Prospective Math Teacher Students' Graphical Shape Thinking: Is it Static or Dynamic?

Fikri Apriyono[1*], Didik Sugeng Pambudi[2], Nurcholif Diah Sri Lestari[3], Kiswara Agung Santoso[4]

{fikrimath@uinkhas.ac.id[1]*, didikpambudi.fkip@unej.ac.id[2], nurcholif.fkip@unej.ac.id[3], kiswara.fmipa@unej.ac.id[4]}

Universitas Islam Negeri Kiai Haji Achmad Siddiq Jember, Postgraduate Program The University of Jember, Indonesia[1]*, The University of Jember, Indonesia[2,3,4]

Abstract. Graphical reasoning is a fundamental skill in mathematics education, particularly for prospective teachers who must interpret and construct mathematical representations effectively. This study examines graphical thinking tendencies among prospective mathematics teachers, distinguishing between static and dynamic approaches. Using a mixed-methods approach, data were collected through quantitative surveys and qualitative interviews to analyze students' reasoning patterns. The findings reveal that while many students exhibit emergent dynamic thinking, a significant portion remains undecided, relying on visual structures rather than conceptual relationships between variables. The persistence of static reasoning suggests a need for pedagogical interventions to enhance students' transition toward dynamic graphical understanding. The study underscores the importance of inquiry-based learning, technology-enhanced graphing tools, and real-world applications in fostering deeper engagement with mathematical representations. Future research should explore longitudinal studies to assess the long-term impact of instructional strategies on graphical reasoning development.

Keywords: graphical thinking, mathematics education, static and dynamic thinking, prospective teachers, graph interpretation

1 Introduction

Many students in higher education today engage in the process of shaping graphics by utilizing modern technology, such as sophisticated design software [1], [2]. However, students exhibit varying levels of Graphical Shape Thinking (GST), which influences their ability to comprehend and construct graphical representations[3], [4]. Prospective secondary teachers struggle with understanding big ideas critical to formulating connections among representations, such as conic curves, and identifying basic metric relations encoded in

ACIE 2024, October 14-15, Jember, Indonesia
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DOI 10.4108/eai.14-10-2024.2355505

algebraic expressions [5]. Prospective teachers exhibit different levels of geometric thinking, influenced by their past experiences and the van Hiele theoretical model [6]. They often start with pictorial representations but struggle to move beyond a rule-based conception of mathematics [7]. Prospective Math Teacher Students' have diverse Graphical Shape Thinking abilities including static or dynamic thinking.

Graphical Shape Thinking involves interpreting or constructing a graph as dynamically generated, useful across various fields such as science, technology, engineering, and mathematics [4]. Their Graphical Shape Thinking approach can be divided into two, namely a static approach and a dynamic approach [3]. Students using the static approach see graphs as physical objects that can be manipulated visually, such as rotating or changing shapes without considering the quantitative context behind them [8]–[10]. In contrast, the dynamic approach involves deeper thinking, where graphs are understood as representations of quantitative relationships, such as changes in variables that interact with each other in a more complex system [3], [4]. Dynamic graphical thinking is known as emergent graphical shape thinking. Emergent Graphical Shape Thinking (EGST) involves interpreting or constructing a graph as dynamically generated, and it is not spontaneously engaged in by most college students and USA teachers [4]. Students who emergent graphical shape thinking are able to see graphics as representations of relationships between variables that change continuously, so they can understand how changes in one variable affect the overall shape of the graphic. This approach is much more relevant in design contexts that require interpretation of data or complex visualizations, such as graphs of mathematical functions or scientific models.

However, the constrained comprehension of static graphical reasoning exhibited by students frequently constitutes an impediment to the exploration of more sophisticated quantitative representations. This phenomenon holds particular significance within the framework of real-world applications, notably in domains such as mathematics, engineering, and data science, wherein an adept understanding of dynamic graphs is imperative [4], [11], [12]. In the realm of graph design, particularly in relation to data visualization or mathematical graphs, students who adopt a static perspective are inclined to concentrate solely on the manipulation of the physical form, without grasping how variations in variables can influence the overall representation of the graph [13]. This static perspective limits students' ability to discern features of representation systems, such as the Cartesian coordinate system, and hinders their mathematical reasoning [13]. Graphs are essential tools for communicating data, and the aesthetics of graphs play a crucial role in their transparency and readability, influencing how they are perceived and understood [14].

