Path Selection Logit Model Introducing Influence of Regret on Relative Utility Difference

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Abstract: This study extends the conventional stochastic regret minimization model and incorporates it into the path choice utility function as an expected regret psychological influence factor. This happens because of the limited explanatory capacity of the purely rational path choice-based model regarding actual travel. The weakness in the classic model's inference of the intensity of regret is fixed by the updated model, which employs the utility difference rather than the actual amount of the utility value. Two IIA characteristics—the path objective utility and the subjective regret psychological intensity brought on by the absolute utility difference—can be lessened by combining the improved utility function with the generalized logit model. The outcomes of the arithmetic examples demonstrate that the new model more accurately captures the influence of subjective regret psychology in path selection while resolving the IIA problems of the logit model. Furthermore, the enhanced model may simultaneously address the distributional implications of the random utility maximization and random regret minimization models, improving its capacity to elucidate actual choice behavior. When calibrated and applied to conventional network arithmetic allocation, the enhanced model's accuracy can reach 9.85%, capturing the network traffic flow allocation raw more accurately.

Keywords: Transportation planning and management; Psychology of regret; Path selection; Generalized Logit model.

1 Introduction

Travel path selection in transportation networks is a critical issue in traffic flow allocation [1]. Traditional path selection behavior is generally based on Expected Utility Theory (EUT) and Random Utility Maximization (RUM) [2-3] and travelers always choose the path with the highest perceived utility or lowest cost. Relevant studies have shown [4-5] that choice behavior is not entirely rational, and the choice decision is not only related to the characteristics of alternative things but also affected by many factors such as the psychological factors of the chooser, personal preferences, and the related environment. Therefore, Bounded Rationality (BR), Prospect Theory (PT) and Regret Theory (RT) have been successively introduced into choice models [4-6].

The classical psychological theory of regret [6-7] suggests that individuals measure the utility of a candidate solution by considering the solution characteristics themselves and imagining the possible better benefits of the alternative solution, forming regret expectations. Coricelli et

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al. [8] showed that decision-makers regret the same active regions of the brain as they did better make the choice decision, which also suggests that there is a correlation between regret and choice behavior correlate. Based on this, Chorus et al. [9] proposed the Random Regret Minimization (RRM) model, which identifies that the choice decision is affected by comparing candidate options with the single best gain option. Literature [10-13] and others have also improved the regret measure by proposing improved extended models. Since the direct reference of the RRM model to network flow allocation is not satisfactory [14-16], some literature [17-20] proposed to take utility maximization and regret minimization into consideration at the same time when the path selection model is constructed, but how to reasonably realize the combination of the two types of models and apply them to traffic flow allocation is a topic worth exploring.

In this paper, by improving the classical regret minimization model and incorporating it into the path utility function as an influence factor of expected regret psychology on decision-making, we argue that path choice is not only related to utility but also affected by regret psychology. Based on this, this improved utility function's generalized logit allocation model is analyzed.

2 Random Regret Minimization Model

2.1 Classical model

The RRM model in the literature[10] is a representative model for the basis of the psychological calculation of regret, which obtains the calculated value of regret for option k by comparing the utility value of that option with that of the alternative R_k , see equation (1):

$$R_k = \sum_{m \neq k, m \in R} ln(1 + exp(t_m - t_k)) = \sum_{m \neq k, m \in R} ln(1 + exp(s_k)) \tag{1}$$

$s_k = t_m - t_k$, which expresses the utility difference between the choice option k and the candidate option m. When $t_m - t_k < 0$, the utility value of the losing option is lower than that of the choice option, the model calculates a value that is the opposite of the regret emotion and expresses the degree of elation over the choice option.

2.2 Improved model

In the literature[19-21], the regret function is used as an influence term of the utility function, combined with the utility calculation. Referring to the way it is combined with the negative value of travel time as the utility of route choice, the RRM model has two shortcomings: when $(t_k - t_m) < 0$, the "regret value" expresses the degree of elation, which is less influential in the model[21], and the measurement of the expected degree of delight is not necessary for the transport route choice; the calculation of regret value in the model is only related to the absolute difference of utility but does not consider its relative difference, which is not consistent with the actual situation of decision-making in route selection.

