An iterative Power Allocation Alogrithm for Energy Efficiency Optimization in Massive MIMO Systems

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Abstract

In this paper, a transmitting power allocation strategy for users jointed together pre-coding is presented to eliminate inter-users interference and improve the energy efficiency of Massive MIMO systems. The power allocation model is formularized as energy efficiency optimization problem of the users' power and circuit power consumption. We demonstrate that energy efficiency is a convex function of users' power. In order to reduce the complexity of traditional power allocation algorithm, an iterative power allocation algorithm based on convex optimization theory is proposed to figure out the optimal value of each user power. The simulation results show that the iterative algorithm can effectively improve the energy efficiency of Massive MIMO systems.

1. Introduction

The Internet of Things is an important part of the new generation of information technology. It can connect objects and bring convenience to people's lives. The fifth generation of mobile communication technology (5G) will make the Internet of things have more powerful communication capabilities, so that the Internet of Things is more widely used. In the 5G research and development, energy efficiency has aroused people's attention. Improving energy efficiency is not only conducive to reducing the interference of co-channel users, but also reducing the environmental pollution. MIMO is one of the most important technology to improve spectral efficiency [1]. Massive MIMO can significantly improve the energy efficiency and spectral efficiency by increasing the number of antennas at the base station [2]. But as the number of base station antennas increases, the loss of the RF chains will also increases [3]. Therefore, the power loss

of RF chains must be taken into account in analyzing the energy efficiency of Massive MIMO systems [4]. At the same time we should pay attention that the less number of RF chains is used, the smaller capacity of the system will be [5].

In recent years, there are many energy efficiency studies on MIMO systems [6] [7]. For example, in [6], it studied how to improve the energy efficiency of sensors in MIMO systems based on Alamouti diversity. Radio resource allocation is an effective way to improve energy efficiency [8]. In [9], the optimal energy efficiency allocation algorithm in OFDMA system was studied. In [10], the optimal energy efficiency algorithm is studied in the distributed antenna system. In [11], the energy efficiency of the uplink multiuser systems is analyzed. It is proved that the energy efficiency function is a quasi-concave function of the power matrix, and based on this analysis, an iterative power allocation algorithm is proposed.

The existing research results show that most power allocation algorithms usually assume that the base station

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allocates power to users evenly, ignoring the energy efficiency optimization problem when users allocated power unequally. Based on these researches, this paper eliminates the interference between users by decomposing the channel matrix and applying pre-coding technique. And then a new power consumption model is founded, which takes the users' power and circuit power into account. After we deduce the expression of energy efficiency, the convexity of energy efficiency is proved, based on convex optimization, an iterative power allocation algorithm is proposed to solve the energy efficiency optimization problem. The simulation results show that the iterative algorithm can effectively improve the energy efficiency of Massive MIMO system.

2. System model

In Massive MIMO downlink systems, we assumed that the number of base station antennas is N and the receiver has K single antenna users. The system model is shown in figure 1.
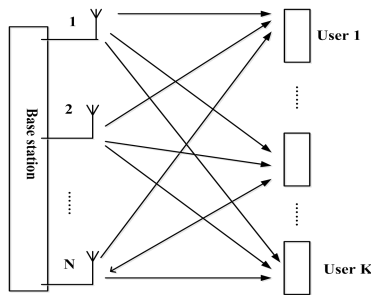

Figure 1. The model of massive MIMO system

The signal received by the client is,

$$\mathbf{y} = \mathbf{HQPx} + \mathbf{n} \tag{1}$$

Where **y** and **x** are the received and transmitted signal vectors of $K \times 1$ respectively[1], **H** is the Rayleigh fading channel matrix of $K \times N$, **Q** is the pre-coding matrix of $N \times K$, **n** is the noise vector of $K \times 1$, $\mathbf{P} = diag\left\{\sqrt{p_1}, \sqrt{p_2}, ..., \sqrt{p_K}\right\}$ represents the , power allocated to each user of $K \times K$.