A study conducted by Moore [3] supports this view by finding that many students tend to use static graphical shape thinking, which in turn causes them difficulties in understanding emergent graphical shape thinking such as mathematical function graphs. This shows that there is still a big challenge in learning to understand and shape graphics to encourage students to move from a static approach to a dynamic approach, which emphasizes more on the quantitative understanding underlying graphical visualization. The move from static to emergent graphic form thinking represents a significant advancement in how information is visualized and understood, leveraging interactivity, cognitive alignment, and technological innovation to enhance learning and comprehension [4], [15], [16]. Therefore, more research is needed that focuses on developing dynamic graphical thinking to improve students' skills in exploring and utilizing graphics in a broader and more applicable context.

Although previous studies have explored graphical thinking, research on how students transition from static to dynamic thinking remains limited. Most existing studies focus on general graphical reasoning rather than investigating how prospective teachers develop and refine their graphical thinking abilities over time. Addressing this research gap is essential for designing effective instructional approaches that promote deeper engagement with graphical representations. Understanding how students perceive and interpret graphs can provide valuable insights for mathematics educators, helping them develop pedagogical strategies that encourage a more conceptual, dynamic understanding of graphs.

This study aims to investigate the graphical reasoning tendencies of prospective mathematics teachers, categorizing their thinking into static, dynamic, or transitional perspectives. Using a mixed-methods approach, the research explores students' graphical reasoning patterns through quantitative surveys and qualitative interviews. The findings will contribute to the development of evidence-based teaching strategies, enabling educators to bridge the gap between static and dynamic graphical understanding. By addressing these challenges, this study seeks to enhance mathematical pedagogy, equipping future educators with the necessary skills to teach graphical concepts effectively and prepare students for the increasing reliance on visual data interpretation in mathematics and related disciplines.

2 Method

This research employed a mixed-methodological framework, integrating both quantitative and qualitative strategies. Quantitative information was gathered via a survey designed to assess students' proclivities towards static versus dynamic reasoning in the interpretation of graphs. The survey investigated students' inclinations and practices in executing graphical tasks. Qualitative insights were derived from comprehensive interviews conducted with students who were selected based on the outcomes of the survey. The primary objective of the interviews was to delve into the cognitive processes of students as they approached graphical problem-solving, considering both static and dynamic perspectives. The application of a mixed-method approach affords a more holistic understanding of students' cognitive patterns. The quantitative component yields statistical data that quantifies tendencies towards static or dynamic reasoning, whereas the qualitative component offers profound insights into the various factors that shape their graphical cognition. Quantitative information was amassed through a survey that centered on students' preferences in engaging with graphical tasks. Students enrolled in college-level mathematics education were designated as participants due to their positioning at a critical juncture in skill acquisition that may significantly influence their future, particularly regarding the instruction of graphical understanding to students.

The participants in this research comprised mathematics education students at an islamic university, specifically from the first, third, fifth, and seventh semesters. A total of 157 students engaged in the survey, while the informants for the in-depth interviews included 2 students categorized as static thinkers and 2 students classified as dynamic thinkers. The selection of these students was predicated on their direct involvement in the application and cognitive processing of graphical representations during their academic pursuits. Their experiences, especially concerning graphical tasks, are pertinent for assessing tendencies in static (manipulation of physical forms) or dynamic (comprehension of quantitative relationships) thinking.

The following is a display of the instrument given through the form to 157 mathematics education students:

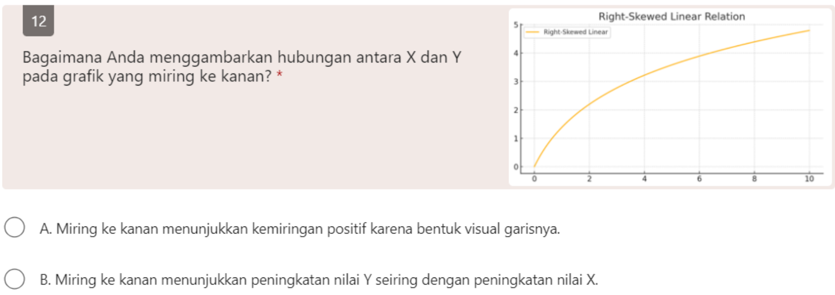

Fig. 1. Graphical Shape Thinking instrument display question 12

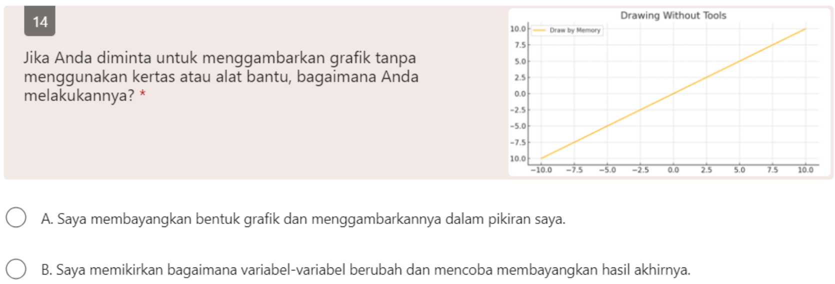

Fig. 2. Graphical Shape Thinking instrument display question 14

Static and dynamic group categorization formula:

