

# Spectral Efficiency of Energy Harvesting Random Cognitive Radio Networks in Dual-slope Model

Saifur Rahman Sabuj<sup>1,\*</sup>, Rakiba Rayhana<sup>2</sup>, and Aldrin Nippon Bobby<sup>#</sup>

<sup>1</sup>Electrical and Electronic Engineering, BRAC University, Bangladesh

<sup>2</sup>Electrical Engineering, The University of British Columbia, Canada

## Abstract

This research article evaluates the performance of an energy harvesting random cognitive radio network employing a dual-slope path-loss model in terms of spectral efficiency. It derives the mathematical expression of spectral efficiency by means of stochastic geometry for secondary receivers present in the network in active mode. The numerical result highlights a rise in the efficiency by 20.7% for an increase of 1 dBm in transmit power at a power splitting ratio of 0.5.

**Keywords:** Energy harvesting, random cognitive radio network, dual-slope path-loss model, spectral efficiency, stochastic geometry.

**Index Terms:** Energy harvesting, random cognitive radio network, dual-slope path-loss model, spectral efficiency, stochastic geometry.

## 1. Introduction

The usage of wireless devices like notebooks, tablets, phablets and cellular phones has been escalated these days. One of the most vital components of these devices is the battery which needs to be energized and substituted at regular intervals. That's why designers are constantly seeking to enhance the durability of batteries for a better service. The manifestation of wireless energy transfer from several radio frequency (RF) sources was initially developed during 1899 by Nicola Tesla [1]. However, base stations (BSs) found in mobile networks are not considered as beneficial sources because they extract a very small amount of energy. Therefore, cognitive radio (CR) network together with wireless energy harvesting technology is a probable solution to gathering energy from authorized and unauthorized RF sources [2] allowing the secondary users meaning the ones who are unauthorized to keep the energy in reserve for recharging batteries.

This paper defines a performance metric known as spectral efficiency ( $\text{nat/s/Hz}$ ) which analyzes the performance of energy harvesting CR networks having dual-slope path-loss model. In earlier work [3], investigation on harvested energy, outage probability,

and maximization of harvested energy in active mode was explored.

## 2. System Model

### 2.1. Network Model

We assume that the downlink scenario of a CR network consists of a primary transmitter (PT) as well as several primary receivers (PRs), secondary transmitters (STs) and secondary receivers (SRs). The position of operating PTs and STs are allocated according to stochastic geometry, specifically a homogeneous Poisson point process (PPP) with density of  $\lambda_{pt}$  and  $\lambda_{st}$ , respectively. The PRs and SRs follow an independent homogeneous PPP too, with density of  $\lambda_{pr}$  and  $\lambda_{sr}$ , respectively. For the sake of simplicity, all operating STs (cognitive macro-BS) are considered outside the primary exclusion region as they are not permitted to employ licensed spectrum due to interference [4, 5]. An intended SR is placed exactly at the origin (0, 0).

### 2.2. Path-loss Model

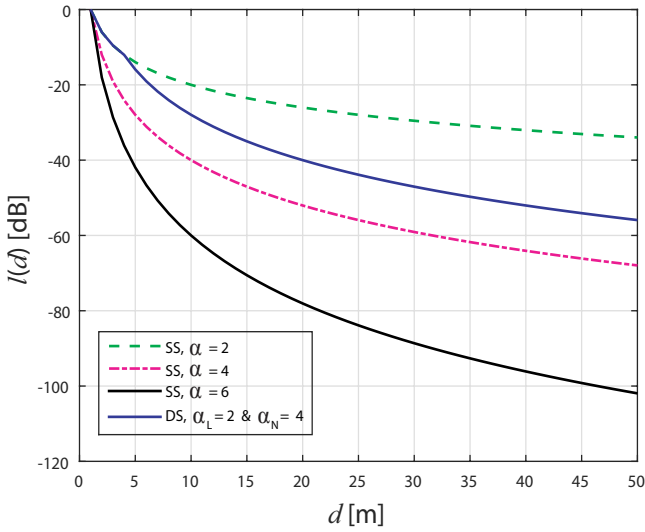
As stated in the report of 3rd Generation Partnership Project (International Telecommunication Union Radio Communication Sector), the dual-slope path-loss model proves to be very effective in performing mobile

\*Corresponding author. Email: s.r.sabuj@ieee.org

networks analysis [6]. All signals in this study are going to be transmitted through the Rayleigh fading channel. The channel power gains are independent of each other. They follow the same exponential distribution with unity mean. Based on [7], the dual-slope path-loss model is defined as

$$l(d) = \begin{cases} \|d\|^{-\alpha_L}, & \|d\| \leq d_c \\ \kappa \|d\|^{-\alpha_N}, & \|d\| > d_c \end{cases} \quad (1)$$

where  $\kappa = d_c^{\alpha_N - \alpha_L}$ ,  $\alpha_L$  and  $\alpha_N$  denote the path-loss exponents for line-of-sight and non-line-of-sight propagation, respectively, and  $d_c$  is the critical distance.



