Momentum effect based on stochastic dominance theory —— Evidence from Chinese Shanghai Stock Exchange A-share

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Abstract. This paper employs portfolio optimization models based on second-order stochastic dominance and "super-convex" third-order stochastic dominance, comparing them with an equal-weight portfolio optimization model. Through the computation of out-of-sample indicators for portfolio evaluation, it is found that, in the Shanghai Stock Exchange A-share market, the portfolio based on stochastic dominance theory exhibits a significant momentum effect, leading to substantial excess returns when the formation period is relatively long compared to the benchmark returns.

Keywords: Stochastic dominance; Portfolio optimization; Momentum effect

1 Introduction

The Efficient Market Hypothesis (EMH) suggests that unless there is market manipulation, investors cannot obtain returns higher than the market average by analyzing past prices. However, since the 1980s, many empirical studies have shown that there are many phenomena in the stock market that do not conform to the EMH. Among them, momentum effect is a more typical one, which is manifested by the trend of stock returns to continue in the original direction of movement, that is, stocks with higher returns over a period of time in the past will still have higher returns than stocks with lower returns in the past in the future.

The earliest study of momentum effect was conducted by Jegadeesh and Titman[1], who found that the strategy of buying stocks with good past performance and selling stocks with poor past performance (JT price momentum strategy) in the US stock market can generate significant positive returns during a holding period of 3 to 12 months. Since the discovery of momentum effect by Jegadeesh and Titman, it has been widely studied in financial markets. Many studies have confirmed the robustness of momentum strategies[2][3][4]. However, for new emerging stock markets such as China, traditional momentum strategies do not seem to be significant[5][6]. A major reason may be that traditional momentum strategies only consider the historical returns of stocks and ignore other risk indicators. The most extensively studied portfolio optimization model in existing literature is the mean-variance (MV) framework[7][8]. This model considers not only the returns of stocks but also the variance of those returns. However, a limitation of this model is that it does not take into account the non-normal distribution of returns, as returns may not necessarily follow a normal distribution[9]. To overcome this limitation, this paper aims

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to investigate the momentum effect of investment strategies based on the stochastic dominance (SD) theory in the Chinese Shanghai Stock Exchange A-share market.

SD theory is an approach that can provide partial ordering based on partial information about investors' utility functions, initially proposed by Lehmann[10]. This theory does not require any assumption about the distribution of asset returns. Building on these advantages, many scholars have applied SD theory to portfolio optimization. For example, Post and Kopa[11] improved upon existing "superconvex" SD conditions by relying on properties of the lower partial moments and developed portfolio optimization model with "superconvex" third-order SD (SCTSD) constraints. Portfolios constructed based on this model were compared to the Center for Research in Security Prices all-share index, and it was found that they achieved an average annual increase in out-of-sample returns of 7%. Post et al.[12] proposed a portfolio optimization approach based on a combination of SD decision criteria and empirical likelihood estimation. When applied to momentum strategies in the stock industry, this method produced significant improvements in out-of-sample performance compared to heuristic diversification and MV optimization. Kouaissah[13] proposed a two-step optimization problem, using a stochastic dominance method in the first optimization step to determine the effective assets of risk-averse investors, thereby greatly improving investors' out-of-sample returns. However, few scholars have used investment strategies based on the SD theory to explore the existence of momentum effects in the Chinese Shanghai Stock Exchange A-share market.

The following of this paper is organized as follows. In section 2, we will introduce SD theory, and portfolio optimization models with SD constraints. In section 3, we will analyze the empirical results. In section 4, we will summarize and present the conclusions.

2 Portfolio optimization with stochastic dominance constraints

In SD theory, the most widely used are primarily three types: first-order SD (FSD), second-order SD (SSD), and third-order SD (TSD). Among SD rules, the most commonly used ones in the literature are SSD and TSD. SSD assumes that investors are risk-averse, while TSD assumes that investors have a decreasing absolute risk aversion.