Table 1 Grouping criteria for static and dynamic type of Graphical Shape Thinking ability

Criteria	**Classification**
If % of Answer A ≥ 70%	Static Category
If % of Answer B ≥ 70%	Dynamic Category
If 30% ≤ % Answer A < 70% and 30% ≤ % Answer B < 70%	Undecided Category
$\%\textbf{Answer } \boldsymbol{A \text{ or } B} = \left(\frac{\textbf{Answer } \boldsymbol{A \text{ or } B}}{\mathbf{40}}\right) \times \mathbf{100\%}$	

The process of data collection was executed in multiple stages. The initial stage involved a comprehensive desk review to delineate the issue based on existing literature pertaining to static and dynamic graphical thinking, thereby elucidating the theoretical framework of the study. Subsequently, a survey was administered to gather quantitative data regarding students' tendencies in graphical thinking, encompassing both static and dynamic dimensions. The final stage entailed conducting in-depth interviews to further investigate students' cognitive processes when addressing graphical problems. These methodologies for data collection are meticulously designed to yield a comprehensive understanding, with surveys furnishing quantifiable data and interviews offering profound insights into the context and strategies associated with graphical thinking.

3 Result

The following is the distribution of the results of a questionnaire with 40 questions filled in by 157 mathematics education students in tertiary institutions

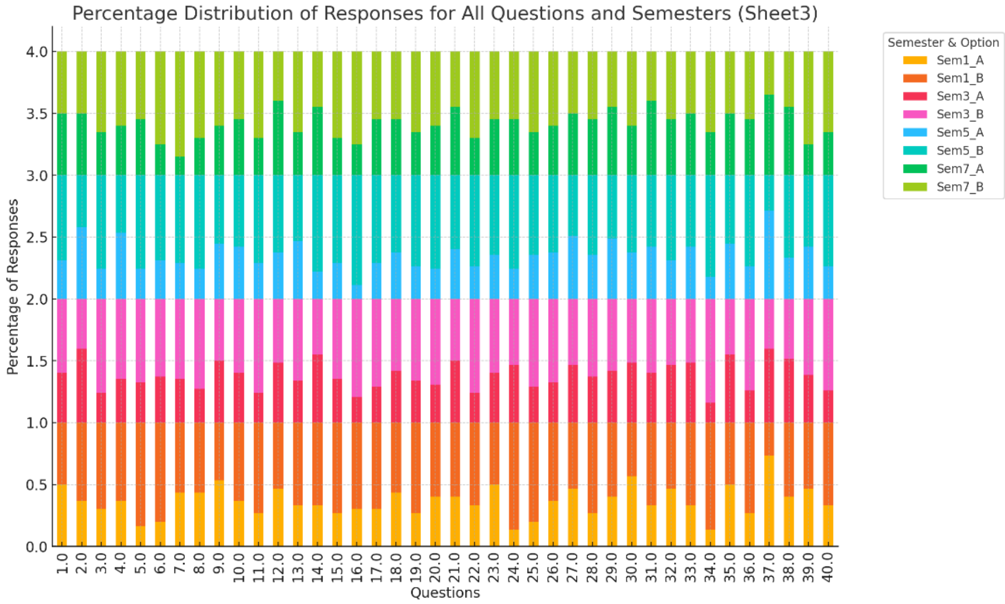

Fig. 3. Distribution of all respondents on all questions

As illustrated in the graphical representation, the data distribution noted corresponds to the previously referenced chart, comprising a total of 40 inquiries, subsequently addressed by participants from semesters 1, 3, 5, and 7. Moreover, the chart clarifies that the responses from each cohort of semesters 1, 3, 5, and 7, classified as A (static category) and B (dynamic category), are represented in percentage format.