**Figure 1.** Comparison among three single-slope (SS) and one dual-slope (DS) for the path-loss exponents of 2, 4 and 6.

Figure 1 depicts the characteristic of path loss with respect to distance for single-slope and dual-slope model. It incorporates a curve for single-slope model with a path-loss exponent of 2 (SS<sub>1</sub>), that with the exponent of 4 (SS<sub>2</sub>), the same with the exponent of 6 (SS<sub>3</sub>) and dual-slope model with a line-of-sight path-loss exponent of 2 as well as a non-line-of-sight exponent of 4 (DS). SS<sub>3</sub> falls out if all curves are compared to one another due to staying far away from the others with the most amount of path loss. Among the rest, SS<sub>1</sub> exhibits the least amount of loss and SS<sub>2</sub> gives the most while DS stays in the middle. DS has almost a constant amount of approximately 12 dB loss less than SS<sub>2</sub>. It goes exactly along with SS<sub>1</sub> having the same loss as SS<sub>1</sub> has from the beginning up to a distance of 4 m. Afterwards, DS drops off SS<sub>1</sub> with a variable loss with respect to the distance. For instance, it is about 8 dB below SS<sub>1</sub> at a distance of 10 m. Again, it is about 20 dB below SS<sub>1</sub> at 40 m.

### 2.3. Time Slot Structure

For secondary transmission, each frame of ST under CR network includes one sensing slot and one transmission slot. In sensing slot, ST approaches the authorized frequency band to identify  $M$  unoccupied and  $N - M$  as occupied channels among  $N$  number of authorized channels. In transmission slot, ST transmits own data with associated SR utilized by one unoccupied channel from  $M$  unoccupied channels.

### 2.4. Design of Receiver

A dual mode power-splitting (PS) receiver is designed which can be effective in both active mode (on-mode) and inactive mode (off-mode) [3]. During active mode, there is an active communication between ST and SR. Each SR is provided with a PS device. So the signal power is received at SR side, is divided in such a way that information decoder (ID) gets  $\rho$  of it and the rest i.e.  $1 - \rho$  portion goes to the energy harvester (EH). During inactive mode, ST does not operate on information transmission. So PS is always associated to EH node at the SR.

### 3. Spectral Efficiency

In this section, the spectral efficiency of intended SR is analyzed for active mode.

In active mode, the signal-to-interference-plus-noise ratio (SINR) at the ID of SR can be expressed as

$$\text{SNIR} = \frac{\rho P_s h_s l(d_o)}{\rho(I_{ss} + I_{sp} + \sigma_s^2) + \sigma_c^2}, \quad (2)$$

where  $l(d_o) = d_c^{-\alpha_L} + \kappa(d_o - d_c)^{-\alpha_N}$ .  $\rho$ ,  $P_s$ ,  $h_s$  and  $d_o$  are PS ratio, transmission power of ST, fading channel gain in intended ST-SR pair and distance between intended ST-SR pair, respectively.  $\sigma_s^2$  and  $\sigma_c^2$  are the variances of additive white Gaussian noise at SR and circuit noise at the ID of SR, respectively. In (2), it is presumed that intended SR is exterior the critical distance. If intended SR is interior the critical distance,  $l(d_o)$  is considered  $\|d_o\|^{-\alpha_L}$ .  $I_{ss}$  is the interference power experienced by SR due to the presence of STs except intended ST.  $I_{sp}$  is that due to the presence of PTs.  $I_{ss}$  and  $I_{sp}$  are mentioned below.

$$I_{ss} = \sum_{i \in \Phi_{st} \setminus \{o\}} P_{si} h_{si} l(d_{si}), \quad (3)$$

$$I_{sp} = \sum_{i \in \Phi_{pt}} P_{pi} h_{pi} l(d_{pi}), \quad (4)$$

where  $o$  is the intended ST.  $h_{si}$  and  $d_{si}$  denote the fading channel gain from intended SR to interfering STs and the distance between intended SR and the interfering STs.  $P_p$ ,  $h_{pi}$  and  $d_{pi}$  denote the transmit power of PT,

the fading channel gain from intended SR to interfering PTs and the distance between intended SR and the interfering PTs.