Using singular value decomposition (SVD),

$$\mathbf{H} = \mathbf{U\Sigma V}^H \tag{2}$$

U and **V** are the unitary matrices of $K \times K$ and $N \times N$ respectively, **Σ** is the diagonal matrix of $K \times N$,

$$\mathbf{\Sigma} = diag\{\lambda_1, \lambda_2, ..., \lambda_K, 0, ..., 0\} \tag{3}$$

At the receiver, we use Zero Forced (ZF) receiver to eliminate the interference between users, the received signal can be rewritten as

$$\mathbf{y'} = \mathbf{Wy} = \mathbf{WHQPx} + \mathbf{Wn} \tag{4}$$

$$\mathbf{W} = (\mathbf{U}^H\mathbf{U})^{-1}\mathbf{U}^H \tag{5}$$

W is the linear receive of $K \times K$. Assuming that the transmitter has known the channel state information, we decomposes the modal and uses the pre-coding at the transmitter, that is $\mathbf{Q} = \mathbf{V}$. At the receiver ZF pre-coding is used. Modal decomposition diagram shown in Figure 2.
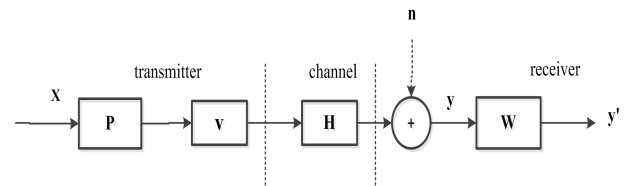

Figure 2. Modal decomposition

The received signal is reduced to,

$$\mathbf{y'} = \sum \mathbf{Px} + \mathbf{n'} \tag{6}$$

Where $\mathbf{n'} = \mathbf{Wn} = (\mathbf{U}^H\mathbf{U})^{-1}\mathbf{U}^H$, **n** is a Gaussian white noise with a zero mean and a variance of σ^2. The signal-to-noise ratio of the user k is given by,

$$\eta_k = \frac{\lambda_k^2 p_k}{\sigma^2} \tag{7}$$

Then the total capacity of the system can be written as,

$$C = \sum_{k=1}^{K} B \log_2(1 + \frac{\lambda_k^2 p_k}{\sigma^2}) \tag{8}$$

3. Power consumption model

The total power consumption is defined as,

$$p_{sum} = \frac{p_{out}}{\alpha} + Np_c + Kp_r \tag{9}$$

Where, the first part is the power amplifier consumption, it is related to the transmitter output power p_{out} and amplifier efficiency α. The latter two parts of (9) are the power consumption of the RF chain. p_c and p_r are the power consumption at the transmitter and the receiver respectively, the output power is determined by

$$p_{out} = \sum_{k=1}^{K} p_k \ . \tag{10}$$

4. Definition of Energy Efficiency and Proof of Its Convexity

Energy efficiency is defined as the total capacity of the system divided by the total power consumption, which can be expressed as,

$$U = \frac{C}{p_{sum}} = \frac{\sum_{k=1}^{K} B \log_2 (1 + \frac{\lambda_k^2 p_k}{\sigma^2})}{\sum_{k=1}^{K} p_k} \ . \tag{11}$$

4.1. Proof of convexity of energy efficiency

Based on the energy efficiency optimization model established in this chapter, with the increase of the user's power, the system energy efficiency increases first and then decreases. The energy efficiency is the convex function of the user's power. That is there is an optimal user power distribution can make the maximum energy efficiency.

Proof: In order to analyze the relationship between the power allocated by the user and the energy efficiency of the system, the energy efficiency function is written as a function of p_k,

$$y(p_k) = \frac{\sum_{k=1}^{K} B \log_2 (1 + \frac{\lambda_k^2 p_k}{\sigma^2})}{\sum_{k=1}^{K} p_k} \ . \tag{12}$$

In order to judge the relationship between the energy efficiency function and the user power in (12), we can first find the partial derivative of the function with the respect to p_k, determine the nature of its derivative function. The derivative function after derivative is (13) ,

$$\frac{\partial y(p_k)}{\partial p_k} = \frac{B \frac{\lambda_k^2}{\ln 2 (\sigma^2 + \lambda_k^2 p_k)} \left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + Np_c + Kp_r \right) - \frac{1}{\alpha} \sum_{k=1}^{K} B \log_2 (1 + \frac{\lambda_k^2 p_k}{\sigma^2})}{\left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + Np_c + Kp_r \right)^2} \ . \tag{13}$$

In order to determine the positive or negative properties of (13), the denominator is definitely greater than zero, only need to judge the positive or negative of the molecules. Because the positive or negative of the

molecule is not easy to judge, the molecule can be expressed as the (14).

$$g(p_k) = B \frac{\lambda_k^2}{\ln 2 (\sigma^2 + \lambda_k^2 p_k)} \left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + Np_c + Kp_r \right) \\ - \frac{1}{\alpha} \sum_{k=1}^{K} B \log_2 (1 + \frac{\lambda_k^2 p_k}{\sigma^2}) \ . \tag{14}$$

The partial derivative of $g(p_k)$ with the respect to p_k is the (15).