From the set of 40 questions, we have organized them into four distinct categories based on the type of graphical thinking they assess. The first category is Linear Graphs and Simple Patterns, which includes questions related to understanding straight lines or simple shapes, as well as reflections and rotations. Specifically, this category covers linear graphs and simple patterns (Questions 1, 5, 8, 20, 40), reflections and rotations (Questions 9, 30), and graphs with specific features such as inflection points or peaks (Questions 6, 16, 24, 27, 31). The second category, Growth, Exponential, and Curved Graphs, examines how students interpret graphs involving growth or curves, including exponential and parabolic functions. This group consists of exponential and growth graphs (Questions 2, 3, 7, 23, 26) and parabolas, cubic functions, and other curved graphs (Questions 15, 17, 29). The third category, Wave and Sinusoidal Patterns, focuses on graphs with wave-like or sinusoidal patterns. Questions in this group explore oscillating graphs and include wave and sinusoidal patterns (Questions 4, 18, 28). Finally, the fourth category is General and Complex Graph Interpretation, which deals with interpreting more complex or unusual graphs. This category includes general graph interpretation (Questions 10, 12, 14, 19, 35) and complex graphs with multiple variables or uncommon patterns (Questions 33, 34, 36, 38, 39). This categorization aims to assess various aspects of graphical thinking, from simple linear forms to more intricate and dynamic representations.

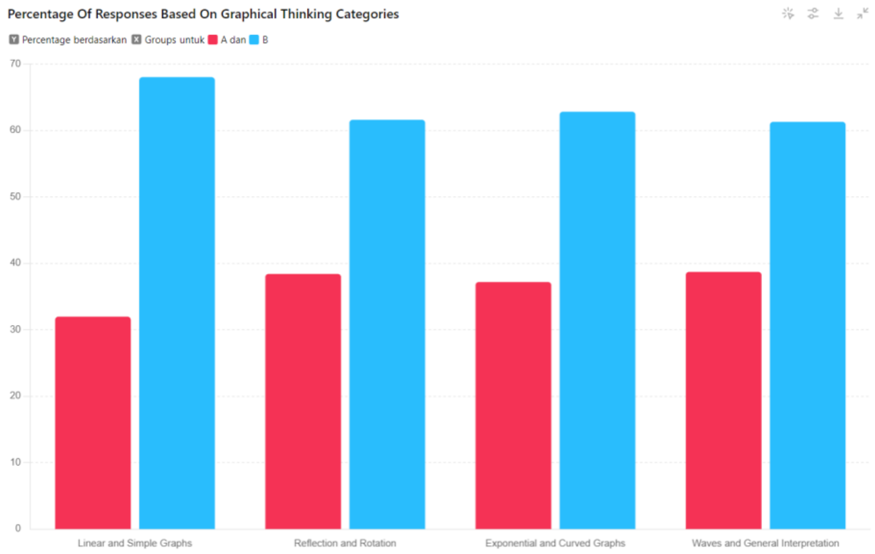

Fig. 4. Respondents' answers are divided into four categories of graphs

The detailed breakdown of responses shows a strong preference for option "B" across all categories. In Linear and Simple Graphs, 59.24% of participants chose "B," while 40.76% selected "A." The Reflection and Rotation category shows a more pronounced difference, with 70.87% selecting "B" and only 29.13% choosing "A." Similarly, Exponential and Curved Graphs had 72.56% of participants opting for "B," reflecting a significant preference for dynamic interpretations, with just 27.44% choosing "A." In Waves and General Interpretation, 66.85% favored "B," while 33.15% chose "A." These results suggest that participants generally lean toward more interpretive or dynamic thinking ("B"), particularly when dealing with more complex graphs, such as exponential and curved shapes, as well as reflection or rotation scenarios.

