima and the other children sitting at table B get in fraction form. This indicates that students' understanding and application of the concept of fractions are good, but their reasoning regarding the concept of fractions remains low.

4.1.5 Problem 5

Table 10 Students answer of problem 5

Score 1 (Understanding)

Fig 10. Student Answer with Score 1 on Problem 5

Students have not written the information and questions given in the problem entirely and correctly. This indicates that they do not fully understand the situation. The strategy in the solution steps presented by students is incorrect, as they add up the temperature conditions for the ice cream to be considered frozen, which should not be necessary because the temperature is information used to draw conclusions or determine the answer to the question in the problem. Furthermore, students also add up the initial temperature of the two ice creams and the time it takes for the ice cream to rise by 1°C. The strategy presented by the students in the solution is certainly incorrect and flawed. This indicates that students' understanding, application, and

reasoning regarding the concept of fractions remain low, as they are unable to solve and answer problems accurately and completely.

Score 2 (Applying)

Fig 11. Student Answer with a Score of 2 on Problem 5

Students have written the information and questions given in the problem correctly, but have not yet completed them. The solution presented by the students is incorrect because they misunderstood the situation. Students immediately subtract 1 °C from each ice cream. This is wrong because 1°C represents the increase in temperature of the ice cream every minute. Students also misunderstand the meaning of temperature increase in the problem. This is proven by students subtracting the initial temperature of each ice cream by 1°C and -1°C, which shows how much the temperature increases for 8 minutes, and then adding it to the initial temperature of each ice cream. Next, students draw conclusions about the final conditions of the two ice creams. However, these conclusions are incorrect because the strategy used to determine the final temperature of each ice cream is flawed. This indicates that students' understanding is good, but their application and reasoning regarding the concepts of integers and fractions are still limited.

Score 3 (Reasoning)
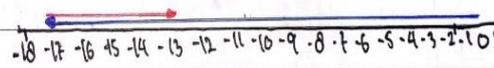
The student has written the information in the problem accurately and correctly, which indicates that the student has a good understanding of the situation. The steps in the solution given by the student are correct, where the student looks for the increase in temperature of each ice cream when

Fig 12. Student Answer with a Score of 3 on Problem 5
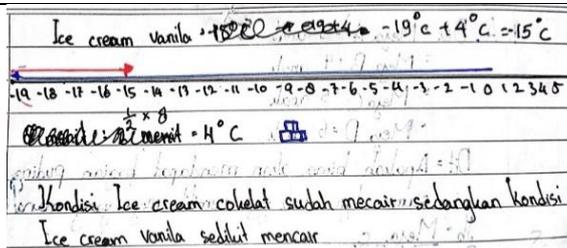
it reaches the cashier. Next, the student adds the temperature increase to each ice cream's initial temperature, and the results obtained are correct. Furthermore, the student can reasonably and correctly conclude the condition of both ice creams. This demonstrates that the student's understanding, application, and reasoning regarding integers and fractions are very good, as the student has been able to comprehend and apply the concepts of integers and fractions, and reason to create accurate and complete solutions to the given problems.

Analysis of Students' Numeracy Ability Based on Understanding, Application, and Reasoning Indicators

The analytical framework for numeracy skills in this study follows the taxonomy of the Ministry of Education and Culture, which divides numeracy skills into three main indicators: understanding, application, and reasoning. These three indicators reflect the progression of mathematical thinking from simply recognizing symbols and numerical facts to the ability to develop strategies and draw conclusions in complex contexts [4]. The following is an in-depth analysis of each indicator, linked to student performance on each test item.

- **Understanding**
  The ability to recognize, understand, and use various symbols and basic numbers to solve practical problems in everyday life. Understanding is at the beginning of the numeracy literacy dimension, representing the recognition of information without complex manipulation or strategies.
  1. Question 1: Students with a score of 1 correctly wrote the number of marbles, but multiplied all the numbers without context, even including irrelevant numbers (20 – 2 – 6), indicating that their understanding was literal procedural, not contextual conceptual.
  2. Question 2: Students copied the numbers and directions from the problem text, but constructed the number line incorrectly (positive numbers to the left of 0, the number 0 written twice), indicating a failure to establish the spatial relationship between numbers and basic numerical symbols.
  3. Question 3: Understanding of units was incomplete. Many students failed to convert kilograms to grams because they did not recognize that the fractions in the problem contained structured numerical meaning.