Suppose there are N different risky base assets. The joint probability distribution of their returns $F(r_1, r_2, \cdots, r_N)$ are discrete with a discrete joint probability distribution with T mutually exclusive and exhaustive scenarios. Let r_{it} denote the return rate of asset i under scenario t with $p_t, t = 1, \cdots, T$, which satisfies $\sum_{t=1}^{T} p_t = 1$. A portfolio is a convex combination of base assets characterized by a vector $\lambda = (\lambda_1, \cdots, \lambda_N)$, where λ_i denote the proportion of asset \$i\$ in the portfolio and $\sum_{i=1}^{N} \lambda_i = 1$. The return of the portfolio is random and depends on the return of base assets as well as their proportion, which is denoted by $X = \sum_{i=1}^{N} \lambda_i r_{it}, t = 1, \cdots, T$. Let Y to denote the return of a given benchmark which are also discrete with scenario $s = 1, \cdots, T_b$ with probability p_s. The return rate of Y under scenario s is y_s and we assume that $y_1 \leq y_2 \leq \cdots \leq y_{T_b}$. Without loss of generality, we assume that return rate is bounded and is in the range $[a, b]$, $-\infty < a \leq b < \infty$. This section introduce portfolio optimization models aim at identifying the best portfolio that dominates the benchmark portfolio by SSD and TSD.

2.1 Portfolio optimization with second-order stochastic dominance constraints

Let $F(x)$ and $G(x)$ be the cumulative probability distribution of random variable X and Y. SSD is related to risk measure the first-order lower partial moment (LPM), which is defined to be $F^{(2)}(x) = \int_a^x F(t)\,dt$ and $G^{(2)}(x) = \int_a^x G(t)\,dt$.

Definition 1 (SSD). X dominates Y by SSD if and only if $F^{(2)}(x) \leq G^{(2)}(x)$ for all values x, and there is at least some x_0 for which a strong inequality holds.

For a given portfolio λ, its first-order LPM at x equals:

$$F_\lambda^{(2)}(x) = \int_a^x F_\lambda(t)\,dt = \mathbb{E}_{F_\lambda}\left[(x - \sum_{i=1}^n \lambda_i r_{it}) I_{\lambda,t}(x)\right] = \sum_{t=1}^T p_t (x - \sum_{i=1}^n \lambda_i r_{it})\, I_{\lambda,t}(x) \tag{1}$$

where $I_{\lambda,t}(x)$ is an indicator function that takes value 1 when $\sum_{i=1}^n \lambda_i r_{it} \leq x$, and 0 otherwise. Dentcheva and Ruszczyński[14] proposed a linear programming model for portfolio optimization with SSD constraints.

$$\begin{aligned} &\max_{\lambda,\theta} \sum_{t=1}^T p_t \sum_{i=1}^N \lambda_i r_{it} \\ s.t.\ \ &\theta_{st} \geq y_s - \sum_{i=1}^n \lambda_i r_{it}, t = 1, \cdots, T; s = 1, \cdots, T_b \\ &\sum_{t=1}^T p_t \theta_{st} \leq G^{(2)}(y_s), s = 1, \cdots, T_b \\ &\sum_{t=1}^T p_t \sum_{i=1}^N \lambda_i r_{it} \geq \sum_{s=1}^{T_b} p_s y_s \\ &\sum_{i=1}^N \lambda_i = 1 \\ &\theta_{st}, \lambda_i \geq 0, t = 1, \cdots, T; s = 1, \cdots, T_b, i = 1, \ldots, N \end{aligned} \tag{2}$$

2.2 Portfolio optimization with third-order stochastic dominance constraints

TSD is another commonly used SD rules, which is proposed by TSD was first proposed by Whitmore[15]. It is defined based on second-order LPM, which is defined to be $F^{(3)}(x) = \int_a^x F^{(2)}(t)\,dt$.

Definition 2 (TSD). X dominates Y by TSD if and only if $F^{(3)}(z) \leq G^{(3)}(z)$ for all values z, and$\mathbb{E}_F[X] \geq \mathbb{E}_G[Y]$ and there is at least one strict inequality.

