

Research on Group Decision-Making Method under Incomplete Information and Its Application in Service Quality Evaluation of Nursing Homes

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Abstract: With the increasing complexity of decision-making problems, group decision-making problems that consider the similarity of opinions and the lack of evaluation information among DMs have become increasingly important. An interesting phenomenon is that decision makers often fail to provide complete evaluation information. Therefore, the research focus of this article is the group decision-making method under complete information. First of all, this paper proposes a new method for calculating the similarity of decision-maker's opinions based on the Jaccard coefficient, and on this basis, a method for constructing a decision-maker's opinion similarity network is proposed. Secondly, this paper proposes a decision-maker clustering method based on Louvain algorithm. In addition, an evaluation information completion method is proposed based on the KNN interpolation method, which can complete the missing parts of the evaluation information provided by the decision-maker. Then, a new two-stage feedback adjustment mechanism is constructed to guide the consensus reaching process. Based on this, this paper proposes a group decision-making method under incomplete information based on Louvain algorithm. Finally, the effectiveness of the method proposed in this paper is verified through a case study of service quality evaluation in nursing homes that includes comparative analysis.

Keywords: Incomplete information; Jaccard coefficient; KNN interpolation method; Louvain algorithm; Consensus reaching process

1 Introduction

In the real-life decision-making process, decision-makers (DMs) often need to select an optimal alternative from multiple alternatives. This process is called multi-attribute decision-making (Bączkiewicz et al., 2021^[1]). As decision-making problems become increasingly complex, it becomes increasingly important for multiple DMs to participate in the decision-making process. Therefore, some scholars have begun to pay attention to multi-attribute group decision-making problems (Sun et al., 2022^[2]). When the number of DMs is greater than or equal to 20, the multi-attribute group decision-making problem is also called a group decision-making problem (Meng et al., 2023^[5]). In recent years, there has been abundant research on group decision-making problems (Zhang et al., 2021^[4]; Xuan, 2022^[3]; Meng et al., 2023^[6]).

The current research focuses on group decision-making problems mainly include the following aspects. First, the study of clustering methods for decision makers. Secondly, a framework study on the consensus reaching process. Regarding the first question, existing research is usually based on the social network relationship between DMs and uses community detection methods to obtain subgroups in the decision maker group (Chao et al., 2021^[8]; Bu et al., 2023^[7]). With the deepening of research, some innovative decision-maker clustering methods have been proposed. Liu et al. (2022)^[10] proposed a new DM clustering algorithm based on social network analysis method. Li et al. 2022^[11] proposed a fast expansion algorithm that considers the trust relationship between DMs to cluster the decision maker group. Regarding the second question, some scholars have already conducted research. (2019)^[13] proposed a new framework for the consensus reaching process that considers the DM's confidence based on fuzzy preference relationships. Zhang et al. (2022)^[12] proposed a consensus reaching process framework based on trust evolution.

After sorting out the current research on group decision-making problems, it is not difficult to find that there are still the following issues that require further research. First of all, due to differences in knowledge levels, personal experiences, etc. among different DMs, decision makers often cannot provide complete evaluation information during the evaluation process. Therefore, there is usually a problem of missing evaluation information in group decision-making processes. Secondly, considering that group decision-making processes require full consideration of the opinions of each individual in the DM group, the missing parts of individual opinions can often be obtained through the opinions of other individual DMs. Therefore, it is necessary to study group decision-making problems where evaluation information is missing.

To sum up, the research content of this paper mainly includes the following points: (1) Based on the Jaccard coefficient, a new calculation method for DM's opinion similarity is proposed, and on this basis, a novel opinion of DM similarity is proposed What is a network and how to build it. (2) Based on the KNN interpolation method, an evaluation information completion method is proposed, which can complete the missing parts of the evaluation information provided by the DM through the individual opinions of other DMs. (3) By constructing a new two-stage feedback mechanism, a novel framework for the consensus reaching process is proposed.

