Space-Time Coding and LMMSE Receiver in OTFS Modulation

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Abstract. Orthogonal Time Frequency Space (OTFS) modulation is a two-dimensional modulation method that is developed in the delay-Doppler domain that is used in doubly-dispersive fading channels. At the transmitter, OTFS utilizes additional transform operations, and the receiver is used above the conventional multicarrier modulation operations, which attains greater performance rather than OFDM. In the delay-Doppler domain, OTFS along with phase rotation by utilizing transcendental numbers proved to attain entire diversity. The proposed work uses space-time coding (STC) in a MIMO configuration. Also, to attain complete transmit diversity in OTFS, the Alamouti code structure is used. Thus, the usage of STC-OTFS along with phase rotation will attain complete diversity in delay-Doppler and spatial domains. In addition, the low complexity LMMSE receiver is employed that uses Sparsity and the quasi-banded shape of the matrices which are included in the demodulation procedure. Hence, the proposed work achieves higher performance than the existing methods.

Keywords: OTFS modulation, Space-Time coding, Full diversity, LMMSE receiver

1. Introduction

In high-Doppler systems like mmWave communications, V2X communications, and high-speed trains, wireless communication models in the 5th gen are expected to operate and provide increased throughput and reliability in fastly changing channel situations. The fading of the channel in that situation will be a doubly dispersive nature along with high Dopplers [1]. OTFS modulation will attain higher performance than conventional systems like OFDM. The main attribute that differentiates OTFS modulation from conventional scheme is that

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information symbols are multiplexed in OTFS in the domain of delay-Doppler, whereas conventional schemes are multiplexed in the domain of time-frequency. In the domain of delay-Doppler, the benefit of signalling is that a channel will quickly change in time manifests, which can be viewed as an invariant channel. This constant channel will obtain experienced by every symbol in a transmission frame of OTFS which will simplify the equalizer design and decrease the overhead on channel estimation in fastly time-changing channels.

For a vehicle speed ranging from 30 - 500 Kmph at 4GHz band, OTFS modulation will achieve notably improved error performances than OFDM. Due to high carrier frequency, OTFS will perform effectively in the 28GHz band, whereas Doppler's will be more for moderate/low speed of vehicle [2]. In traditional OTFS along with numerous transmit antennas, OTFS with STC is not considered. It also describes an OTFS phase rotation strategy [3] that utilizes transcendental numbers and the complete diversity is obtained which is provided by the delay-Doppler channel.

The structure of this paper is as follows. Section 2 explains the Literature work. Section 3 describes the OTFS Modulation. Section 4 discusses the proposed work. Section 5 highlights the result followed by the conclusion in Section 6.

2. Literature Review

An existing method describes the Doppler-resilient orthogonal signal-division multiplexing (D–OSDM) for underwater acoustic (UWA) communication. In UWA transmission, it will create a secure communication environment. The author described the signal procedures at the transmitter and the D–OSDM receiver; thereby the performance is calculated by conducting experiments and simulations [4]. The results indicate that the D–OSDM will give high-quality and low-power UWA transmissions in channels along with huge Doppler spread and delay.

The discrete-time formulation of the OTFS model based on OFDM is reported in [5]. The author discussed the window functions deployment at the transmitter of OTFS in real conditions, thus limiting the windowing state on the receiver side. The study is made on the impact of the channel in discrete time, which will provide a deeper understanding of OTFS design. The suggested method leads to simpler demodulator and modulator architectures than in the existing model.

An existing method investigates the OTFS diversity by assuming a delay–Doppler channel and rectangular waveforms. After launching the effective diversity (ED) concept, which will be more effective than existing diversity in a huge amount of transmitted symbols. The author examined the OTFS conditions which will reach complete ED for QAM symbols [6]. Thus, the simulation result shows that OTFS can attain complete ED with huge signal constellations.

To 5th gen New Radio standard, this method describes new encoding technique and a high-throughput low-complexity encoder framework to quasi-cyclic low-density parity-check (QC-LDPC) codes [7]. In keeping the permutation information's quantized value for every submatrix rather than the entire matrix of parity check, the memory used by this method is less. Also, sharing methods are used to minimize the hardware complexity.