$$\frac{\partial g(p_k)}{\partial p_k} = \frac{-B \lambda_k^4}{\ln 2 (\sigma^2 + \lambda_k^2 p_k)^2} \left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + Np_c + Kp_r \right) \\ + \frac{B \lambda_k^2}{\ln 2 (\sigma^2 + \lambda_k^2 p_k) \alpha} - \frac{B}{\alpha \ln 2} \frac{1}{1 + \frac{\lambda_k^2 p_k}{\sigma^2}} \frac{\lambda_k^2}{\sigma^2} \\ = \frac{-B \lambda_k^4}{\ln 2 (\sigma^2 + \lambda_k^2 p_k)^2} \left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + Np_c + Kp_r \right) < 0 \tag{15}$$

(15) is a number less than zero, so $g(p_k)$ is a decreasing function with respect to p_k ,because

$$g(p_k)|_{p_k=0} = B \frac{\lambda_k^2}{\sigma^2} (Np_c + Kp_r) > 0 \ . \tag{16}$$

$$g(p_k)|_{p_k=+\infty} = \frac{B}{\alpha} \left(\frac{K}{\ln 2} - \lim_{p_k \to \infty} \log 2 \left(1 + \frac{\lambda_k^2 p_k}{\sigma^2} \right) \right) < 0 \ . \tag{17}$$

Since the first part of (17) is a constant and the latter part is infinite when p_k tends to infinity, so (17) is less than zero. That is, with the increase of p_k , the value of $\partial y(p_k) / \partial p_k$ is greater than zero and then less than zero. In other words $y(p_k)$ monotonically increasing first and then monotonically decreasing. So there is an optimal p_k to make the efficiency function obtain the maximum value.

In summary, with the increase of user power, the system energy efficiency increases first and then decreases, that is, there is an optimal user power to make the efficiency obtain the maximum value. Therefore, through the power distribution algorithm, we can find the optimal value of the energy efficiency function.

5. The iterative power allocation algorithm

The energy efficiency function is the convex function of p_k. Therefore, the optimal value of user power can be obtained by solving the derivative of the function U with the respect to p_k, then letting the derivative function equal to zero, we can obtain,

$$p_k = \begin{cases} \dfrac{\alpha B}{U \ln 2} - \dfrac{\sigma^2}{\lambda_k^2} & if \ \dfrac{\alpha B \lambda_k^2}{\sigma^2 \ln 2} > U \\ 0 & else \end{cases} \quad . \quad (18)$$

(18) has gotten the expression of the user power optimal value, but the expression of U implies p_k, we can not get the optimal value of the user power directly. Therefore, an iterative power allocation algorithm is proposed to obtain the optimal value. Define:

$$f(p_k) = \sum_{k=1}^{K} B \log_2 (1 + \frac{\lambda_k^2 p_k}{\sigma^2}) - U \left(\frac{\sum_{k=1}^{K} p_k}{\alpha} + N p_c + K p_r \right) . \quad (19)$$

(18) and (19) can get the energy efficiency of the system by updating and iterating constantly. When the maximum energy efficiency is obtained, (19) should be zero, so the iteration continues until (19) is sufficiently small to meet the convergence requirement. This power allocation algorithm is summarized in Table 1.

Table 1. An iterative power allocation algorithm

Step 1:	assuming that the average allocation of power that is $p_k^1 = p_{out}/K$, take p_k^1 and $U^1 = 0$ into (19) to get f^1.
Step 2:	take $U^2 = f^1$ into (18) to get p_k^2, then take p_k^2 and U^2 into the (19) to calculate the energy efficiency f^2.
Step n:	let $U^n = f^{n-1}$, take U^n into (18) to get p_k^n, then take p_k^n and U^n into the (19) to calculate f^n at the same time, if $f^n < 0.001$, then the iteration ends. Otherwise continue to step n+1.

6. Simulation results

The following is the simulation results to verify the effectiveness of the algorithm. Assuming that the base station bandwidth is 10 kHz, the power consumption of each RF chain at the transmitter and receiver is 100 mW. The noise power is -114dBW / MHz and the power amplifier efficiency is 0.5. Define the equal power allocation algorithm is that the transmitted power is allocated equally for each user ,