2 Preliminaries

2.1 Group decision-making

Assume a set of DMs $E = \{E_1, E_2, \dots, E_M\}$, a DM weight vector $\omega = \{\omega_1, \omega_2, \dots, \omega_M\}$, where ω_k is the weight of the k th ($k = 1, 2, \dots, M$) DM. Assume that the criterion set $C = \{C_1, C_2, \dots, C_n\}$, $w = (w_1, w_2, \dots, w_n)$ is the criterion weight vector, where w_j represents the weight of the criterion C_j . Suppose $X = \{X_1, X_2, \dots, X_m\}$ is a set of alternatives. $P^k = (p_{ij}^k)_{m \times n}$ is evaluation information of DM E_k .

The collective decision matrix $P^C = (p_{ij}^C)_{m \times n}$ can be calculated using Eq. (1).

$$P_{ij}^C = \sum_{k=1}^M \omega_k P_{ij}^k \quad (1)$$

Assuming that $\varphi(X_i)$ is the comprehensive score of the alternative, it can be calculated according to Eq. (2).

$$\varphi(X_i) = \sum_{j=1}^n w_j P_{ij}^C \quad (2)$$

2.2 Graphs and Networks

In order to reduce the complexity of group decision-making problems, it is necessary to divide the DM group into subgroups. For example, a network can be constructed based on the similarity of opinions among DMs. In this network, nodes represent DMs, and edges represent the similarity of opinions between DMs. Graph theory is a powerful tool for analyzing networks. The concepts of graphs and networks are described below.

Definition 1 (Li et al., 2023^[9]) Assume that $G = (V, E)$ is an undirected graph, $V = \{v_1, v_2, \dots, v_n\}$ is a set composed of n nodes, and $E = \{E_1, E_2, \dots, E_M\}$ is a set composed of m edges.

Definition 2 (Li et al., 2023^[9]) Assuming that $G = (V, E)$ is an undirected graph and

$A = (a_{ij})_{n \times n}$ is the corresponding adjacency matrix, then $a_{ij} = \begin{cases} 1, (v_i, v_j) \in E \\ 0, otherwise \end{cases}$.

Definition 3 (Zhang et al., 2018^[15]) Given an undirected graph $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$ is the node set of graph G , $E = \{E_1, E_2, \dots, E_M\}$ is the edge set of graph G , and ST_i is the degree of node v_i , then $ST_i = \sum_{k=1, k \neq i}^n \phi(v_i, v_k)$. Among them, when there is a connecting edge between node v_i and node v_k , $\phi(v_i, v_k) = 1$; otherwise, $\phi(v_i, v_k) = 0$.

3 Clustering method for DMs based on Louvain algorithm

3.1 Construction of opinion similarity network

Definition 4 Suppose $P^k = (p_{ij}^k)_{m \times n}$ is evaluation information of E_k ($k = 1, 2, \dots, M$) of alternative X_i under criterion C_j . \hat{p}_{ij}^k indicates that the attribute value p_{ij}^k in the decision matrix P^k is currently known, \bar{p}_{ij}^k indicates that the attribute value p_{ij}^k in the decision matrix P^k is currently unknown, and the evaluation information missing adjacency matrix $P_b^k = (p_{bij}^k)_{m \times n}$ can be obtained using Eq. (3):

$$P_{bij}^k = \begin{cases} 1, & P_{ij}^k \rightarrow \tilde{P}_{ij}^k \\ 0, & P_{ij}^k \rightarrow \check{P}_{ij}^k \end{cases} \quad (3)$$

Before constructing the DM opinion similarity network, the opinion similarity between DMs is first calculated. This similarity is measured by the Jaccard coefficient, which captures the overlap of evaluation information between DMs. This paper focuses on the similarity between DMs under incomplete information, in order to better describe the similarity in evaluation habits and professional fields between DMs. Use Jaccard coefficient to obtain incomplete information position similarity.