Generally, spectral efficiency of SR is defined as the transmission rate at which an information is successfully transmitted between ST-SR pair over per unit bandwidth (unit is named as *nat/s/Hz*). Thus, the spectral efficiency of SR can be expressed as

$$\begin{aligned}
 S &= \int_0^{\infty} \Pr[\ln(1 + \text{SNIR}) > t] dt \\
 &= \int_0^{\infty} \Pr\left[\frac{\rho P_s h_s l(d_o)}{\rho(I_{ss} + I_{sp} + \sigma_s^2) + \sigma_c^2} > E(t)\right] dt \\
 &= \int_0^{\infty} \exp\left[-\frac{E(t)\{\rho(I_{ss} + I_{sp} + \sigma_s^2) + \sigma_c^2\}}{\rho P_s l(d_o)}\right] dt \\
 &= \int_0^{\infty} \frac{1}{1+z} \exp(-\vartheta_1 z) \mathcal{L}_{I_{ss}}\left(\frac{z}{\vartheta_2}\right) \mathcal{L}_{I_{sp}}\left(\frac{z}{\vartheta_2}\right) dz, \quad (5)
 \end{aligned}$$

where  $E(t) = e^t - 1$ ,  $\vartheta_1 = \frac{\rho\sigma_s^2 + \sigma_c^2}{\rho P_s l(d_o)}$  and  $\vartheta_2 = P_s l(d_o)$ .  $\mathcal{L}_{I_{ss}}(\cdot)$  and  $\mathcal{L}_{I_{sp}}(\cdot)$  are the Laplace transforms of the probability density functions of  $I_{ss}$  and  $I_{sp}$ . The Laplace transforms of  $\mathcal{L}_{I_{ss}}(z(P_s l(d_o))^{-1})$  and  $\mathcal{L}_{I_{sp}}(z(P_s l(d_o))^{-1})$  are written as

$$\mathcal{L}_{I_{ss}}(z(P_s l(d_o))^{-1}) = \exp(-\pi \lambda_{st} Q_s \vartheta_3), \quad (6)$$

$$\mathcal{L}_{I_{sp}}(z(P_s l(d_o))^{-1}) = \exp(-\pi \lambda_{pt} Q_p \vartheta_4), \quad (7)$$

where  $Q_s$  is the probability of an unoccupied channel being successfully selected which is defined as  ${}^M C_1 / {}^N C_1$ .  $Q_p$  is the probability of an unoccupied channel being utilized which is defined as  $1 / {}^N C_1$ .  ${}^x C_y$  is a selection of  $y$  channels from  $x$  channels.

$$\begin{aligned}
 \vartheta_3 &= z d_c^{2-\alpha_L} l(d_o)^{-1} + 2(\kappa z l(d_o)^{-1})^{2-\alpha_N} I(L_s, \alpha_N), \quad \vartheta_4 = \\
 &= z P_p (P_s l(d_o))^{-1} d_c^{2-\alpha_L} + 2(\kappa z P_p (P_s l(d_o))^{-1})^{2-\alpha_N} I(L_p, \alpha_N), \\
 I(L_s, \alpha_N) &= \int_{L_s}^{\infty} \frac{y}{1+y^{\alpha_N}} dy, \quad L_s = \frac{d_c}{(\kappa z l(d_o)^{-1})^{2/\alpha_N}}, \quad I(L_p, \alpha_N) = \\
 &= \int_{L_p}^{\infty} \frac{y}{1+y^{\alpha_N}} dy \quad \text{and} \quad L_p = \frac{d_c}{(\kappa z P_p (P_s l(d_o))^{-1})^{2/\alpha_N}}.
 \end{aligned}$$

*Proof:* The detail proof is provided in [3].

Replacing the value of  $\mathcal{L}_{I_{ss}}(z(P_s l(d_o))^{-1})$  and  $\mathcal{L}_{I_{sp}}(z(P_s l(d_o))^{-1})$  into (5), the spectral efficiency can be expressed as

$$S = \int_0^{\infty} \frac{1}{1+z} \exp(-\vartheta_1 z - \pi \lambda_{st} Q_s \vartheta_3 - \pi \lambda_{pt} Q_p \vartheta_4) dz. \quad (8)$$