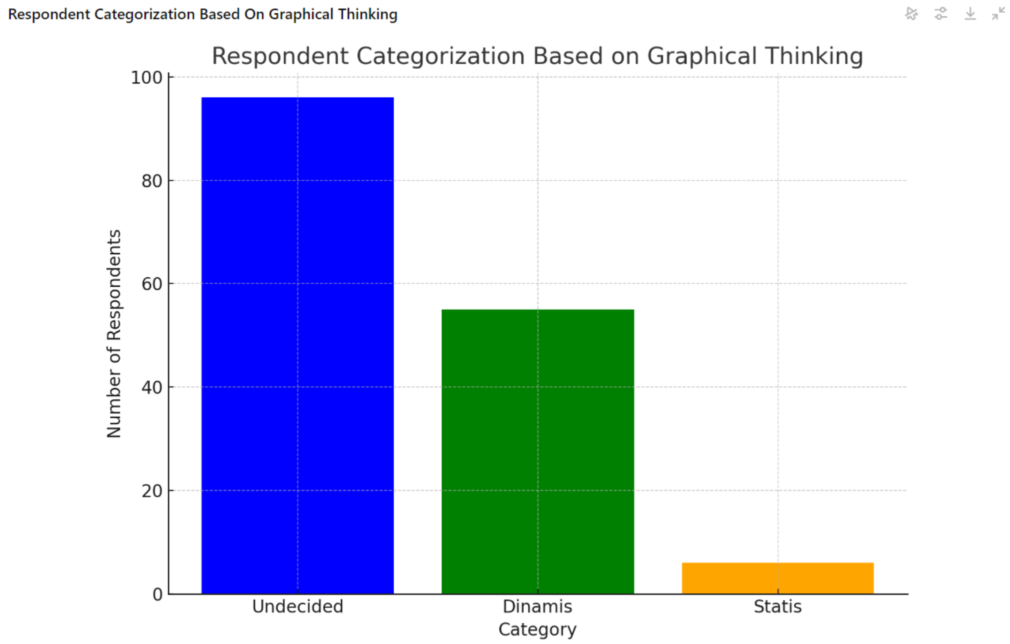

Fig. 5. respondents who are divided into three categories

The bar chart displays the categorization of respondents into three groups: *Static*, *Dynamic*, and *Undecided*. From the total respondents, a majority fall into the Undecided category, accounting for 76.43% (120 out of 157 respondents). The Dynamic category, representing those who predominantly selected "B" (70% or more), makes up 20.38% (32 respondents). Lastly, the Statis group, consisting of respondents who selected "A" for 70% or more of the questions, constitutes only 3.18% (5 respondents). This distribution indicates that most participants exhibited a balanced approach to interpreting graphical questions, with a smaller portion leaning heavily towards dynamic or static thinking styles.

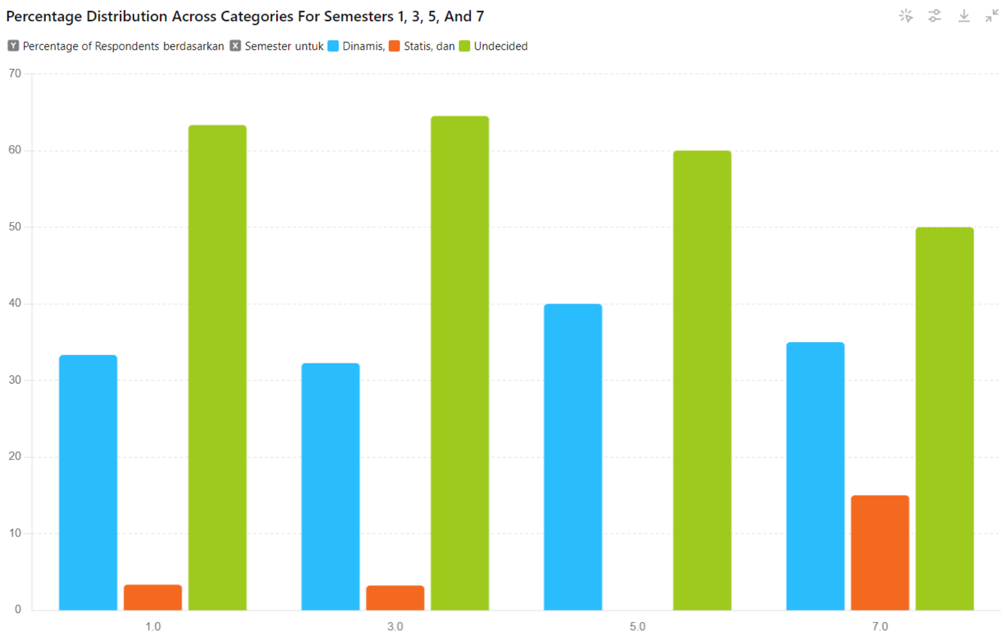

Fig. 6. Respondents who are divided into four semester categories

The data illustrates the percentage distribution of respondents across the *Dynamic*, *Static*, and *Undecided* categories for first, third, fifth, and seventh semester. In first semester, a majority of 63.33% of respondents fall into the undecided category, while 33.33% are categorized as dynamic, and only 3.33% are in the static group. This suggests that most first-semester students have a mixed approach to graphical thinking. In the third Semester, the trend is similar, with 64.52% in the undecided category, 32.26% in dynamic, and 3.23% in static. As students advance to Semester 5, the proportion of dynamic respondents increases to 40%, while the Undecided category slightly decreases to 60%, and there are no respondents in the static category. This shift indicates a growing preference for dynamic interpretations of graphs as students gain more experience. By Semester 7, the dynamic category remains prominent at 35%, with 50% of respondents still 'undecided' and a higher proportion (15%) falling into the 'static' category. This suggests that while dynamic thinking is favored, some students still exhibit static thinking in the later stages of their education. Overall, the data shows a gradual shift toward dynamic thinking, with a consistent yet small presence of static approaches across all semesters.