Definition 5 Suppose $P^k = (p_{ij}^k)_{m \times n}$ is evaluation information of E_k ($k=1,2,\dots,M$) of alternative X_i under criterion C_j . The adjacency matrix of missing evaluation information is $P_b^k = (p_{bij}^k)_{m \times n}$. The location similarity of incomplete information can be expressed by Eq. (4):

$$w_{kk'} = \frac{\sum_{i=1}^m \sum_{j=1}^n P_{bij}^k P_{bij}^{k'}}{|mn| - \sum_{i=1}^m \sum_{j=1}^n \phi(p_{bij}^k, p_{bij}^{k'})} \quad (k, k' = 1, 2, \dots, M, k \neq k') \quad (4)$$

where $|mn|$ is the matrix dimension, that is, the total number of coordinates, and

$$\phi(p_{bij}^k, p_{bij}^{k'}) = \begin{cases} 1, & p_{bij}^k = 0 \text{ and } p_{bij}^{k'} = 0 \\ 0, & \text{otherwise} \end{cases}$$

represents the number of positions where information is

missing in the evaluation information provided by the two DMs.

Definition 6 Suppose $P^k = (p_{ij}^k)_{m \times n}$ is evaluation information of DMs E_k and $E_{k'}$ of alternative X_i under criterion C_j . The evaluation information missing adjacency matrix is $P_b^k = (p_{bij}^k)_{m \times n}$, and $w_{kk'}$ is the incomplete information position similarity between the opinions of DMs E_k and $E_{k'}$. Then the comprehensive similarity $S'_{kk'}$ between the opinions of DMs E_k and $E_{k'}$ can be calculated using Eq. (5).

$$S_{kk'} = \sqrt{w_{kk'} \cdot \frac{\sum_{i=1}^m \sum_{j=1}^n P_{ij}^k P_{ij}^{k'}}{\sum_{i=1}^m \sum_{j=1}^n (P_{ij}^k)^2 + \sum_{i=1}^m \sum_{j=1}^n (P_{ij}^{k'})^2 - \sum_{i=1}^m \sum_{j=1}^n P_{ij}^k P_{ij}^{k'}}} \quad (5)$$

where $S_{kk'} \in [0, 1]$, $k, k' = 1, 2, \dots, M, k \neq k'$. If $k = k'$, then $S'_{kk'} = 0$.

By calculating the comprehensive similarity $S_{kk'}$ ($k, k' = 1, 2, \dots, M$) between all DMs, the comprehensive similarity matrix $S = (S_{kk'})_{m \times m}$ of the DMs' opinions is finally obtained. Then, a reasonable similarity threshold is set to convert this comprehensive similarity matrix into an opinion similarity network, and an undirected graph $G = (V, E)$ is constructed, in which each DM is represented as a node in the network, and the weight of the edge represents the opinions between them. Similarity. Through this network, the visual similarity relationships between DMs can be obtained, which provides a basis for further clustering of DMs.

3.2 Clustering process of DMs

In order to solve the problem of incomplete information in group decision-making, the Louvain algorithm is used to perform subgroup clustering of the DM's opinions. The Louvain algorithm is a commonly used method for detecting community structure in networks. Its core idea is to allocate nodes to communities to maximize connectivity within the community. This section will use the Louvain algorithm to divide a large group of decision makers into different subgroups, where the DMs within each subgroup have significant opinion similarities. The construction of this subgroup structure helps reduce the impact of incomplete information on decision-making and provides a more organized and collaborative framework for group decision-making.

Given network $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$ is a set of n nodes representing DMs, and $E = \{E_1, E_2, \dots, E_m\}$ is a set of m edges representing similarity relationships. Use Louvain to divide the nodes in the network G into n' subgroups, namely $SG = \{SG_1, SG_2, \dots, SG_{n'}\}$. The specific process is as shown in Algorithm 1.

Algorithm 1: Obtain the optimal community division result.

Input: Decision maker similarity matrix S and initial network G .

Output: The community division result $SG = \{SG_1, SG_2, \dots, SG_g, \dots, SG_{n'}\}$.

Step1: Treat each DM as a separate community and create an initial community division $SG = \{SG_1, SG_2, \dots, SG_n\}$.

Step2: In order to obtain the most compact division within the community, try to move each DM v_k from its current community SG_k to the subgroup $SG_{k'}$, where each of its neighbor nodes $v_{k'}$ is located.