The improved RRM model, abbreviated as MRRM (Modified Random Utility Maximization Model, MRUM) model, is shown in equation (2)

$$R_k = \sum_{m \neq k,\ k、m \in R_\omega, t_k \geq t_m} \left\{ ln\left(1 + exp\left(\frac{(-t_m)-(-t_k)}{|-t_k|}\right)\right) \right\} = \sum_{k、m \in R_\omega} g_k(C) \tag{2}$$

The corresponding modification is: when $(t_k - t_m) < 0$, the regret value is not calculated; the absolute difference of utility S_k is changed to the relative difference of utility $g_k(C)$ satisfies the inequality (3)、(4), and its regret value increases with the increase of the difference of utility: when the difference of utility is slight, the increment of the regret value is small; when the difference of utility is significant, the increment of the regret value is arge.

$$g_k'(C) = \frac{exp(C)}{1 + exp(C)} > 0 \tag{3}$$

$$g_k''(C) = \frac{exp(C)}{(1 + exp(C))^2} > 0 \tag{4}$$

As shown in Figure 1, the regret value function generated by the MRRM model for different walk times t has a significant change compared to the RRM model. Its regret value depends not only on the absolute difference in utility s_k, but also on the relative difference in utility $\frac{s_k}{|-t_k|}$, and the regret value is inversely proportional to the absolute value of the utility of the choice option.

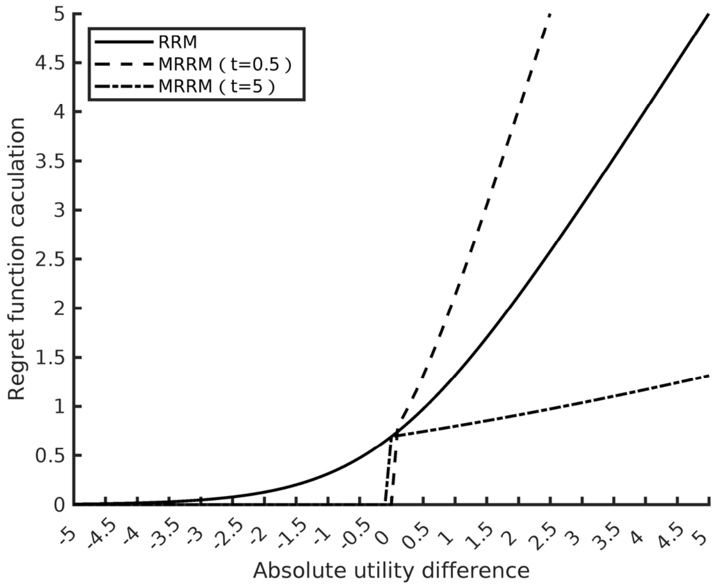

Fig.1 Comparison between improved model and classical model

3 Generalised logit allocation model including improved regret function

The traveling time of path k of the MRRM model is weighted and summed with the expected regret function to form the new expected path choice utility h_k^{rs} , see equation (5):

$$h_k^{rs} = t_k^{rs} + \gamma \sum_{m \neq k, m \in R_\omega, t_k \geq t_m} \left(ln \left(1 + exp\left(\frac{t_k^{rs} - t_m^{rs}}{t_k^{rs}} \right) \right) \right) \tag{5}$$

Refer to the literature[20-21] to add influence weights $\gamma(\gamma \geq 0)$ to the regret function; the values of the weights need to be calibrated, and the empirical weights range [0.68, 2.22]. When $\gamma=0$ is used, equation (9) degenerates into a pure path utility calculation. Assuming that the deviations of the traveler's perceived utility from the actual utility are independent of each other and obey the same Gumbel distribution[22-24], according to the Multi-Nomial Logit (MNL) model, the probability of choosing path k can be written as equation (6), which is abbreviated as MNL-MRRM, or the Generally Logit (GL) model [25], see equation (7), abbreviated as GL-MRRM.

$$P_k^{rs} = \frac{exp(-\theta(t_k^{rs} + \gamma \sum g_k(C)))}{\sum_{i \in R_\omega} exp(-\theta(t_i^{rs} + \gamma \sum g_i(C)))} \tag{6}$$

$$P_k^{rs} = \frac{exp(-(b_k^{rs}(t_k^{rs} - t_0^{rs}) + \sum_l g_{kl}^{rs} + \gamma \sum g_k(C)))}{\sum_{i \in R_\omega} exp(-(b_i^{rs}(t_i^{rs} - t_0^{rs}) + \sum_l g_{il}^{rs} + \gamma \sum g_i(C)))} \tag{7}$$

$$g_{kl}^{rs} = c_{kl} \left(1 - exp\left(-\frac{(b_l^{rs} - b_{kl}^{rs})(t_k^{rs} - t_0^{rs})}{J_{rs} c_{kl}} \right) \right) \tag{8}$$