Another method explains the P-path doubly-dispersive channel in OTFS with fewer delays and Doppler shifts. In this method, the input-output relation is derived using a basic matrix for pulse-shaping waveforms, and then generalized to arbitrary waveforms. The OTFS input-output connection has a basic sparse framework that allows the utilization of low-complexity detection techniques [8] .Finally, the OTFS performance is compared with various pulse-shaping waveforms, resulting in reducing overall error performance.

The pilot-aided channel method to OTFS is reported in [9]. In the OTFS framework, the data symbols, guard, and pilot in the delay–Doppler plane were arranged to eliminate interference among data and pilot symbols on the receiver side. The author also arranged the symbols to OTFS through multipath channels along with fractional as well as integer Doppler shifts. Thus, the suggested OTFS scheme is better than the OFDM frame.

This study highlights the low-complexity with message passing (MP) detection technique that is used for massive OTFS. With given Doppler and delay parameters, the author described OTFS demodulation and modulation for the delay–Doppler channels along with an arbitrary number of routes [10]. As the path of the fractional Doppler generates the Inter Doppler Interference (IDI), the author adopted the MP detection technique for compensating the IDI effect to further increase the performance. The results show that a higher performance was obtained in OTFS than the OFDM method under numerous channel states.

The OTFS design exploits complete diversity in frequency and time, and it is combined with equalization which will convert the time-varying channels like OFDM to a time-independent channel[11].This channel diversity will permit OTFS to simplify the system operation and also enhance the performance, especially in high Doppler models, small packets, and enormous antenna arrays. Thus, the result indicates that the performance of the block error rate is improved in OTFS than in the OFDM settings.

The suggested OTFS scheme uses a linear complexity iterative rake detector is reported in [12]. The combining algorithms of the linear diversity such as selection, as well as equal gain combination, and maximum ratio combining (MRC) are used to enhance the SNR. By using the input-output relation, the author suggested this detector depending on the MRC scheme. As a result, the suggested detector's BER performance outperforms the conventional OFDM approach.

Another technique suggested a channel estimation method based on sparse Bayesian learning (SBL).Then, the SBL method is described and a sparse signal prior design is constructed as a hierarchical Laplace prior. In the prior model, the parameter is updated by using the expected maximum (EM) technique [13]. The simulations indicate that the suggested method outperforms in the anti-noise interference, pilot power utilization, and pilot overhead.

3. OTFS Modulation

In OTFS modulation, the delay-Doppler domain is utilized to multiplex the modulation symbols. Figure 1. depicts the block diagram for the OTFS scheme. The post and pre-processor will constitute the OTFS modulator and any multicarrier TF modulation can be used.

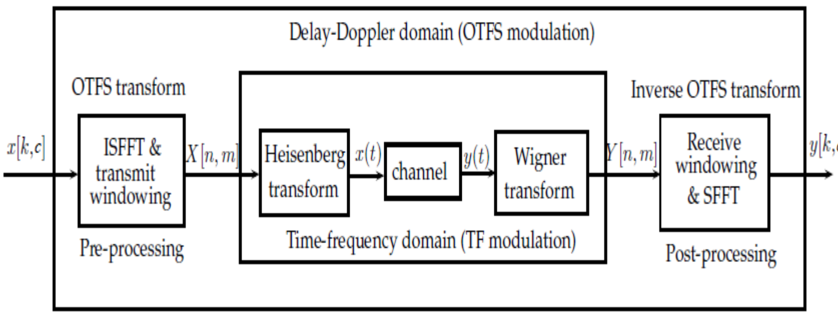

Fig.1. OTFS scheme

3.1 Time-Frequency modulation

To acquire a 2D grid Λ, the TF plane will be sampled at intervals of Δf and T, which is stated in the equation (1)

$$\Lambda = \{(nT, m\Delta f), n = 0, \dots, N-1, m = 0, \dots, M-1\} \quad (1)$$

In the TF domain, the signal in a packet will burst with NT duration and $M\Delta f$ bandwidth is occupied. The Heisenberg transform is used to convert the TF signal *X[n, m]* to the time domain signal *x(t)*.Then, the matched filter output is obtained at the receiver side.