4. Questions 4 and 5: Some students wrote down basic information but did not proceed with the calculation process. In problem 5, for example, the student added the initial temperature and the "freezing condition" without realizing the informative function of the freezing temperature as an evaluative parameter, rather than a value to be operated on.

The student's understanding was at the level of literal recognition of the data and mathematical symbols. This suggests that their interpretation of the problem's context was still limited, and their understanding often fell into the trap of recognizing numbers without considering their numerical meaning.

- **Application**

    Students' ability to use mathematical strategies (e.g., arithmetic, conversion, distribution, measurement) appropriately to solve contextual problems. The application requires data manipulation using logical procedures, although this is not fully accompanied by argumentative reflection.
    1. Problem 1: A student with a score of 2 was able to add up the total number of marbles and divide them to obtain the average, but did not solve the final distribution as asked. This demonstrates the use of a partial strategy without consistency throughout.
    2. Problem 2: The student constructed a number line and calculated relative distances in a nearly correct, though incomplete, direction. This demonstrates the use of a visual model to approximate a contextual answer.
    3. Problem 3: The strategy for adding food ingredients was successfully implemented, but unit and fraction conversions were inaccurate. The error in the flour fraction demonstrates a conceptual mistake in unit transformation.
    4. Problem 4: The calculation of the number of children after the configuration change was performed correctly, but the distribution of the pudding was not recorded, indicating that the application stopped before the concept of proportionality was realised.
    5. Question 5: Many students can calculate the total temperature change, but make errors in the use of time and temperature direction. A common error is directly subtracting 1°C from the initial temperature, indicating that the meaning of the change per minute is not fully understood.

    Students have limited procedural skills in applying mathematical concepts. They can identify the necessary operations, but often stop midway or apply steps inconsistently with the context.

- **Reasoning**

    The ability to use analysis results and strategies to draw logistical conclusions, disseminate information, and make data-based predictions or decisions.
    1. Question 1: Only a small number of students achieved whole reasoning (score 3). They were able to determine a fair redistribution of marbles by calculating the mean and difference, and explain who should give what to whom. This reflects logical reasoning and distribution skills.

2. Question 5: Students who received a score of 3 were able to calculate the total temperature increase, add it to the initial temperature, and then deduce the final temperature state up to the freezing point. This demonstrates integrative skills in constructing a logistical narrative from quantitative information.

3. Questions 2, 3, and 4: Almost no students were able to reason correctly. In these questions, students only developed partial strategies without drawing explicit conclusions. For example, in question 4, none of the students solved the division of pudding into proportional fractions.

Students' reasoning skills were limited. This indicates that they are not yet accustomed to constructing complete numerical reasoning and lack experience in reflective thinking or providing solutions based on numerical data. The following is a summary of the distribution of abilities based on indicators.

Table 11. Summary of the Distribution of Abilities Based on Indicators

Indicator	General Characteristic based on the Findings	Dominant Question Item	Highest Percentage
Understanding	Students tended to copy data without constructing contextual meaning	1, 2, 3	±59%
Applying	Students demonstrated partial strategies and followed logical procedures, but solutions were often incomplete	2, 3, 4, 5	±14%
Reasoning	Students consistently applied strategies that led to coherent and logical conclusions	1, 5	±3%

4.2 Analysis Based on Learning Theory (Piaget, Bruner, Vygotsky)

4.2.1 Analysis Based on Piaget's Theory: Stages of Cognitive Development

According to Piaget theory [21], most students are still in the concrete operational stage of development. This means they can manipulate concrete objects and carry out basic procedures, but struggle with abstract thinking and developing logical strategies for more complex or non-routine problems. Analysis of the answers revealed that many students were able to complete only the initial steps of the problem without reaching a complete logical conclusion. This was evident in problems 1 and 5, where a small percentage of students successfully demonstrated logical reasoning and complete solutions (score 3). This suggests that only a small proportion of them have reached the formal operational stage, characterised by the ability to think systematically and predictively. However, in problems 2 (number lines) and 3–4 (fractions and proportions), many students demonstrated conceptual errors and illogical strategies. They, for example, placed positive numbers to the left of zero on the number line or multiplied whole numbers in problems without any basis.

At this stage, students tend to focus on things that can be directly observed and engage in manipulating tangible objects. This is reflected in students' difficulties in solving numeracy problems that require deeper reasoning or that do not have a single solution, such as those that test reasoning skills [11]. For example, students often struggle to connect the concepts of integers or fractions to real-life contexts, indicating a limitation in their abstract thinking skills. Therefore, in numeracy, many students struggle to fully integrate information and apply higher-order mathematical reasoning to solve more complex problems. Overall, according to Piaget theory, the abilities of students in the concrete operational stage are still limited to a basic understanding and application of simple mathematical procedures.