TSD is more consistent with investors' psychology. Based on **Definition 2**, Post and Kopa[11] proposed the concept of SCTSD. Given a portfolio λ, its second-order LPM at x equals:

$$F_\lambda^{(3)}(x) = \int_a^x F_\lambda^{(2)}(t)\,dt = \mathbb{E}_{F_\lambda}\left[(x - \sum_{i=1}^n \lambda_i r_{it})^2 I_{\lambda,t}(x)\right] = \sum_{t=1}^T p_t (x - \sum_{i=1}^n \lambda_i r_{it})^2\, I_{\lambda,t}(x) \tag{3}$$

Similary, second-order LPM of the benchmark portfolio equals:

$$G^{(3)}(x) = \int_a^x G^{(2)}(t)\,dt = \mathbb{E}_G[(x - y_s)^2 I(y_s \leq x)] \tag{4}$$

where $I(y_s \leq x)$ is an indicator function that takes value of 1 if $y_s \leq x$ and otherwise 0. Based on LPM, Post and Kopa[11] defines a series of data-dependent tolerance parameters $\varepsilon_s, s = 1, \cdots, T$ in a way that $\varepsilon_1, \varepsilon_2 \coloneqq 0$ and:

$$\varepsilon_s := \frac{G^{(3)}(y_s)}{G^{(3)}(y_{s-1})+2G^{(2)}(y_{s-1})(y_s-y_{s-1})} - 1, s = 3, \cdots, T_s \tag{5}$$

With these parameters, Post and Kopa[11] introduced an approximation of TSD, termed as SCTSD.

Definition 3 (SCTSD). Portfolio λ dominates the benchmark portfolio Y by SCTSD if and only if:

$$\begin{gathered} (1+\varepsilon_s)F_\lambda^{(3)}(y_s) \leq G^{(3)}(y_s), s = 1, \cdots, T_b \\ \sum_{t=1}^{T}(p_t \sum_{i=i}^{N} \lambda_i r_{it}) \geq \sum_{s=1}^{T_b}(p_s y_s) \end{gathered} \tag{6}$$

Post and Kopa[11] transforms the portfolio optimization model with SCTSD constraints into a manageable quadratic constrained programming model, as detailed below:

$$\begin{aligned} & \max_{\lambda,\theta} \sum_{t=1}^{T} p_t \sum_{i=1}^{N} \lambda_i r_{it} \\ s.t.\ & \theta_{st} \geq y_s - \sum_{i=1}^{N} \lambda_i r_{it}, t = 1, \cdots, T; s = 1, \cdots, T_b \\ & (1+\varepsilon_s) \sum_{t=1}^{T} p_t \theta_{st}^2 \leq G^{(3)}(y_s), s = 1, \cdots, T_b \\ & \sum_{t=1}^{T} p_t \sum_{i=1}^{N} \lambda_i r_{it} \geq \sum_{s=1}^{T_b} p_s y_s \\ & \sum_{i=1}^{N} \lambda_i = 1 \\ & \theta_{st}, \lambda_i \geq 0, t = 1, \cdots, T; s = 1, \cdots, T_b, i = 1, \dots, N \end{aligned} \tag{7}$$

3 Empirical analysis

3.1 Data

This paper considers all 2154 stocks of Shanghai A-share listed before December 31, 2022, and selects the Shanghai Stock Exchange A share Index (SSEA) as the benchmark. We consider a period from January 1, 2002 to December 31, 2022. The daily return of each stock is the daily change in the stock's price on that day. All data above are from the iFinD APP.

In this paper, we conducted preliminary processing and screening of all stocks. Stocks do not have price fluctuation limits on the first trading day, and the price fluctuations are significantly higher than subsequent trading days. Therefore, when calculating the weights of the portfolio, we do not consider the price fluctuations of stocks on the first trading day to avoid the impact of this special value on the solution of the entire portfolio optimization model. In the selection of stocks, we exclude ST stocks and *ST stocks to avoid the impact of such stocks on the optimal investment portfolio due to poor performance or even possible delisting.