Step3: Calculate the change ΔQ of modularity Q for each node transfer. Modularity Q is an indicator used to measure the closeness within a community. The specific calculation is

$$Q = \frac{1}{2} \sum_{k,k'} S_{kk'} \sum_{k,k'} \left(S_{kk'} - \left(b_k b_{k'} / 2 \sum_{k,k'} S_{kk'} \right) \right) \delta(c_k, c_{k'}).$$

where $S_{kk'}$ represents the weight of the edge between node k and node k' .

$b_k = \sum_{k'} S_{kk'}$ is the sum of the weights of all edges connected to the node k . The same is

true for $b_{k'}$; $\sum_{k,k'} S_{kk'}$ is the sum of the number of all edges. $\delta(c_k, c_{k'})$ indicates

whether node k and node k' are in the same community. If node n and node m are in a community, the value of $\delta(c_i, c_j) = 1$, otherwise $\delta(c_i, c_j) = 0$.

Step4: If $\Delta Q > 0$, move node k to the community where k' is located, otherwise it remains unchanged.

Step5: Repeat Step2 to Step4 until $\Delta Q = 0$ or ΔQ is very small.

Step5: Output the final community division result $SG = \{SG_1, SG_2, \dots, SG_g, \dots, SG_{n'}\}$.

The main advantage of the Louvain algorithm is that it can automatically detect the community structure without pre-setting the number of communities, so it is very suitable for community

clustering of group decision-making problems. The iterative process of the algorithm ensures the robustness of the results and can cope with group decision-making under incomplete information.

4 A novel group decision-making method under incomplete information

4.1 Missing evaluation information estimation

For information completion in matrices with incomplete information, we introduce the sequential K-nearest neighbor (KNN) ^[14] interpolation method to estimate the missing values in order based on the proportion of missing information. In finding the DM's nearest neighbors, the most commonly used method is to calculate the similarity between decision evaluation information.

Definition 7 Suppose that DM E_k is missing a coordinate p_{ij}^k . In the group SG where E_k is located, there are $2m$ DMs, then $k = m$. If there are $2m+1$ DMs, then $k = (2m+1-1)/2$. Taking the coordinates $p_{ij}^{k'}$ of m DMs $E_{k'}$ with the greatest similarity to E_k , the missing coordinates of d_k can be obtained by using Eq. (6) to obtain the missing information:

$$p_{ij}^k = \frac{1}{m} \cdot \sum_{m=1}^m V_{ij}^{k'} \quad (6)$$

For any DM with an incomplete information matrix, this method can be used to complete the decision-making information, thereby obtaining a complete decision-making matrix and more accurate calculation results. When there are not enough DMs in the community to provide decision information for a certain DM, the DM information with the highest similarity in adjacent communities can be obtained.

4.2 Consensus measure

According to the definition in Section 2.1, in order to determine whether the k th DM's opinion in round t is consistent with the collective opinion, we can use Eq. (7) to calculate the degree of consensus $CD^{k,t}$ between opinion of DM E_k in round t and the collective opinion.

$$CD^{k,t} = \frac{\sum_{i=1}^m \sum_{j=1}^n |p_{ij}^{k,t} - p_{ij}^{C,t}|}{mn} \quad (7)$$

The consensus level $GCD^{g,t}$ of community SG_g in round t can be calculated according to Eq. (8), where $\#SG_g$ represents the number of DMs in the community SG_g .

$$GCD^{g,t} = \frac{\sum_{k=1}^{\#SG_g} CD^{k,t}}{\#SG_g} \quad (8)$$

4.3 Opinion adjustment process

In order to judge whether the opinions of community SG_g in the round reach consensus with the collective opinions, we set the consensus threshold ε , where $\varepsilon \in (0,1]$. If \mathcal{G} exists and $GCD^{g,t} < \varepsilon$ is satisfied, it means that the collective opinions of round t have not reached a consensus, and DM E_k needs to adjust his opinions. If \mathcal{G} is satisfied for any $GCD^{g,t} \geq \varepsilon$, it means that the collective opinion of round t reaches a consensus. The updated opinion of DM E_k in community SG_g can be obtained through Eq. (9).