3.2 OTFS Modulation

The TF domain symbol *X[n, m]* is mapped to a delay-Doppler domain's *x[k,c]* information symbol which is defined in eq (2)

$$X[n,m] = \frac{1}{MN} \sum_{k=0}^{N-1} \sum_{c=0}^{M-1} x[k,c] e^{j2\pi\left(\frac{nk}{N} - \frac{mc}{M}\right)} \quad (2)$$

X[n, m] is the TF modulated for transmission across the channel. The Wigner filter is used to convert the received signal *y(t)* to *Y[n,m]*.

3.3 Input-Output relation in OTFS

Considering a channel of delay-Doppler along with P reflector taps, each of which has a Doppler, delay and fade coefficient. For the *P* path channel, the relation for the input-output is written as (3)

$$y[k,c] = \sum_{i=1}^{P} h_i' x\left[\left((k-\beta_i)\right)N,\ \left((l-\alpha_i)\right)M\right] + v[k,c] \quad (3)$$

Where α_i and β_i are considered as integers. The relation for the input-output is vectorized as

$$y = Hx + v \quad (4)$$

3.4 MIMO-OTFS

The vector channel (4) is extended in a MIMO-OTFS scheme with transmit (n_t) and receive (n_r) antennas, and the relation of the input-output is stated as (3)

$$y_{MIMO} = H_{MIMO} x_{MIMO} + v_{MIMO} \quad (5)$$

Where, v_{MIMO} denotes noise vector. The transmit and received vector of MIMO-OTFS is $x_{MIMO} = [x_1^T, x_2^T, \dots, x_{nt}^T]^T \in \mathbb{C}^{n_t NM \times 1}$, as well as $y_{MIMO} = [y_1^T, y_2^T, \dots, y_{nt}^T]^T \in \mathbb{C}^{n_t NM \times 1}$.

4. Proposed work

4.1 Space-Time Coded OTFS

For OTFS, an alternate input-output relation is introduced, which will be significant for constructing space-time coding to OTFS. It will also make the space-time code diversity analysis to be easier. Figure 2 depicts the STC-OTFS scheme.

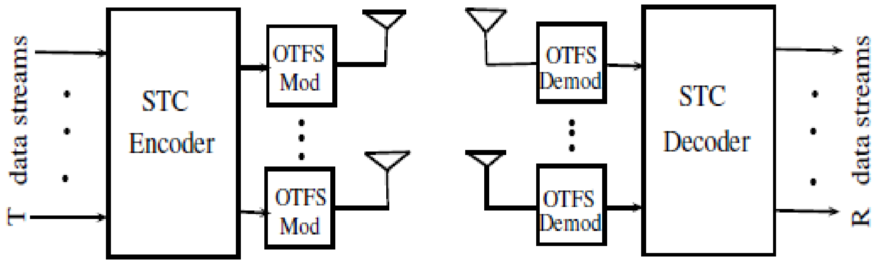

Fig.2. STC-OTFS

4.1.1 Alternative Input-Output relationship for STC-OTFS

The vectorized formula is given in Eqn (4) is considered. In (10), the channel matrix H contains *P* non-zero entries in every column and row because of modulo operations represented in (2). Also, it is observed from (2) that every transmitted symbol is subjected to a similar channel gain. Hence, the vectorized model is given in eq (6)

$$y^T = h'X + v^T \qquad (6)$$

Where y^T, v^T and h′ is $1 \times MN$ received vector, noise vector, as well as vector have non-zero elements *P*.X denotes $MN \times MN$ matrix.

4.1.2 Encoding and Decoding

Considering a 2×1system and the STC is constructed by applying the Alamouti code framework, which is generalized for matrices. The Alamouti STC-OTFS codeword matrix of dimension2×2 MN is generated, as shown in eq(7)

$$\tilde{X} = \begin{bmatrix} X_1 & -X_2^H \\ X_2 & X_1^H \end{bmatrix} \qquad (7)$$

The OTFS transmit vectors equivalent to X_1 and X_2 are sent between two antennas during the initial frame duration in the STC-OTFS system. Whereas in the second duration, the vectors of $-X_2^H$ and X_1^H are sent between two antennas.