$$p_k = \frac{p_{out}}{K} \quad . \quad (20)$$

Figure 3 shows the relationship between the power consumption and the number of antennas. From the figure 3, we can see that, with the increase of the transmit antenna number, the total power consumption of the system is constantly increasing, and the more users at the receiver, the greater the power consumption. This is because under this paper's power consumption model, the total power consumption is a linear relationship with the number of transmitting antennas and users
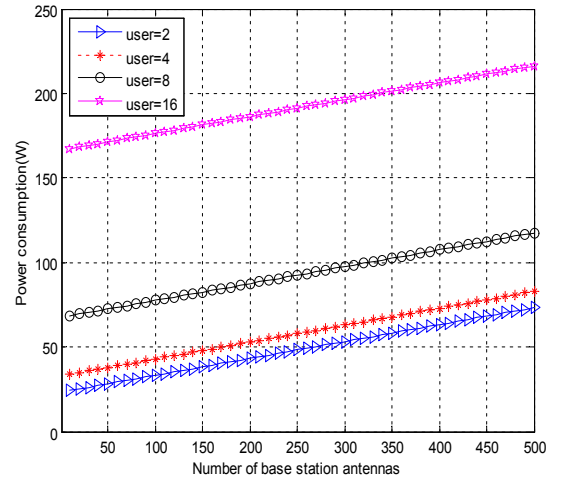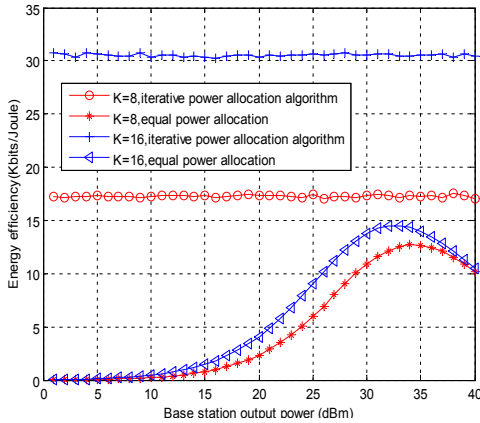

Figure 3. Relationship between power consumption and transmitting antennas number

Figure 4 is an energy efficiency comparison between the iterative power allocation algorithm and the equal power allocation algorithm when the number of base station antennas is 128. It can be seen that the energy efficiency of the iterative power allocation algorithm is higher than the equal allocation power. Because the user's power in iterative power allocation algorithm is optimal, it is independent with the output power of base station, and it can be approximately a straight line. However when the user allocates the output power equally, the energy efficiency increases first and then decrease, that is because the power consumption increases with the increase of output power, but when the output power is less than 32dBm, the system capacity increases more than the power consumption, the energy efficiency increases. When the output power is greater than 32dBm, the system capacity increases less than the power consumption, so energy efficiency reduces. At the same time we can see

that in the same algorithm the more users, the greater the energy efficiency.

Figure 4. Energy efficiency comparison between iterative power allocation algorithm and equal power allocation

Figure 5 shows the relationship between the number of base station antennas and the energy efficiency. It can be seen that with the increase of base station antenna number, the energy efficiency increases first and then decrease, in other words, there exists an optimal number of base station antennas to maximize energy efficiency. This is because when the number of base station antennas is small, with the increase of the transmitting antenna number, the RF chain consumes less power and the system capacity increases more quickly, so the energy efficiency increases. However, when the number of base station antennas is large, the loss of the RF chain becomes large and the capacity increases slow relatively, so the energy efficiency reduces. And the more number of users, the greater energy efficiency of the system.
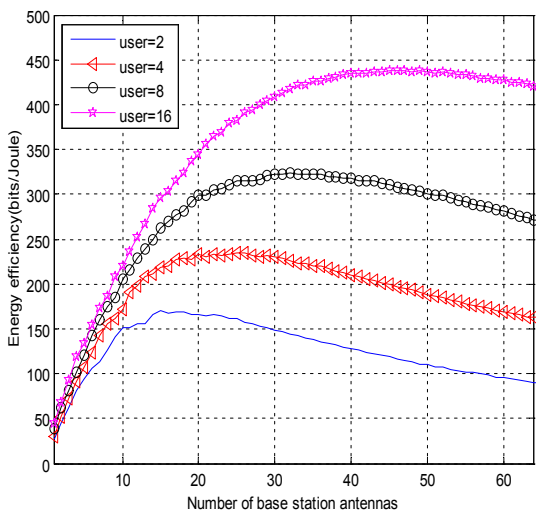
Figure 5. Energy efficiency of the iterative power allocation algorithm

7. Summary

In this paper, an iterative power allocation algorithm is proposed to improve the energy efficiency of Massive MIMO systems. Firstly, a new power model is founded, which takes the users ' power and circuit power into account. Based on this model, we deduce the expression of energy efficiency and an iterative power allocation algorithm is proposed by using convex optimization to obtain the optimal value of user power. Then the influence of the algorithm on the total power consumption of the system is analyzed. The power efficiency of the proposed power algorithm and average power allocation is compared. The simulation results show that the proposed iterative power allocation algorithm can significantly improve the energy efficiency of the system.

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