$$p_{ij}^{k,t+1} \in \left[\min(p_{ij}^{k,t}, p_{ij}^{C,t}), \max(p_{ij}^{k,t}, p_{ij}^{C,t}) \right] \quad (9)$$

4.4 Selecting alternatives

Assuming that the collective opinion has reached a consensus in round t , we can use Eq. (2) to calculate the comprehensive score $\varphi(X_i)$ of all alternatives based on the collective decision matrix $P^C = (p_{ij}^C)_{m \times n}$. Usually, the alternative with the highest overall score is the optimal alternative.

4.5 The main flow of the method

The main process of this paper is as described in Algorithm 2.

Algorithm 2: Group decision-making method under incomplete information.

Input: Incomplete decision matrix (evaluation information) provided by each DM.

Output: the collective decision-making matrix and final solution ranking after reaching consensus on the collective opinions.

Step1: DM E_k gives the decision matrix $P^k = (p_{ij}^k)_{m \times n}$, where $k = 1, 2, \dots, M$.

Step2: Calculate the similarity matrix $S = (S_{kk'})_{m \times m}$ using incomplete information. Based on incomplete preference information, Eq. (5) is used to calculate the similarity between DMs, and finally the similarity matrix between all DMs is obtained.

Step3: Constructing opinion similarity networks G .

Step4: Use **Algorithm 1** to obtain the optimal community division result of the DM.

Step5: Using Eq. (6), the missing evaluation information of each DM is obtained.

Step6: Using Eq. (7) to calculate the degree of consensus $CD^{k,t}$ between opinion of DM E_k in round t and the collective opinion.

Step7: If \mathcal{G} exists and $CD^{g,t} < \varepsilon$ is satisfied, it means that the collective opinions of round t have not reached a consensus, and DM E_k needs to adjust his opinions. If \mathcal{G} is satisfied for any $CD^{g,t} \geq \varepsilon$, it means that the collective opinion of round t reaches a consensus. The updated opinion of DM E_k in community SG_g can be obtained through Eq. (9).

Step8: When the opinions of all communities have reached consensus with the collective opinion, the comprehensive score of each alternative is calculated using Eq. (2). The alternative with the largest comprehensive score is the optimal alternative.

5 Case study

As China's population continues to age, the need for nursing homes for the elderly has become increasingly apparent. Recently, we invited 25 experts (decision makers) to evaluate the service quality of four nursing homes (X_1, X_2, X_3, X_4). These decision makers evaluated the four nursing homes in terms of service time (C_1), service attitude (C_2), and service sustainability (C_3).

Step 1: The decision matrix $P^k = (p_{ij}^k)_{m \times n}$ provided by DM E_k ($k = 1, 2, \dots, M$) is as follows.