By considering the MIMO-OTFS model in (5), the decoding complexity can be observed. H_1 and H_2 denote the channel matrices between the transmit antennas, then the received vectors between frame duration are

$$y_1 = H_1 x_1 + H_2 x_2 + v_1 \quad (8)$$

$$y_2 = -H_1(\hat{x}_2)^* + H_2(\hat{x}_1)^* + v_2 \quad (9)$$

On the receiver side, conjugation and permutation are used on y_2. For the effective channel matrix, i.e. the problem of decoding for x_1 and x_2 are decomposed into 2 individual orthogonal problems, thus the complexity of decoding is kept as similar to that of SISO-OTFS.

4.1.3 Diversity Analysis

Here, the asymptotic diversity of the STC-OTFS method is considered. Assume that $\tilde{X}_i$ and $\tilde{X}_j$ be the unique codeword matrices for STC-OTFS and the difference matrix is $\tilde{\Delta}_{ij} \triangleq \tilde{X}_i - \tilde{X}_j$. The 2×1 STC-OTFS's diversity order is given as

$$\rho_{STC-OTFS} = min\{_{i,j} min_{i \neq j} rank(\tilde{\Delta}_{ij}), 2P\} \quad (10)$$

4.1.4 Phase Rotation in STC-OTFS

By utilizing the phase rotation design with the STC-OTFS, complete spatial and delay-Doppler diversity is obtained. In the phase rotated STC-OTFS codeword matrix, the difference matrix is defined as $\tilde{\Delta}_{ij}^{\phi}$. The 2×1 phase rotated STC-OTFS's asymptotic diversity is stated in eq (11)

$$\rho_{STC-OTFS} = min\{_{i,j} min_{i \neq j} rank(\tilde{\Delta}_{i,j}^{\phi}), 2P\} \quad (11)$$

4.2 Low Complexity LMMSE receiver for OTFS

Further, to increase the performance of OTFS, the LMMSE receiver is used. The LMMSE equalization will be operated as two phase equalizer. In the initial phase, the LMMSE equalization is operated in order to get $r_{ce} = H_{eq} r$. A matched filter receiver of an OTFS is used in the next phase to get $\tilde{d} = A^{\dagger} r_{ce}$. The $\tilde{d} = A^{\dagger} r_{ce}$ is easy to implement which needs $\frac{MN}{2} log_2(N)$ complex multiplications (CMs). Ina direct implementation of $r_{ce} = H_{eq}r$ needs inversion of $\psi = HH^{\dagger} + \frac{\sigma_n^2}{\sigma_d^2} I$and multiplication of $H^{\dagger}$ that requires O($M^3 N^3$) CMs. To decrease the $r_{ce} = H_{eq}r$complexity, the matrices structure which is required in channel equalization is described.

4.2.1 Structure of $\psi = [HH^{\dagger} + \frac{\sigma_n^2}{\sigma_d^2} I]$

$$HH^{\dagger} = \sum_{p=1}^{p} h_p \Delta^{k_p} \Pi^{l_p} \sum_{s=1}^{p} \bar{h}_s \Delta^{-k_s} \Pi^{-l_s} \quad (12)$$

Where, the circulant matrix is denoted by Π. Because of the shift's cyclic nature, Ψ is quasi-banded along with 2α—1 bandwidth. Also, Ψ is sparse to a conventional wireless channel.

4.2.2 Low complexity LU factorization of Ψ

For accomplish this LU factorization, the Ψ is partitioned as

$$\underbrace{\begin{bmatrix} T_{Q\times Q} & B_{Q\times\theta} \\ S_{\theta\times Q} & C_{\theta\times\theta} \end{bmatrix}}_{\Psi} = \underbrace{\begin{bmatrix} \mathcal{L}_{Q\times Q} & O_{Q\times\theta} \\ V_{\theta\times Q} & F_{\theta\times\theta} \end{bmatrix}}_{L} = \underbrace{\begin{bmatrix} \mathcal{U}_{Q\times Q} & E_{Q\times\theta} \\ O_{\theta\times Q} & G_{\theta\times\theta} \end{bmatrix}}_{U} \quad (13)$$