$$c_{kl} = \ln\left(\frac{b_{kl}^{rs} + (J_{rs} - 1) b_l^{rs}}{J_{rs} b_l^{rs}} \right) \tag{9}$$

$$b_k^{rs} = \frac{1}{J_{rs} - 1} \sum_{l \neq k} b_{kl}^{rs} \tag{10}$$

t_0^{rs}denotes the most minor travel time utility of all paths between OD pairs; b_{kl}^{rs} depicts the degree of correlation between paths k and l between OD pairs, where the more significant the value, the lower the degree of correlation; at $b_{kl}^{rs} \to \infty$, paths k, and l are completely uncorrelated and must satisfy the requirement that $b_{kl}^{rs} > 0, b_{kl}^{rs} = b_{lk}^{rs}$。 g_{kl}^{rs} is the term of the utility of path l on the probability of choosing path k.

4 Model validation

4.1 Special networks

The network tested is shown in Figure 2, including two OD pairs and four road sections, with specified walk times of 125, 120, 10, and 5 min for paths 1, 2, 3, and 4, respectively.

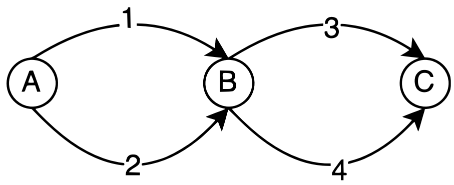

Fig.2. Test network

Examining and comparing the different choice probabilities of $A \to B$ and $B \to C$, the results are shown in Table 1. The MNL and MNL-RRM models calculate the exact path choice probabilities for $A \to B$ and $B \to C$, obviously, the result is unreasonable. The MNL model bases its choice probability calculation on the absolute difference in utility, which yields limited results; the MNL-RRM model incorporates an expected regret function, also consists of a fundamental difference in utility, and cannot help the model improve the IIA characteristics of MNL. Although there is a 5-minute difference in travel time between path 1 and path 2, as well as between path 3 and path 4, the probability of selecting path 1 and 2 only differs by 4.0-4.2%, while the probability of selecting path 3 and 4 is between 50-100%. According to practical experience, the probability of choosing the longer traveling time path 1 in $A \rightarrow B$ should be much larger than the longer traveling time path 3 in $B \rightarrow C$. MNL and MNL-RRM models are unable to express such differences in choices.

Table 1. Comparison of path selection probabilities among different models

Path selection model		OD pair/section			
		(A,B)		(B,C)	
		1	2	3	4
MNL		0.0067	0.9933	0.0067	0.9933
MNL-RRM	$\gamma = 1$	0.0025	0.9975	0.0025	0.9975
	$\gamma = 2$	0.0020	0.9980	0.0020	0.9980
	$\gamma = 3$	0.0013	0.9987	0.0013	0.9987
	$\gamma = 4$	0.0010	0.9990	0.0010	0.9990
	$\gamma = 5$	0.0008	0.9992	0.0008	0.9992
MNL- MRRM	$\gamma = 1$	0.0045	0.9955	0.0026	0.9974
	$\gamma = 2$	0.0042	0.9958	0.0022	0.9978
	$\gamma = 3$	0.0037	0.9963	0.0020	0.9980
	$\gamma = 4$	0.0034	0.9966	0.0018	0.9982
	$\gamma = 5$	0.0033	0.9964	0.0018	0.9982
GL		0.3486	0.6514	0.1645	0.8355
GL-RRM	$\gamma = 1$	0.2065	0.8935	0.1069	0.8931
	$\gamma = 2$	0.1786	0.8214	0.0761	0.9239
	$\gamma = 3$	0.1580	0.8420	0.0490	0.9510
	$\gamma = 4$	0.1484	0.8516	0.0401	0.9599
	$\gamma = 5$	0.1444	0.8556	0.0399	0.9601
GL-MRRM	$\gamma = 1$	0.3275	0.6725	0.1503	0.8497
	$\gamma = 2$	0.2978	0.7022	0.1449	0.8551
	$\gamma = 3$	0.2907	0.7093	0.1372	0.8628
	$\gamma = 4$	0.2890	0.7110	0.1288	0.8712
	$\gamma = 5$	0.2882	0.7118	0.1176	0.8824

