Subway Track System: Fixed Station Spacing based Optimization Model

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Abstract. To achieve dual-carbon target and the fixed station spacing, this paper introduced an optimization model to achieve shortest running time and the lowest energy consumption. For this purpose, our work proposed an optimization mathematical model. The analytical results show the correctness and effectiveness of the model, and also provide theoretical and data support for the further optimization and improvement of the subway running scheme. This study has significance for improving the efficiency of subway operation and reducing the operating cost, providing theoretical support and technical guidance for the sustainable development of the subway industry, and is expected to promote the benign development of urban public transport.

Keywords: control strategy; Energy-saving operation; Urban rail transit; model

1 Introduction

With the acceleration of the urbanization process, a large number of people continue to migrate to the city, resulting in the rapid expansion of the scale of our country's cities, and the population of large and medium-sized cities increased rapidly. Compared with small and medium-sized cities, the pressure of public transportation and the difficulty of energy supply in large and medium-sized cities show a progressive increase. Therefore, urban underground railway has become the first choice of urban public transportation construction because of its fast running and controllable traffic time. With the improvement of subway lines, subway has become the first choice and dependence of urban residents. Passengers' demand for speed and punctuality has become the basic goal of subway route and time scheduling. At the same time that passengers travel more and more convenient, the energy consumption of the subway also gradually increases. With the idea of ecological civilization construction and the gradual implementation of dual-carbon goal strategy, energy saving and emission reduction has become a new goal of urban public transport management. Therefore, minimizing energy consumption under the condition of ensuring the shortest running time is a new topic in the design of urban subway running scheme. This has an active role in promoting the optimization of transport capacity and giving full play to the advantages of the subway network. Therefore, it is of great

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theoretical significance and engineering application value to study the optimal strategy of subway route.

2 Problem description

The running process of the train between the two platforms is shown in Figure 1. Its operation is more complex, and there is a maximum speed limit during operation. Based on the maximum speed limit, trains usually have four operating conditions: traction, cruise, idle, and brake, as shown in Figure2.

Fig. 1 Diagram of train operation process.

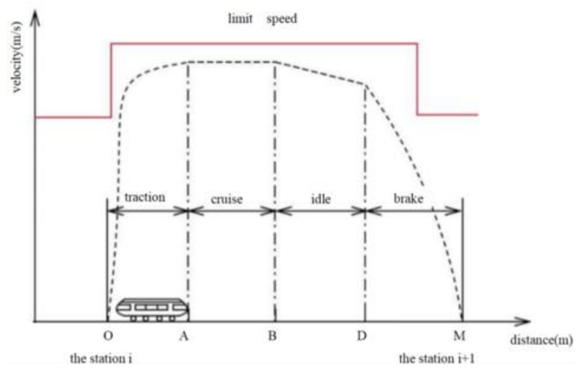

Fig. 2 Diagram of train operation condition.

Traction is a process of energy consumption. Traction forces need to consume electrical energy to overcome the resistance and do work, which is converted into kinetic energy of the train and generates some heat energy. Therefore, in this stage, the engine provides traction. In this stage, the engine is in the state of energy consumption, and the train is in the state of acceleration. In the cruise stage, the train moves at a constant speed, and the resultant force is zero. At this time, whether the train needs traction or braking depends on the total resistance of the train at that time. In the idle running stage, the train is neither traction nor braking, and does not need the engine to supply energy, its running state depends on the total resistance of the train, and the engine does not need to consume energy. In the braking stage, the train slows down and does not need the engine to supply energy, and the engine is still in a state of no energy consumption. If the distance between stations is short, the train generally adopts the strategy of "traction → idling → braking". If the distance between stations is long, the train usually uses traction until it approaches the maximum speed limit, then alternates between cruising, idling, traction, and braking is applied when approaching the platform.

If the train accelerates at maximum traction in the acceleration stage, adopts uniform speed and idling as much as possible in the middle stage, and uses maximum braking force when the train coming to the station, the energy consumption of the train will be minimal in this operation.

In the same journey, trains using different driving strategies will usually produce different time and energy consumption. This paper studies the strategy of trains running between the station i^{th} and the station $i+1$.

Assume that the train is running on a horizontal track, the distance between the i^{th} station and the station $i+1$ is S， the upper limit speed of the train is $v_{\max}$ ， the mass of the train is m ， the rotating mass factor of the inertia of the rotating parts is ρ，the maximum traction force is $F_{d\max}$ ， and the maximum braking force is $F_{s\max}$ .Body resistance meets Davis resistance： $f = 2.0895 + 0.0098v + 0.006v^2$.