$$\begin{aligned}
 P^{1,1} &= \begin{pmatrix} 0.450 & 0.683 & 1.000 \\ 1.000 & 0.000 & 0.267 \\ 0.000 & 0.106 & 0.545 \\ 0.370 & - & - \end{pmatrix} & P^{1,2} &= \begin{pmatrix} 0.000 & 0.175 & 0.429 \\ 0.900 & 1.000 & 0.771 \\ - & - & 1.000 \\ 0.350 & 0.807 & 0.000 \end{pmatrix} & P^{1,3} &= \begin{pmatrix} 1.000 & 0.640 & 0.770 \\ 0.750 & - & 0.500 \\ 0.250 & - & 0.350 \\ - & 0.320 & 0.000 \end{pmatrix} \\
 P^{1,4} &= \begin{pmatrix} 0.000 & 0.179 & 0.431 \\ 0.755 & 1.000 & 0.760 \\ 1.000 & 0.000 & 1.000 \\ 0.322 & 0.877 & 0.000 \end{pmatrix} & P^{1,5} &= \begin{pmatrix} 0.440 & - & 1.000 \\ 1.000 & 0.000 & 0.000 \\ - & 0.106 & 0.450 \\ 0.420 & 1.000 & 0.000 \end{pmatrix} & P^{1,6} &= \begin{pmatrix} 0.460 & 1.000 & 0.640 \\ 0.745 & 0.245 & 0.000 \\ 1.000 & 0.450 & 1.000 \\ 0.000 & 0.000 & 0.320 \end{pmatrix} \\
 P^{1,7} &= \begin{pmatrix} 0.475 & 0.430 & 1.000 \\ 1.000 & - & 0.260 \\ 0.000 & - & 0.457 \\ 0.520 & 1.000 & 0.000 \end{pmatrix} & P^{1,8} &= \begin{pmatrix} 1.000 & 0.610 & 0.780 \\ 0.730 & 0.000 & 1.000 \\ 0.260 & 1.000 & 0.360 \\ 0.000 & 0.350 & 0.000 \end{pmatrix} & P^{1,9} &= \begin{pmatrix} 0.550 & 0.683 & 1.000 \\ 1.000 & 0.000 & 0.255 \\ 0.000 & 0.240 & 0.550 \\ 0.380 & 1.000 & 0.000 \end{pmatrix} \\
 P^{1,10} &= \begin{pmatrix} - & - & 0.429 \\ 0.950 & 1.000 & 0.771 \\ - & - & 1.000 \\ 0.321 & - & 0.000 \end{pmatrix} & P^{1,11} &= \begin{pmatrix} 0.000 & - & 0.000 \\ 1.000 & 0.487 & 0.623 \\ 0.470 & 1.000 & 0.450 \\ 0.566 & 0.135 & 1.000 \end{pmatrix} & P^{1,12} &= \begin{pmatrix} - & 0.482 & 1.000 \\ 0.000 & - & 0.000 \\ - & 0.000 & 0.347 \\ - & 1.000 & - \end{pmatrix} \\
 P^{1,13} &= \begin{pmatrix} 1.000 & 0.951 & 0.392 \\ 0.341 & 1.000 & 1.000 \\ 0.231 & 0.000 & 0.420 \\ 0.000 & 0.560 & 0.000 \end{pmatrix} & P^{1,14} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.500 & 0.328 \\ 0.762 & 1.000 & 0.317 \\ 0.649 & 0.000 & 1.000 \end{pmatrix} & P^{1,15} &= \begin{pmatrix} 1.000 & 0.960 & 0.346 \\ 0.545 & - & - \\ - & 0.000 & - \\ 0.000 & 0.320 & 0.000 \end{pmatrix} \\
 P^{1,16} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.369 & 0.213 \\ 0.871 & 1.000 & 0.650 \\ 0.258 & 0.876 & 1.000 \end{pmatrix} & P^{1,17} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.545 & 0.874 \\ 0.231 & 1.000 & 0.334 \\ 0.333 & 0.667 & 1.000 \end{pmatrix} & P^{1,18} &= \begin{pmatrix} 1.000 & - & - \\ - & 1.000 & 1.000 \\ - & 0.000 & 0.471 \\ 0.000 & - & - \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
P^{l,19} &= \begin{pmatrix} - & 0.000 & 0.000 \\ 1.000 & 0.345 & - \\ - & - & 0.774 \\ 0.361 & 0.221 & - \end{pmatrix} & P^{l,20} &= \begin{pmatrix} 1.000 & 0.637 & 0.456 \\ 0.364 & 0.000 & 1.000 \\ 0.589 & 0.247 & 0.000 \\ 0.000 & 0.036 & 0.841 \end{pmatrix} & P^{l,21} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.451 & 0.258 \\ 0.324 & 1.000 & 0.314 \\ 0.587 & 0.056 & 1.000 \end{pmatrix} \\
P^{l,22} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.545 & 0.874 \\ 0.231 & 1.000 & 0.334 \\ 0.333 & 0.667 & 1.000 \end{pmatrix} & P^{l,23} &= \begin{pmatrix} 1.000 & 0.344 & 0.915 \\ 0.251 & 1.000 & 1.000 \\ 0.346 & 0.000 & 0.751 \\ - & - & - \end{pmatrix} & P^{l,24} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.343 & 0.620 \\ 0.514 & 1.000 & 0.424 \\ 0.630 & 0.171 & 1.000 \end{pmatrix} \\
P^{l,25} &= \begin{pmatrix} 1.000 & 0.035 & 0.144 \\ 0.254 & 1.000 & 1.000 \\ 0.364 & 0.000 & 0.365 \\ 0.000 & 0.221 & 0.000 \end{pmatrix}
\end{aligned}$$

Step 2-3: The opinion similarity network G is shown in Fig. 1.