The calculated probability values of the GL model include the correction term for the effect of alternative paths on the probability of choosing a way, which considers the relative utility difference, the GL-MRRM model not only considers the relative difference of utility but also considers the relative difference of utility composing the expected regret function, which makes the two models able to distinguish between the probability of choosing between $A \rightarrow B$ and $B \rightarrow C$. The calculated values in Table 1 show that the choice probabilities of GL and GL-MRRM models under different regret weights can clearly distinguish the choice probabilities of $A \rightarrow B$ and $B \rightarrow C$, and their calculated results are close to the commonsense estimation. Although the MNL-RRM and GL-RRM models can also distinguish the different choice probabilities of the two paths, the MNL-RRM model is affected by the IIA characteristics, and the calculated results are not reasonable. However, the GL-RRM model's calculation results are better than the MNL-RRM model, and the effect of IIA characteristics generated by the regret function has been brought into the calculation results. Its theoretical explanation needs to be more reasonable.

The GL-MRRM model adds the regret function and its weights into the utility term, increasing its degrees of freedom; in that case, the model can be better calibrated based on the measured data. From the calculation results in Table 1, the regret psychological influence weight γ affects the GL-MRRM model path selection. As the value of γ increases, the choice probability is more favorable to the shortest path; as the value of γ increases, the incremental influence of γ on the total choice probability decreases. This phenomenon reflects the role of expected regret psychology in choice decisions.

4.2 Sioux Falls Network

Based on the Sioux Falls network, its corresponding OD matrix, and the actual traffic distribution data[25-26], the traffic distribution is assigned and comparatively verified using this paper's GL-RUM, GL-RRM, and GL-MRRM models. The topology of the Sioux Falls network is shown in Figure 3. It comprises 24 nodes and 76 paths, with 528 OD pairs and a travel demand of 360600.

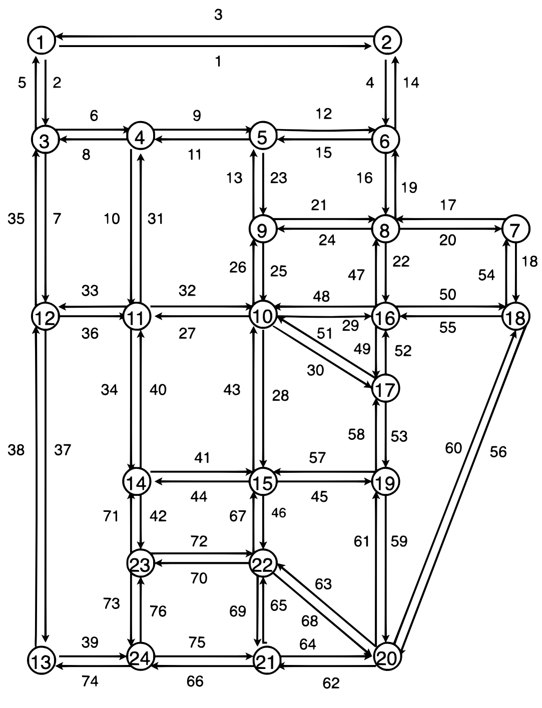

Fig.3. Sioux Falls network

The BPR function, $t_a(v_a) = t_a^0(1 + \alpha(\frac{v_a}{c_a}))^{\beta}, \alpha = 0.15，\beta = 4$. calculates the traveling time of the road segment. Due to the excessive number of paths in the Sioux Falls network, using the k-shortest circuit method[24] to generates k-loop-free paths before flow allocation. The study of Bekhor et al.[29] suggests k's value: the smaller value of k in the Sioux Falls network coincides with the actual choice of travelers, so in this paper, we choose k=6 to find six shortest paths for each OD pair[27], generating a total of 3168 approaches, and the equilibrium state of the road network is solved by using the Method of Successive Weight Averages (MSWA) [23] for three models.

4.2.1 Calibration of regret impact in GL-MRRM mode

The Simulated Annealing (SA) algorithm[28] calibrated the degree of regret impact γ . The Mean Absolute Percentage Error (MAPE) value of the roadway flow was assigned by the GL-MRRM model, and the actual roadway flow was used as the objective function, see equation (11):

$$M = \frac{1}{J}\sum_{t=1}^{J}\left|\frac{A_t - F_t}{A_t}\right| \tag{11}$$

A_t is the measured value of the t road section, F_t is the model-assigned flow value, and J is equal to seventy-six. Figure 4. gives the objective function optimization curve of the simulated annealing algorithm solution results; the metropolis criterion makes the SA algorithm have probabilistic jump characteristics in the optimization process, which can effectively jump out of the local optimum, the optimal value of the objective function decreases from the initial 0.1936 to the minimum of 0.0985 in the 100th generation, which is a reduction of 9.51%, and the final γ value is calibrated to 1.6766. After calibration, the mean MAPE value of the model-assigned road segment flow was at 9.85%. In addition, the average error variance of the model-calculated road sections is 0.2914.