3 Optimization model

3.1 Physical model of train motion

In this paper, the train is regarded as a simple point and the model of the simple point is adopted, and its motion follows Newton's second law. Its physical model as follows[1].

$$F_d = F_f + F_i + F_w + F_j \tag{1}$$

Where: F_d is the train traction force; F_f is rolling resistance; F_i is slope resistance; F_w is air resistance; F_j is acceleration resistance.

Since the train is running on a horizontal track, F_i is zero. The rolling resistance F_f and air resistance F_w satisfy the Davis resistance equation $F_r(v) = F_f + F_w = 2.0895 + 0.0098v + 0.006v^2$ given in this paper（v is train speed）. Combining the train mass m 、driving force F_d 、braking force F_s and rotating mass factor ρ given in this paper, Eq.(1) can be concretized as follows.

$$m\rho \frac{dv}{dt} = F_t - F_b - f \tag{2}$$

3.2 Train trajectory under the condition of minimum time

1) Determination of constraints

a)Acceleration phase (from point O to point A) During the acceleration phase, traction reaches its maximum, braking force is zero, and the train accelerates forward.

$$F_d - F_r(v) = \rho m \frac{dv}{dt}$$

$$F_d = F_{d\max} \tag{3}$$

$$\frac{dv}{dt} = \frac{F_{d\max}}{\rho m} - \frac{F_r(v)}{\rho m}$$

$$v = v(t) \quad 0 < t < t_A$$

$$s(t) = \int_0^t v(t)dt$$

$$s_A = \int_0^{t_A} v(t)dt$$

$$t(s) = t(s)\Big|_{0,s} \tag{4}$$

$$v = v(t(s)\Big|_{0,s}) \quad 0 < s < s_A \tag{5}$$

$$E_{OA}(s) = sF_{d\max} \quad 0 < s < s_A \tag{6}$$

$$v_A = v(t_A) \qquad v_O = 0$$

Where E is the energy consumption.

b)Uniform phase (from point A to point B) In this phase, traction is used to overcome the running resistance, the braking force is zero, the acceleration is zero, and the train is moving at a constant speed.

$$v = v(t_A) = v(t_B) \tag{7}$$

$$F_d = F_r(v(t_A)) \tag{8}$$

$$s = (t - t_A)v(t_A) + s_A \quad s_A < s < s_B \tag{9}$$

$$t(s) = \frac{s - s_A}{v(t_A)} + t_A \tag{10}$$

$$E_{AB}(s) = (s - s_A)F_r(v(t_A)) \quad s_A < s \leq s_B \tag{11}$$

c) Idling stage (from B to D) In this stage, the traction force and the braking force are zero, and the train is slowed down by the driving resistance.

$$F_r(v) = \rho m \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{F_r(v)}{\rho m}$$

$$v = v(t) \quad t_B < t < t_D$$

$$s(t) = \int_{t_B}^t v(t)dt + s_B$$

$$t(s) = t(s)\Big|_{s_B,s} + t_B \tag{12}$$

$$v(s)=v(t(s)\big|_{0,s}) \quad s_A < s \le s_B \tag{13}$$

$$F_d = 0 \quad E_{BD} = 0 \tag{14}$$

d) Braking stage (from D to M) In the braking stage, the traction force is zero, the braking force reaches its maximum, and the train is slowed down by the braking force and running resistance until it stops.

$$F_s = F_{s\max} \tag{15}$$

$$F_{s\max} + F_r(v) = \rho m \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{F_{s\max} + F_r(v)}{\rho m}$$

$$v = v(t)$$

$$s(t) = \int_{t_D}^{t} v(t)dt + s_D$$

$$t(s) = t(s)\big|_{s_D,s} + t_D \tag{16}$$

$$v(s) = v(t(s)\big|_{s_D,s}) \quad s_D < s \le s_M \tag{17}$$

$$E_{DM}(s) = F_{s\max}\int_{t_D}^{t_M} v(t)dt \quad s_D < s < s_M \tag{18}$$

3.3 Determination of the optimal change location of train motion state

$$\min_{s_A,s_B} t_A + (t_B - t_A) + (t_D - t_B) + (t_M - t_D) \tag{19}$$

$$s.t.\begin{cases} t_{\mathbf{B}} = t_D \\ S_B = S \\ v \le v_{\max} \end{cases}$$

t_A、t_B、t_M determined by Eq.(4), Eq.(10) and Eq.(12) respectively. Thus, S_A, S_B can be obtained under the condition of the shortest time, and it can be substituted into Eq.(3) to Eq.(18) to get the train motion track.

The speed-distance curve models of acceleration stage, uniform stage, coasting stage and braking stage are determined by Eq.(5), Eq.(7), Eq.(13) and Eq.(17) respectively. The traction/braking force - distance curve model is determined by Eq.(3), Eq.(8), Eq.(14) and Eq.(15) respectively. The time-distance curve model is determined by Eq.(4), Eq.(10), Eq.(12) and Eq.(16) respectively. The energy consumption -distance curve model is determined by Eq.(6), Eq.(11), Eq.(14) and Eq.(18) respectively.