Step 4: Use the Louvain algorithm to divide DMs into groups. Obtain the following clustering results:

$SG_1 : [1, 3, 5, 7, 9, 12, 15]$; $SG_2 : [11,14,16, 17, 21, 22, 24]$; $SG_3 : [8,13,18, 20, 23, 25]$; $SG_4 : [2, 4,10, 19]$; $SG_5 : [6]$. Fig. 2 clearly depicts the group division results.

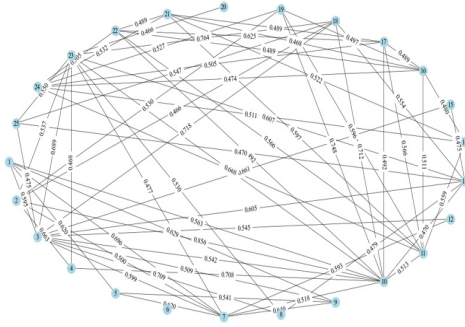


Fig. 1. Similarity network.

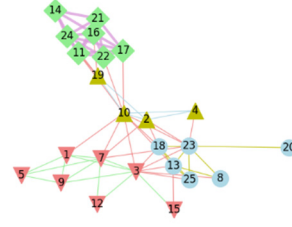


Fig. 2. Group division results.

Step 5: Missing information estimation. Using Eq. (6), the missing elements in the decision matrix of each DM with missing information can be calculated, and the complete decision matrix of each DM can be obtained.

$$\begin{aligned}
P^{l,1} &= \begin{pmatrix} 0.450 & 0.683 & 1.000 \\ 1.000 & 0.000 & 0.267 \\ 0.000 & 0.106 & 0.545 \\ 0.370 & 1.000 & 0.000 \end{pmatrix} & P^{l,2} &= \begin{pmatrix} 0.000 & 0.175 & 0.429 \\ 0.900 & 1.000 & 0.771 \\ 0.673 & 0.000 & 1.000 \\ 0.350 & 0.807 & 0.000 \end{pmatrix} & P^{l,3} &= \begin{pmatrix} 1.000 & 0.640 & 0.770 \\ 0.750 & 0.500 & 0.500 \\ 0.250 & 0.500 & 0.350 \\ 0.000 & 0.320 & 0.000 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
P^{1,5} &= \begin{pmatrix} 0.440 & 0.557 & 1.000 \\ 1.000 & 0.000 & 0.000 \\ 0.000 & 0.106 & 0.450 \\ 0.420 & 1.000 & 0.000 \end{pmatrix} & P^{1,7} &= \begin{pmatrix} 0.475 & 0.430 & 1.000 \\ 1.000 & 0.000 & 0.260 \\ 0.000 & 0.173 & 0.457 \\ 0.520 & 1.000 & 0.000 \end{pmatrix} & P^{1,10} &= \begin{pmatrix} 0.500 & 0.088 & 0.429 \\ 0.950 & 1.000 & 0.771 \\ 0.673 & 0.000 & 1.000 \\ 0.321 & 0.293 & 0.000 \end{pmatrix} \\
P^{1,11} &= \begin{pmatrix} 0.000 & 0.018 & 0.000 \\ 1.000 & 0.487 & 0.623 \\ 0.470 & 1.000 & 0.450 \\ 0.566 & 0.135 & 1.000 \end{pmatrix} & P^{1,12} &= \begin{pmatrix} 0.738 & 0.482 & 1.000 \\ 0.000 & 0.000 & 0.000 \\ 0.125 & 0.000 & 0.347 \\ 0.46 & 1.000 & 0.000 \end{pmatrix} & P^{1,15} &= \begin{pmatrix} 1.000 & 0.960 & 0.346 \\ 0.545 & 0.500 & 0.750 \\ 0.241 & 0.000 & 0.385 \\ 0.000 & 0.320 & 0.000 \end{pmatrix} \\
P^{1,18} &= \begin{pmatrix} 1.000 & 0.492 & 0.843 \\ 0.501 & 1.000 & 1.000 \\ 0.298 & 0.000 & 0.471 \\ 0.000 & 0.440 & 0.000 \end{pmatrix} & P^{1,19} &= \begin{pmatrix} 0.000 & 0.000 & 0.000 \\ 1.000 & 0.345 & 0.623 \\ 0.492 & 1.000 & 0.774 \\ 0.361 & 0.221 & 0.500 \end{pmatrix} & P^{1,23} &= \begin{pmatrix} 1.000 & 0.344 & 0.915 \\ 0.251 & 1.000 & 1.000 \\ 0.346 & 0.000 & 0.751 \\ 0.161 & 0.440 & 0.000 \end{pmatrix}
\end{aligned}$$