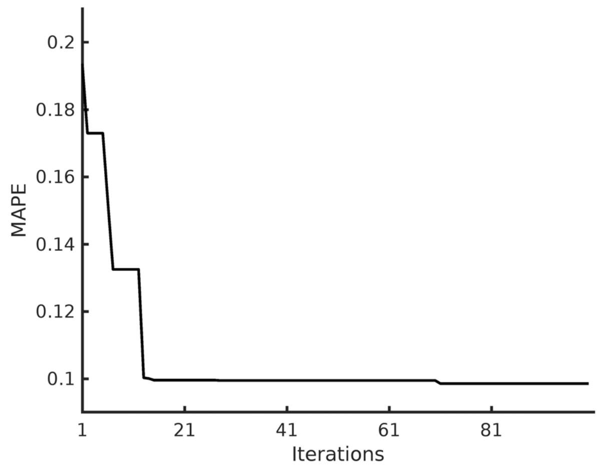

Fig.4. Convergence curve of Simulated Annealing algorithm

4.2.2 Comparison of traffic flow distribution in different models

Based on the distribution model calculations, OD pairs 1-9, 12-5, and 23-3 were selected to analyze the equilibrium solutions of Sioux Falls network flows under three different models. Table 2 shows the results:

Table 2. Comparison of partial path traffic flow allocation with three other models

OD	route	Route flow		
		GL	GL-RRM	GL-MRRM
1-9	1-3-4-5-9	65.48	67.98	66.49
	1-2-6-5-9	65.48	67.98	66.49
	1-3-12-11-10-9	56.95	50.53	60.23
	1-3-4-11-10-9	56.95	50.53	60.23
	1-2-6-8-9	65.48	67.98	66.49
	1-2-6-8-16-10-9*	100.57	298.17	107.76
12-5	12-3-4-5	21.10	31.89	18.09
	12-11-4-5	72.93	80.67	78.97
	12-11-10-9-5*	11.01	2.26	7.99
	12-3-1-2-6-5	72.93	80.67	8.97
	12-13-24-23-14-11-4-5*	11.01	2.26	7.99
	12-11-10-16-8-6-5*	11.01	2.26	7.99
23-3	23-24-13-12-3	11.04	12.60	11.04
	23-14-11-4-3	11.04	12.60	11.04
	23-14-11-12-3	13.89	23.19	16.14
	23-22-21-24-13-12-3	11.04	12.60	11.04
	23-24-13-12-11-4-3	13.89	23.19	16.14
	23-22-15-14-11-4-3	13.89	23.19	16.14

The assigned traffic trends are the same since all three models are generalized logit models. The GL-RRM model assigns significantly different traffic flows to the marked * paths than the other two models, which suggests that the RRM model's computation of the regret value makes the model's traffic flow assignment highly biased in favor of the shortest path, which is not reasonable enough. Overall, the GL model does not influence the regret factor. The regret factor of the GL-RRM model has an excessive influence on the road section when the traveling time is longer or shorter, and the preference for the shortest path is greater than that of the GL-RRM model. The influence of the regret factor of the GL-MRRM model is in between the two models, which has a more substantial explanatory power of the model. It can describe the actual decision-making process of the traveler better.

5 Conclusion

This paper considers the psychology of regret in behavioral science, and the improved utility function of path flow allocation includes the expected regret function. Through the study and comparison of the flow allocation of different path selection models, there are conclusions : (1) When applying the expected regret function to transport allocation, it is appropriate to use relative utility differentials to avoid the IIA characteristics brought about by absolute utility differentials and with a generalized logit model to eliminate or eliminate the selection probability bias brought about by absolute utility differentials; (2) In this paper, the improved regret function eliminates the "elation value" calculated in the original model, which can reasonably describe the impact of regret psychology on decision-making and combines it with the path utility through the weights, the weights need to be determined by the calibration or a priori data, and the calibration value is related to the characteristics of the road network; (3) The generalized logit model, including the improved regret function proposed in this paper, can better express the distribution law of network traffic flow. Although the weight of the regret factor in the choice probability is low, it provides a new perspective for the portrayal of the path selection behavior, which is a corrective to the entirely rational path selection model.

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