In this paper, we employ an equal-weight(EW) portfolio optimization model for comparison. In determining the optimal number of stocks to be held in the EW model, we select the top 1% of stocks based on their return rankings from the filtered stocks as our investment strategy.

3.2 Momentum investment strategy

Investors form portfolios by optimizing based on historical return data over a J-month formation period and rebalance them after the end of a K-month holding period. Specifically, at the beginning of each stage, investors utilize historical return data from the past J months to solve the portfolio optimization model, obtain, and update their portfolios to ensure that the capital allocation ratios after rebalancing match those of the optimal investment portfolio obtained. This adjusted portfolio is held for K months and rebalanced again after this period ends. The entire process is repeated in multiple stages. In our study, we explicitly specify the condition of disallowing short positions, which means that investors can only choose to buy stocks and cannot trade by borrowing stocks.

3.3 Evaluation of out-of-sample performance

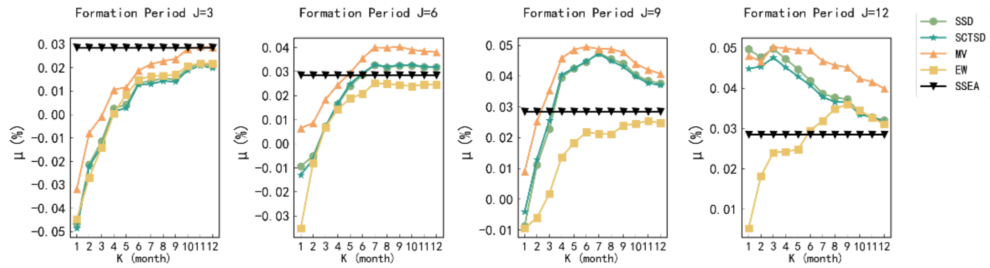

Fig. 1. Mean of Daily Returns of the Investment Strategy

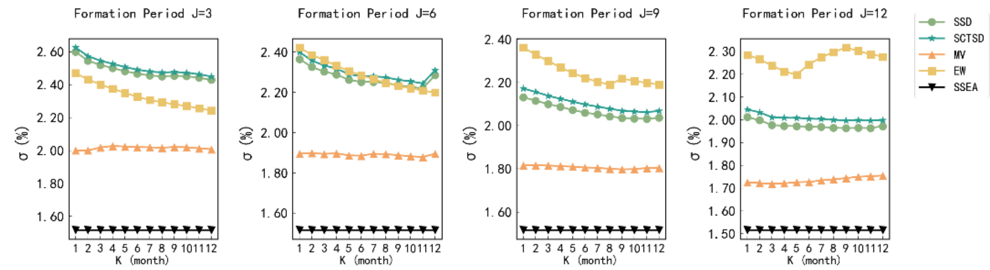

Fig. 2. Standard Deviation of Daily Returns of the Investment Strategy

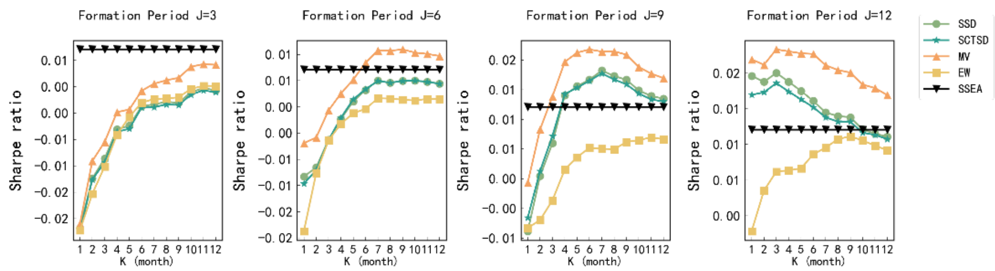

Fig. 3. Sharpe ratio of Daily Returns of the Investment Strategy

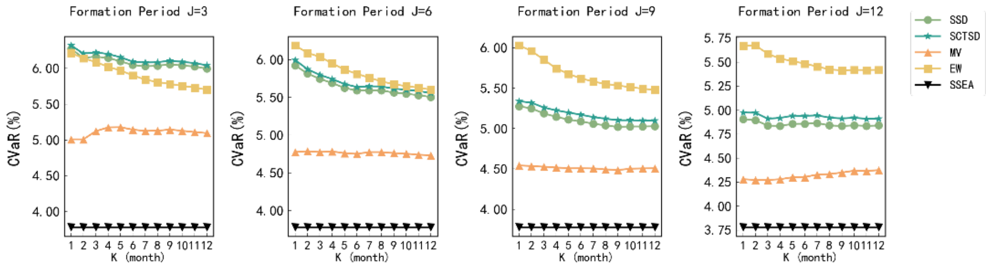

Fig. 4. CVaR of Daily Returns of the Investment Strategy

Fig. 1. - Fig. 4. report descriptive statistics and evaluation indicators for the returns delivered by investment strategies in the holding period without deducting transaction costs. In the in-sample evaluation, as the formation period J increases, the returns of SSD and SCTSD investment strategies continuously decrease, leading to a decrease in the Sharpe ratio. However, in the out-of-sample evaluation, as the formation period J increases, the returns of SSD and SCTSD investment strategies continue to increase, while the standard deviation continuously decreases. This ultimately results in an increase in the Sharpe ratio. From **Fig. 1.**, it can be observed that when $J = 3$, both SSD and SCTSD investment strategies have lower returns compared to the benchmark. However, when $J = 12$, both SSD and SCTSD investment strategies have higher returns than the benchmark. Despite the benchmark having a small standard deviation, when $J = 12$, the benchmark's returns are significantly lower than those of SSD and SCTSD investment strategies, leading to a lower Sharpe ratio for the benchmark compared to SSD and SCTSD investment strategies. The maximum drawdown and the CVaR also significantly decrease with the increase in the formation period J. This indicates that with a longer formation period J, SSD and SCTSD investment strategies exhibit more significant momentum effects and lower risk.