Table 2 distribution of static and dynamic category respondent data based on semester

No	Semester	% Statis (A)	% Dinamis (B)
1	1	36,58	63,42
2	3	38,51	61,49
3	5	35,17	64,83
4	7	42	58

The analysis of student responses across different semesters reveals a consistent dominance of dynamic (B) thinking in graph interpretation. In Semester 1**,** 63.42% of students exhibited dynamic reasoning, while 36.58% demonstrated static (A) thinking. A similar trend is observed in Semester 3, with 61.49% dynamic responses and 38.51% static responses. By Semester 5, dynamic interpretations increased slightly to 64.83%, while static responses declined to 35.17%, indicating a gradual shift toward a more conceptual and relational understanding of graphs. However, in Semester 7, the percentage of static reasoning increased to 42.00%, while dynamic responses declined to 58.00%, suggesting a more balanced distribution compared to earlier semesters.

Table 3 percentage of respondents as a whole which is divided into static and dynamic

Total Statis (A) Responses	Total Dinamis (B) Responses	Percentage Statis (A)	Percentage Dinamis (B)	Percentage Difference (B - A)
2365	3930	37,57	62,43	24,86

The overall difference between Dynamic (B) and Static (A) responses across all respondents is approximately 24.86%, with dynamic responses being higher. This highlights a notable preference for dynamic thinking among the respondents.

The following are the results of interviews with respondents who have answered 40 questions. We select subjects to be interviewed based on several criteria, namely that the respondent must be in a strong static and dynamic group. Strong static and dynamic are seen from the percentage value of answering more than 70%. then the subject is seen working time normally, not too fast or not too slow. So we found a subject with this category right on respondent number 156 for the static one has a score of 77.5% with a working time of 14.15 minutes, respondent number 16 for the dynamic one has a score of 80% with a working time of 14.37 minutes.

Based on the results of the interview with respondent no.156 who is categorized as thinking to form a static graph, the following results are obtained:

Researcher	:	Can you explain your approach to solving problems involving graphs? Do you rely more on visual patterns or shapes to understand them?
Respondent 156	:	I rely more on visual patterns and shapes. The shape of the graph helps me understand the content faster
Researcher	:	So, you feel more comfortable with the appearance of the graph than understanding the relationship between the variables in it?
Respondent 156	:	Yes, that's right. I find it easier to see the visual structure rather than looking for the relationship between variables
Researcher	:	What made you focus more on the visual structure rather than the relationship between variables?
Respondent 156	:	I think the visual structure is clearer and can be seen immediately. I feel more confident when I see a graph that has a clear shape.
Researcher	:	What about graphs that are more complex or changing, such as graphs with many variables or those with inflection points? How comfortable are you with such graphs?
Respondent 156	:	I feel a bit uncomfortable with graphs like that. There are too many variables and changes in it, so it's more difficult to understand.
Researcher	:	Do you think keeping the structure of the graph constant helps you better understand the relationship between the variables?

Respondent 156	:	Yes, it is very helpful. If the structure doesn't change, I can more easily recognize the pattern and understand the relationship between the variables.
Researcher	:	How often do you use knowledge from simple graphs like straight lines when trying to understand more complex graphs?
Respondent 156	:	I often use graphs that I have understood before to help predict patterns in new or more complicated graphs.

The conclusion from the interview with Respondent 156 shows that respondents rely more on the visual structure of the graph in understanding the problem. They feel more comfortable with graphs that have clear visual patterns or shapes, as this gives them confidence in understanding them. Respondents tend to avoid complex or dynamic graphs with many variables, as they find such graphs more difficult to interpret. They also emphasized the importance of keeping the structure of the graph constant to make it easier to recognize patterns and understand relationships between variables. In addition, they often use knowledge from simple graphs, such as straight lines, as a reference to understand more complex graphs. This suggests that the visual approach is more dominant in their understanding process than the quantitative relationship between variables.