4 Performance analysis

4.1 Numerical Results

In this paper, Runge Kutta method is used to solve the model. In numerical analysis, Runge Kutta method[2-6] is a classical algorithm for solving nonlinear ordinary differential equations, and it is an important implicit or explicit iterative method. Because it is a high-precision and single-step algorithm, it is widely used in engineering, including the famous Euler method, which is often used to solve differential equations numerically. Due to the high precision of this algorithm and the adoption of measures to suppress errors, its implementation principle is also complicated. The forms of the general Runge Kuta method as follows.

$$\begin{aligned} & y_{i+1} = y_i + c_1 K_1 + c_2 K_2 + \ldots + c_p K_p \\ & K_1 = hf(x_i, y_i) \\ & K_2 = hf(x_i + a_2 h, y_i + b_{21} K_1) \\ & \ldots\ldots\ldots\ldots\ldots \\ & K_p = hf(x_i + a_p h, y_i + b_{p1} K_1 + \ldots + b_{p,p-1} K_{p-1}) \end{aligned} \tag{20}$$

Where Eq.(20) is called Runge-Kutta method of P-order. The coefficients of a_i, b_{ij}, c_i, we need Taylor expansion of y_{i+1} in Eq.(19) at (x_i, y_i) to determine the parameters by the coefficients of the same terms.

The initial data in this article is[7]: The distance between the i^{th} platform and the platform $i+1$ is 5144.7m. The maximum speed $v_{\max}$ is 100km/h, the mass of the train body m is 176.3t, the rotating mass factor ρ is 1.08, and the maximum tractive force is 310KN. The maximum braking force $F_{s\max}$ is 760KN. The pre-processed data is brought into the above model, the calculation results are obtained based on Matlab software environment, and the curves are derived.

4.2 Simulation Results

Time-distance curve, speed-distance curve, energy-consumption-distance curve and traction braking power-distance curve are shown in Fig.3 to Fig.6 respectively.

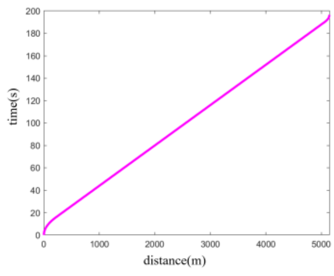

Fig. 3 Time-distance curve.

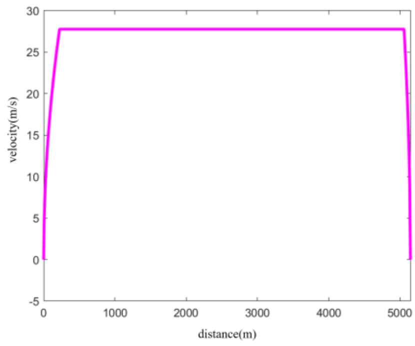

Fig.4 Velocity - distance curve.

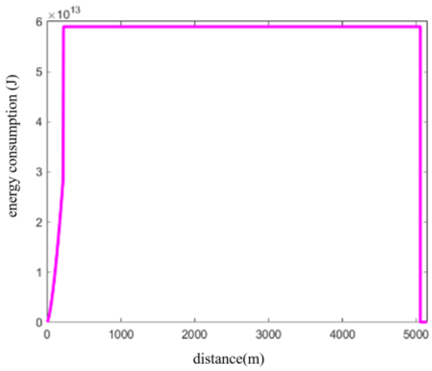

Fig.5 Energy consumption-distance curve.

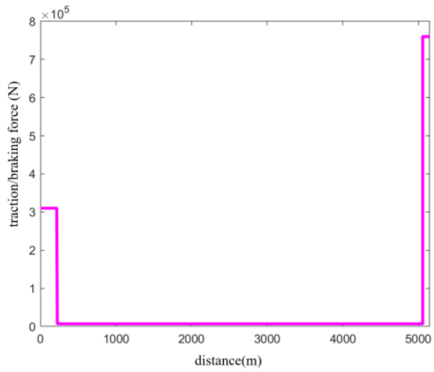

Fig. 6 Traction/braking force-distance curve.