Using this partition (13),

$$T = \mathcal{L}\mathcal{U} \quad (14)$$

$$E = \mathcal{L}^{-1}B \quad (15)$$

$$V = S\,\mathcal{U}^{-1} \quad (16)$$

$$FG = C - VE \quad (17)$$

Where T denotes a banded matrix, its *LU* decomposition is determined by employing a low complexity algorithm. For the lower triangular matrix, $\mathcal{L}^{-1}B$is calculated by a forward substitution algorithm. The (16) can be calculated in two steps. Firstly, $V^{\dagger} = (\mathcal{U}^{\dagger})^{-1}S^{\dagger}$ is calculated, since the lower triangular matrix is $\mathcal{U}^{\dagger}$.Then, V is calculated by obtaining the $V^{\dagger}$hermitian. The direct computation of (17) takes O(θ^2MN) computations, since it has θ <<

MN. G and F are determined by applying LU decomposition of (17), since the upper as well as lower triangular matrix is G&F. Without adding much in complexity, the pivotal Gaussian elimination approach is utilized to perform LU decomposition. It shows that the diagonal elements of F and $\mathcal{L}$will be equal. Hence, L's diagonal values are also equal.

The U and L should be inverted to LMMSE processing. $HH^{\dagger}$is a positive semi-definite matrix, because it is a hermitian matrix. As $\frac{\sigma_n^2}{\sigma_d^2}$>0 to finite SNR range, and the positive definite matrix is Ψ; thereby, it is invertible. L will be non-singular, since its diagonal values are unity. Moreover, the U's non-singularity is an outcome of the non-singularity of Ψ.

4.2.2 Calculation of r

After LU decomposition, r_{ce} is simplified. To calculate r_{ce}, it is initially shifted circularly by delay and then multiplied using point to point multiplication to every path. Rather than computing $\hat{d}$ as $A^{\dagger}r_{ce}$directly, it transforms r_{ce} to a M * N dimension matrix and then performed by (18), which is applied M number of N-point fast Fourier transform FFT procedures.

$$\hat{d} = vec\{RW_N^{\dagger}\} \qquad (18)$$

4.3 LMMSE receiver to OFDM over Time Variant Channel

For OFDM, low complexity receiver in OTFS can be expanded by using $A = I_N \otimes W_M$. Additionally $\hat{d} = (I_N \otimes W_M^{\dagger})r_{ce}$ is calculated using the FFT. Hence, the proposed model uses the LMMSE receiver in the OTFS scheme to improve the performance.

5. Result

The experiment on the BER performance of OTFS is conducted. The simulation parameters are depicted in table 1.

Table 1 - Simulation parameters

Parameters	Values
Maximum speed (km/h)	506.2
MIMO setup	1 × 1, 2 × 2, 2 × 1
Delay-Doppler profile for P = 2	(0,0), (133.3,1.875)
Modulation scheme	BPSK
Sizes of frames	M = 4, N = 2, M = N = 2
Carrier frequency (GHz)	4
Delay-Doppler profile for P = 4	(0,0),(0,1.875), (133.3,0), (133.3,1.875)
Subcarrier spacing (kHz)	3.75

Figure 3 illustrates the performance of BER using SISO-OTFS, 2 × 1, and 2 × 2 STC-OTFS models. P = 2, M =2, N = 2, and BPSK modulation are used in all the systems. The subcarrier spacing and the carrier frequency are also applied. The estimated BER for the 2 × 2 and 2 × 1 STC-OTFS models displays the diversity orders.

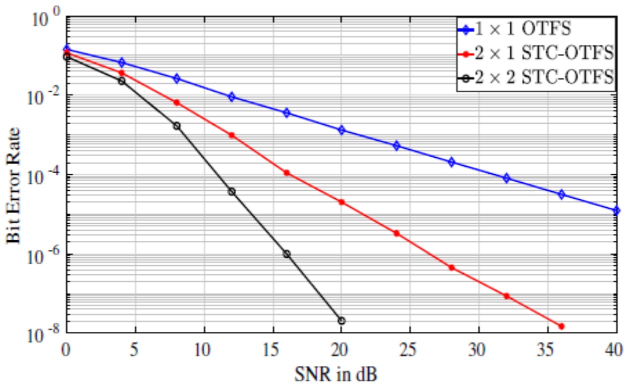

Fig.3 BER performance when M = N = 2

Figure 4.illustratesthe performance of BER using SISO-OTFS, 2 × 1, and 2 × 2 STC-OTFS models. P = 2, N = 2, M = 4, and BPSK modulation are used in all of the systems. From figure 4, it is viewed that the BER performance for every model with N = 2 and M = 4 will exceed the models with M = 2 &N = 2 in figure 3.It indicates that STC-OTFS models with larger frame sizes will attain better diversity orders before reaching their asymptotic diversity of 2_{nr}.