Meanwhile, the results of the interview with respondent no.16 who is categorized as thinking of forming graphs dynamically are obtained as follows:

Researcher	:	When you look at a graph, what do you usually look at first - do you focus on the relationships between variables or the overall shape of the graph?
Respondent 16	:	I tend to focus more on the relationship between the variables
Researcher	:	Can you explain further? What makes you focus more on the relationship between variables?
Respondent 16	:	I always think about how one variable can affect another. I think that is the most important thing to understand a graph.
Researcher	:	How do you interpret the changes in the graph, especially when the variables change dynamically, like in an exponential or sinusoidal graph?
Respondent 16	:	I try to understand how changes in one variable will affect the whole graph. For example, on an exponential graph, I would pay attention to how rapid growth affects the result.
Researcher	:	Do you often imagine how the graph will change if one of the variables is modified? Can you give an example?
Respondent 16	:	Yes, often. For example, in an exponential graph, if the growth variable is faster, the graph will increase faster too.
Researcher	:	How comfortable are you in interpreting graphs that evolve over time or involve complex relationships?
Respondent 16	:	I am quite comfortable with such graphs. I usually use visualization strategies to help me understand how the graph changes over time.
Researcher	:	When interpreting graphs, do you often imagine how one variable affects another? How does this affect your understanding?
Respondent 16	:	Yes, very often. Seeing how the variables interact with each other really helps me understand quantitative changes more

easily.

The conclusion from the interview with Interviewee 16 shows that they focus more on the relationship between variables when analyzing graphs. They prioritize understanding how changes in one variable affect another, which is considered the most important aspect of understanding graphs. Respondents are also comfortable interpreting complex and dynamic graphs, such as exponential or sinusoidal graphs, by visualizing the changes between variables. Respondents often predicted how modifications to one variable would change the overall graph, and they used this approach to better understand graphs that evolve over time. The visualization strategy helps them map the interactions between variables, which facilitates a deeper understanding of the quantitative changes in the graph.

4 Discussion

4.1 Progression of Graphical Thinking in Prospective Mathematics Teachers

The findings reveal a gradual shift toward dynamic graphical thinking among prospective mathematics teachers as they progress through their academic semesters. In the first semester, a significant majority (63.33%) of students fall into the undecided category, suggesting that they exhibit mixed characteristics of both static and dynamic graphical thinking. This aligns with existing research indicating that novice learners often struggle to establish a stable conceptual framework for interpreting graphs, relying on intuitive visual features rather than deeper functional relationships [3], [4]. The relatively low percentage of static thinkers (3.33%) suggests that while some students approach graphs in a purely pictorial manner, most are still in a transitional phase, needing further cognitive development to adopt a dynamic perspective.

By the third semester, the distribution remains similar, with 64.52% of students still classified as undecided, and 32.26% demonstrating dynamic thinking. The persistence of a high percentage in the undecided category indicates that students at this stage are still refining their graphical reasoning skills, possibly influenced by curriculum structure and exposure to mathematical modeling. The lack of a significant decline in undecided responses suggests that students require targeted instructional interventions to strengthen their ability to interpret graphs dynamically.

A notable shift occurs by the fifth semester, where the proportion of dynamic thinkers increases to 40%, and static thinkers are no longer present. This suggests that with increased exposure to advanced mathematical concepts, problem-solving tasks, and instructional scaffolding, students become more adept at recognizing graphs as representations of changing relationships rather than static figures. This trend is consistent with the van Hiele model of geometric thinking, which posits that learners progress from visual recognition to relational reasoning through structured learning experiences[6][17].

However, an unexpected trend emerges in the seventh semester, where 15% of respondents revert to static thinking, while 50% remain undecided, and 35% continue with dynamic reasoning. This increase in static reasoning at later stages may indicate that certain students revert to rote-based approaches when faced with complex graphical tasks, struggling to maintain flexible, conceptual interpretations. Prior studies suggest that even among advanced learners, contextual factors such as instructional style, assessment methods, and cognitive load can influence the stability of conceptual frameworks[5][18]. The persistence of a large undecided category (50%) further supports the need for pedagogical strategies that explicitly emphasize functional relationships, mathematical modeling, and real-world applications of graphs to reinforce dynamic reasoning.

4.2 The Differences Between Static and Dynamic Graphical Thinking Approaches

The research findings also reveal distinct differences between static and dynamic graphical thinking approaches among prospective mathematics teachers. Respondent 156, who exemplifies static graphical thinking, relies heavily on the visual structure and shape of graphs to interpret mathematical relationships. This respondent expresses a preference for simple, well-defined visual patterns, such as straight-line graphs, as they provide a sense of familiarity and confidence in problem-solving. The reluctance to engage with complex, multi-variable graphs suggests a cognitive dependency on recognizing fixed structures rather than exploring dynamic relationships. Additionally, the tendency to apply knowledge from basic graph types as a reference for understanding more advanced representations indicates a rule-based, pattern-recognition strategy rather than a conceptual grasp of functional dependencies. These findings align with previous studies suggesting that early-stage graphical reasoning often emphasizes shape recognition over analytical reasoning, leading to difficulties in interpreting graphs beyond their immediate visual form[3], [4].