4.3 Train Tracks Based on Minimum Time Extension

According to Eq. (19) , the shortest time is $t_{\min}$, in order to test the stability of the model, the minimum time $t_{\min}$ can be extended. In this paper, under the condition of the lowest energy consumption, the minimum time $t_{\min}$ extended 10 seconds. The objective function can be obtained based on formula (6), (11),(14)and (18), as shown below.

$$\min_{s_A,s_B,s_D} E = F_{d\max} s_A + F_r\left(v(t_A)\right)(s_M - s_B) + F_{s\max}\int_{t_B}^{t_M} v(t)\,dt \tag{21}$$

$$s.t.\begin{cases} t_A + (t_B - t_A) + (t_D - t_B) + (t_M - t_D) = t_{\min} + 10 \\ S_{\mathrm{M}} = S \\ v \le v_{\max} \end{cases}$$

Where t_A、t_B、t_M、t_D determined by Eq.(4), Eq.(10)，Eq.(12) and Eq.(16) respectively. Thus, S_A, S_B can be obtained, and substituted into Eq.(3) to Eq.(18) to get the train motion track.

Time-distance curve, speed-distance curve, energy-consumption-distance curve and traction braking power-distance curve are shown in Fig.7 to Fig.10 respectively.

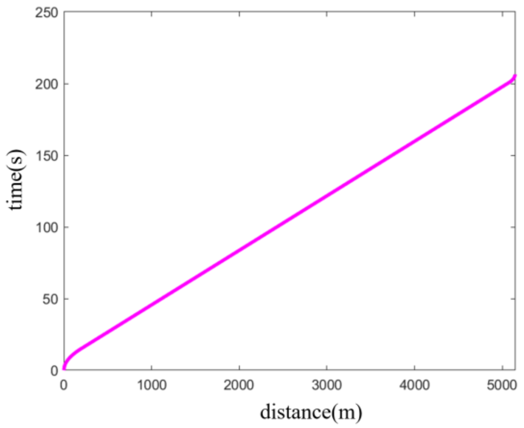

Fig.7 Time-distance curve.

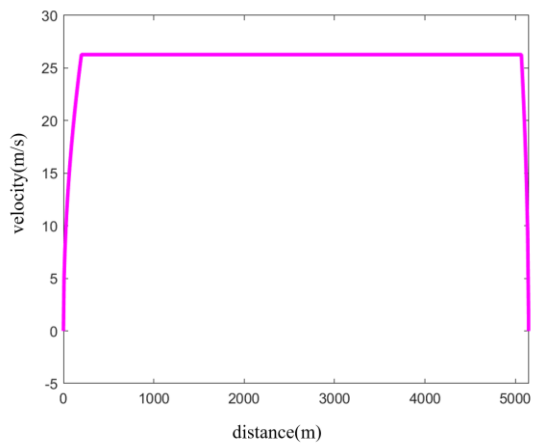

Fig.8 Velocity - distance curve.

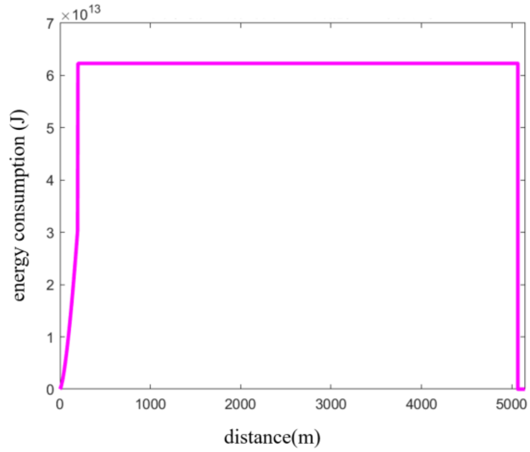

Fig.9 Energy consumption-distance curve.

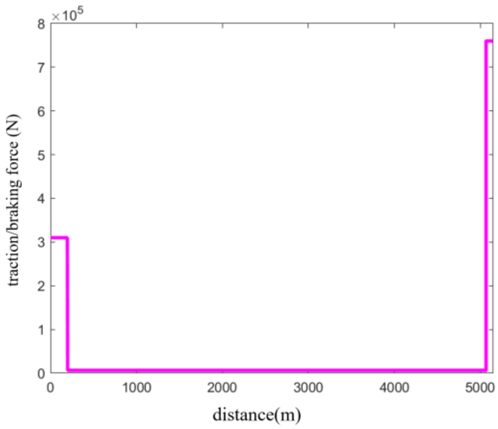

Fig.10 Traction/braking force-distance curve.

From the calculation results in fig.3 to fig.10, the following conclusions can be drawn:

(1) The shortest running time is 196.306s, about 3 minutes.

(2) The train runs through three stages: traction, cruising and braking.

(3)The change of working conditions has no obvious influence on the optimal running track.

As shown above, the theoretical operating conditions of the train are in accordance with the actual conditions, and verify the correctness and feasibility of the model established in this paper.

5. Conclusions

The Runge Kutta algorithm adopted in this paper has high precision, which ensures the accuracy of data calculation. The minimum time is about 3 minutes, which is consistent with the daily ride experience, and verifies the correctness and feasibility of the model and algorithm. The calculation results provide theoretical and data support for the further expansion and optimization of train trajectory.

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