Step 6-8: We set the threshold $\varepsilon = 0.36$. After 2 rounds of adjustments, the consensus levels of all subgroups have reached the required level. The $GCD^{1,2} = 0.4004$, $GCD^{2,2} = 0.3737$, $GCD^{3,2} = 0.3995$, $GCD^{4,2} = 0.3948$, $GCD^{5,2} = 0.4000$, . Therefore, the collective opinion has reached a consensus at this time. The consensus degree of each DM are shown in Table 1.

Table 1. The consensus degree of each DM.

DM E_k	$CD^{k,2}$	DM E_k	$CD^{k,2}$	DM E_k	$CD^{k,2}$
k=1	0.4140	k=10	0.3940	k=19	0.3950
k=2	0.3920	k=11	0.3740	k=20	0.4050
k=3	0.3780	k=12	0.4010	k=21	0.3850
k=4	0.3980	k=13	0.3960	k=22	0.3760
k=5	0.4140	k=14	0.3600	k=23	0.3980
k=6	0.4000	k=15	0.4130	k=24	0.3820
k=7	0.3950	k=16	0.3630	k=25	0.3950
k=8	0.4040	k=17	0.3760		
k=9	0.3880	k=18	0.3990		

The collective decision matrix $P^{C,2} = (p_{ij}^{C,2})_{m \times n}$ is shown below.

$$P^{C,2} = \begin{pmatrix} 0.5360 & 0.4617 & 0.7734 \\ 0.8247 & 0.7161 & 0.7560 \\ 0.5447 & 0.3083 & 0.4687 \\ 0.3953 & 0.6052 & 0.2737 \end{pmatrix}$$

Therefore, according to equation (2), we can get the comprehensive scores $\varphi(X_1) = 0.5904$, $\varphi(X_2) = 0.7656$, $\varphi(X_3) = 0.4406$, $\varphi(X_4) = 0.4247$. Obviously, we obtain the final alternative ranking is $X_2 \succ X_1 \succ X_3 \succ X_4$, and the optimal alternative is X_2 .

6 Conclusions

This paper proposes a novel group decision-making method that can deal with the problem of incomplete evaluation information given by decision makers. Specifically, by introducing the Jaccard coefficient, we proposed an innovative method for calculating the similarity of DM's opinions, and based on this, we constructed a DM's opinion similarity network. To handle the clustering of DMs, we adopt a method based on Louvain algorithm. In the face of incomplete evaluation information, we introduced the KNN interpolation method and successfully completed the missing parts of the evaluation information provided by the DM. In order to guide the consensus-reaching process, we designed a novel two-stage feedback adjustment mechanism to further improve the efficiency and accuracy of group decision-making.

We conducted a case study on the service quality evaluation problem in nursing homes using the proposed method to verify the effectiveness of this method. This paper provides new ideas and methods for dealing with decision-making problems in groups, and is especially suitable for situations where DMs provide incomplete information.

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