As the holding period K increases, when $J = 3$, the returns of both SSD and SCTSD investment strategies consistently increase but remain below the benchmark. It can be anticipated that when transaction costs are deducted during trading, the returns of SSD and SCTSD investment strategies will be significantly lower than those of the benchmark, possibly even becoming negative. When $J = 6$ and 9, the returns of SSD and SCTSD investment strategies first rise and then stabilize. When $J = 12$, the returns of both SSD and SCTSD investment strategies generally stabilize and are higher than the benchmark. Although there are notable fluctuations in the standard deviation of SSD and SCTSD investment strategies when $J = 9$ and 12, the overall trend of Sharpe ratios for SSD and SCTSD investment strategies closely follows the trend of returns. The CVaR also show slight decreases as the holding period K increases. This suggests that with the increase in the formation period J, the momentum effects of SSD and SCTSD investment strategies become more prominent, particularly for shorter holding periods K.

4 Conclusions

This paper investigates the Chinese Shanghai Stock Exchange A-share market using the portfolio optimization models with SD constraints. The research results distinctly indicate that the investment strategy performs remarkably well out-of-sample when the formation period is extended, achieving significant excess returns. Conversely, with a shorter formation period, even with a longer holding period, it is challenging to attain excess returns. As the formation period extends, the returns of the investment strategy gradually increase, while various risk indicators exhibit a declining trend.

For investors, the selection of an investment portfolio with an extended formation period is crucial in the investment process, contributing to obtaining more robust excess returns. Longer formation period strategies often demonstrate greater reliability in the face of market volatility and uncertainty. Therefore, investors should carefully consider the choice of the formation period to better optimize their investment portfolios, achieving more reliable and sustained investment returns.

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