Conversely, Respondent 16, who represents dynamic graphical thinking, demonstrates a fundamentally different approach by prioritizing the relationship between variables rather than focusing solely on the graph's visual structure. This respondent actively analyzes how changes in one variable influence another, reflecting a more advanced level of graphical reasoning. Unlike static thinkers, dynamic thinkers are comfortable with nonlinear, complex graphs such as exponential or sinusoidal functions, as they approach graphs as evolving entities rather than fixed structures[19]. The ability to predict modifications in a graph based on variable interactions suggests a deeper conceptual understanding of mathematical relationships, which is a critical skill for both scientific problem-solving and mathematics education[6].

In line with this, Rabinovich and Varona stated that creative thinking is a dynamic, nonlinear process characterized by internal instability and the generation of new information[20]. This nonlinear dynamic approach is also applied to work and organizational psychology, offering insights into phenomena such as leadership, team behavior, and training transfer[21]. In education, nonlinear dynamic models have been used to explain conceptual change processes in science learning, with convergent and divergent thinking acting as bifurcation factors[22].

These contrasting thought (static and dynamic) processes highlight a significant pedagogical challenge in mathematics education: facilitating the transition from static to dynamic graphical reasoning. Research indicates that students often begin with visual dependency but require structured interventions—such as inquiry-based learning and interactive graphing tools—to develop emergent graphical reasoning[5][23]. Educators must implement instructional strategies that encourage students to engage with graphs as dynamic representations of functional relationships, emphasizing cause-effect reasoning rather than mere shape recognition[24].

Future research should explore instructional techniques that effectively support this cognitive transition, such as technology-enhanced graphing tools, real-world data applications, and collaborative learning environments. By fostering conceptual engagement with variable interactions, educators can equip students with the skills necessary for advanced mathematical reasoning[25] and applied problem-solving across diverse disciplines.

4.3 Implications for Mathematics Education

Overall, the data underscores the importance of curriculum design and targeted interventions in supporting the transition from static to dynamic graphical thinking. While students gradually shift toward dynamic interpretations, a significant proportion remains undecided, suggesting that existing instructional methods may not sufficiently address the conceptual struggles faced by learners. Integrating technology-based graphing tools, inquiry-based learning, and real-world problem-solving activities may help bridge this gap, reinforcing students' ability to engage with graphs beyond visual recognition toward meaningful, functional analysis[26], [27]. Future research should explore longitudinal studies to examine how instructional strategies influence the long-term retention of emergent graphical thinking skills in mathematics education.

5 Conclusion

This study aimed to investigate the graphical thinking of prospective mathematics teachers, specifically whether their thinking is more static or dynamic. Static thinking involves viewing graphs as physical objects without considering quantitative relationships, while dynamic thinking views graphs as representations of changes between variables. The research employed a mixed-methods approach, combining a survey of 157 students and in-depth interviews with selected participants to gain deeper insights.

The findings indicate that the majority of students exhibit dynamic thinking, with 20.38% of respondents showing a preference for this approach. Dynamic thinkers are more adept at interpreting graphs in terms of variable relationships and changes over time, which is essential for real-world applications. On the other hand, only 3.18% of respondents displayed static thinking, focusing on the visual manipulation of graphs without understanding the underlying quantitative relationships. The largest group, 76.43%, fell into the undecided category, reflecting a mixture of both thinking approaches.

Across different semesters, a gradual shift from static to dynamic thinking was observed. In the first semester, 63.33% of students were undecided, with only 33.33% classified as dynamic thinkers. By the fifth semester, dynamic thinking increased to 40%, indicating that students develop more complex graphical reasoning as they progress in their studies.

In addition to the quantitative findings, interviews with students classified as static or dynamic thinkers offered deeper insights into their cognitive processes. A student from the static category, scoring 77.5%, revealed that they often rely on the visual structure of graphs rather than the relationships between variables. They felt comfortable with interpreting simple, familiar graphs but struggled with complex or dynamic graphs involving multiple variables or changes over time. This respondent noted that keeping the graph structure constant helped them, but they found it challenging to understand how variable changes affected the overall graph. Conversely, a student from the dynamic category, scoring 80%, focused clearly on relationships between variables when interpreting graphs. This student frequently imagined how changes in one variable would impact the entire graph and felt more comfortable with dynamic graphs, such as exponential or sinusoidal patterns. They highlighted the importance of understanding how variable interactions shape the graph, aiding in their comprehension of quantitative changes.

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