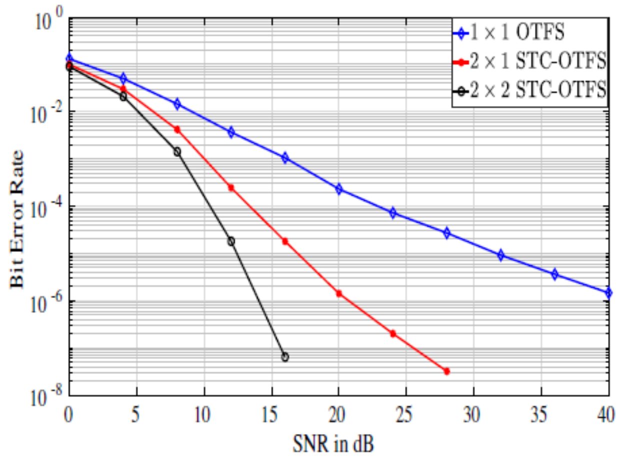

Fig.4 BER performance when M =4, N = 2 for OTFS

Figure 5 depicts the BER performance of OTFS in a MIMO system, with and without phase rotation. It shows that in MIMO channels, phase rotated STC-OTFS will obtain full spatial and delay-Doppler diversity.

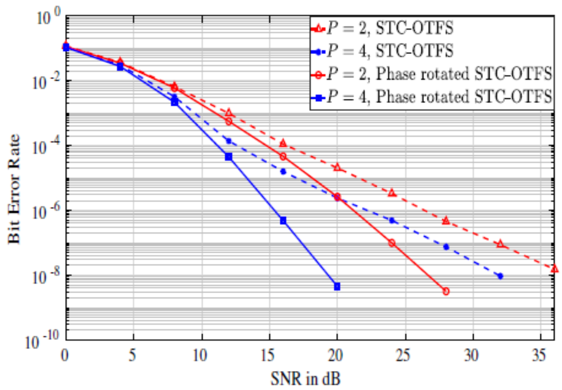

Fig.5 BER performance with and without phase rotation

Then, the complexity is calculated by using complex multiplications (CMs). The proposed receiver contains $O(MN[log_2(N) + \alpha^2 + P^2\beta)$ complexity. Figure 6 depicts the BER comparison with existing and proposed OTFS.

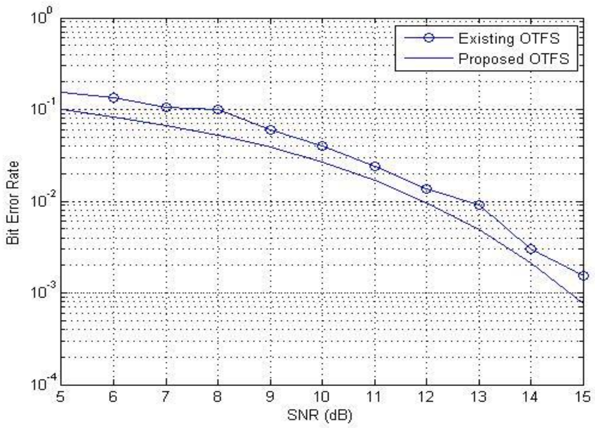

Fig.6 BER comparison

Thus, the LMMSE receiver in our proposed work will attain a significant reduction in complexity without degrading performance while compared to the existing OTFS model.

6. Conclusion

In this work, the STC and LMMSE receiver is used in the OTFS scheme. It demonstrates that in a 2×1 STC-OTFS model, the full spatial transmits diversity order of two is attained. Also, it demonstrated that in a 2×1 STC-OTFS model, the STC along with phase rotation will attain a full spatial and delay-Doppler diversity. Therefore, the proposed model will obtain better diversity performance and the LMMSE receiver will reduce complexity without affecting the performance in OTFS. Simulation on BER performance with SNR is conducted, which shows proposed OTFS achieves greater performance than the existing OTFS scheme.

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