4.2.2 Analysis Based on Bruner's Theory: Learning Representation Mode

Based on Bruner's Learning Theory [22], learning must progress through three stages of representation: enactive (action), iconic (images), and symbolic (mathematical symbols). Referring to the results of the numeracy literacy analysis of students at SMP 14 Medan, many students are only at the basic and proficient levels, meaning they are still at the enactive stage, with a small number beginning to transition to the iconic stage. Bruner's enactive stage focuses on the manipulation of real objects and physical interactions, as evidenced by students' understanding of simple numeracy problems, such as recognising numbers and operating with them in concrete situations. However, students still struggle to transition to the iconic and symbolic stages, as reflected in their difficulty solving problems requiring the application of more abstract concepts.

The analysis shows that most students have not yet mastered symbolic representation, particularly in problems involving unit conversion (problem 3) and modelling fractions in a proportional context (problem 4). For example, students failed to connect visual information to the numerical logic of fractions. They made errors in solving temperature problems due to a lack of understanding of abstract representations of change. In problem 2, which is based on a number line, many students made errors in constructing the east-west direction visualisation and number sequence. This indicates a failure at the iconic representation stage, which should be the bridge to symbolic understanding. On the other hand, high-performing students were able to internalise visual information into logical symbolic strategies, demonstrating a successful transition between modes of representation.

4.2.3 Analysis Based on Vygotsky Theory: Zone of Proximal Development and Scaffolding

Based on Vygotsky's Sociocultural Theory [23], learning occurs in social interactions within the Zone of Proximal Development (ZPD), where students can reach their maximum potential with the assistance of more experienced individuals, such as teachers or peers. In the context of the results and analysis presented in this article, several relevant findings that can be explained using this approach are:

1. Listening in Solving Non-Routine Problems:
   The results show that many students at SMP 14 Medan struggled to solve non-routine problems, reaching only a basic level of understanding and application. This indicates that students are in their ZPD, where they can solve simple problems but struggle with problems that require deeper reasoning or the application of more complex concepts. According to Vygotsky, to overcome these difficulties, students need scaffolding (support) to help them develop their problem-solving skills. Learning that involves

direct guidance or support from more competent peers can help students overcome these difficulties.

2. Errors in Using Mathematical Concepts:
The analysis also shows that students frequently make errors in basic mathematical concepts, such as incorrect multiplication operations or misunderstandings about integers. In Vygotsky's theory, these errors can be minimised through structured scaffolding, where students are given opportunities to learn through social interaction and guided learning. For example, group discussions or guiding questions from teachers can help students understand and correct their errors in using these concepts.

3. Social Interaction in Learning:
Vygotsky emphasised the importance of social interaction in cognitive development. The results show that many students only reach the Basic or Moderate level, indicating that they require additional guidance and interaction to achieve the Advanced level in numeracy literacy. In this case, learning that involves scaffolding in the form of group discussions, cooperation, and collaboration can provide students with opportunities to expand their understanding by sharing ideas and helping each other solve problems.

5 Conclusion

This study aims to analyse the numeracy literacy skills of seventh-grade students at SMP Negeri 14 Medan based on indicators of understanding, application, and reasoning. The results show that the majority of students are at the basic and moderate levels, with only a small number reaching the advanced level. Most students can understand and apply basic mathematical concepts, but struggle to solve more complex numeracy problems, which require deeper reasoning. Based on Piaget's Theory, most students are still in the concrete operational stage, which means they have difficulty with abstract thinking and solving problems that require more complex information integration. In Bruner's Theory, students are more often in the enactive and iconic stages, which focus on manipulating concrete objects and visual representations. However, they still have difficulty transitioning to the symbolic stage, which is necessary for understanding and solving more abstract numeracy problems. Based on Vygotsky's Theory, the results indicate that many students are in the Zone of Proximal Development (ZPD), suggesting a need for scaffolding or additional support in the learning process. Social interaction, direct teacher guidance, and peer collaboration are crucial for helping students overcome their difficulties in solving more complex problems and for strengthening their reasoning skills. Overall, these findings underscore the importance of a more holistic and multi-level learning approach that incorporates concrete, visual, and symbolic representations and provides strong social support to help students achieve higher levels of numeracy.

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