As explained in (8), the spectral efficiency  $S$  is considered in worst case scenario where both interferers (i.e., primary and secondary) are present. Spectral efficiency cannot be derived in closed-form. Substituting the values of  $\alpha_L = 2$  and  $\alpha_N = 4$ , a simple expression is obtained

$$S_0 = \int_0^{\infty} \frac{1}{1+z} \exp(-\vartheta_5 z - \pi \lambda_{st} Q_s \vartheta_6 - \pi \lambda_{pt} Q_p \vartheta_7) dz, \quad (9)$$

where  $\vartheta_5 = \frac{\rho\sigma_s^2 + \sigma_c^2}{\rho P_s l(d_o)}$ ,  $\vartheta_6 = \frac{z}{l(d_o)} + \sqrt{\frac{\kappa z}{l(d_o)}} \tan^{-1}\left(\frac{\kappa z}{d_c^2 l(d_o)}\right)$ ,  $\vartheta_7 = z P_p (P_s l(d_o))^{-1} + \sqrt{\kappa z P_p (P_s l(d_o))^{-1}} \tan^{-1}\left(\frac{\kappa z P_p}{d_c^2 P_s l(d_o)}\right)$  and  $l(d_o) = d_c^{-2} + \kappa(d_o - d_c)^{-4}$ .

**Special Case 1:** If ST-SR pairs are considered to be transmitted over various orthogonal frequencies, the interference ( $I_{ss}$ ) cannot occur. So the mathematical expression of spectral efficiency can be reduced as

$$S_1 = \int_0^{\infty} \frac{1}{1+z} \exp(-\vartheta_1 z - \pi \lambda_{pt} Q_p \vartheta_4) dz. \quad (10)$$

**Special Case 2:** If both pairs (i.e., ST-SR and PT-PR) are considered to be transmitted over various orthogonal frequencies, the interferences ( $I_{ss}$  and  $I_{sp}$ ) cannot occur. So the mathematical expression of spectral efficiency can be written as

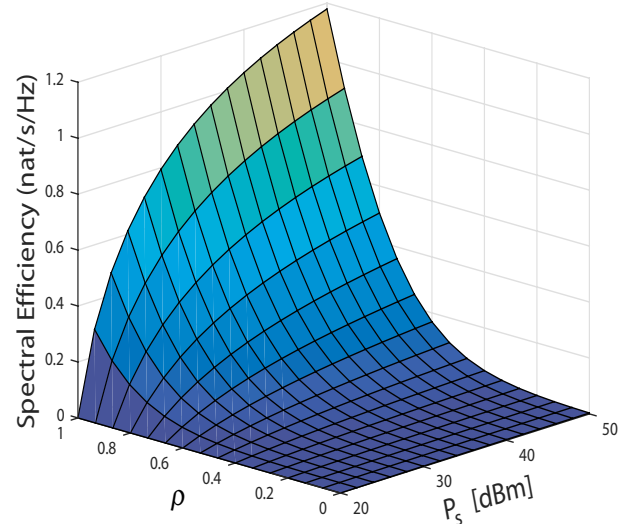
$$\begin{aligned}
 S_2 &= \int_0^{\infty} \frac{1}{1+z} \exp(-\vartheta_5 z) dz \\
 &= \exp(\vartheta_5) \Gamma(0, \vartheta_5), \quad (11)
 \end{aligned}$$

where  $\Gamma(\cdot, \cdot)$  is incomplete gamma function.

*Proof:* The mathematical formulae is provided in [8] for calculation of closed-form expression.

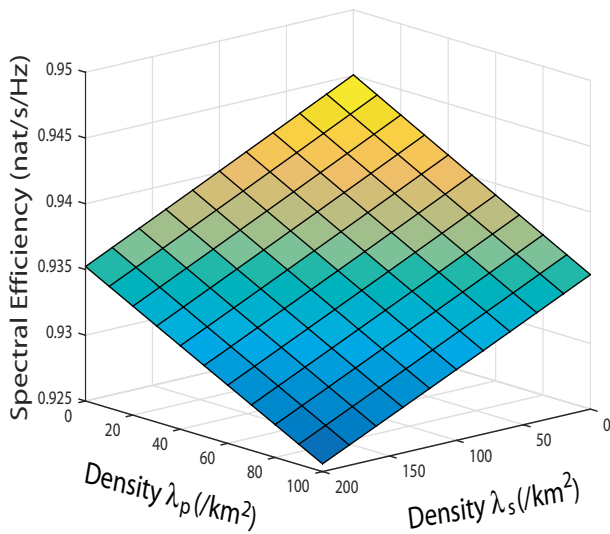
## 4. Numerical Studies

Numerical results are presented in this section to evaluate the spectral efficiency of CR networks. The simulation parameters are set as  $N = 20$ ,  $M = 8$ ,  $d_c = 4$  m,  $d_o = 100$  m,  $\sigma_c^2 = 1$ ,  $\sigma_s^2 = 1$ ,  $\alpha_L = 2$ ,  $\alpha_N = 4$ ,  $P_p = 60$  dBm and  $\rho = 0.5$ .



**Figure 2.** Spectral Efficiency vs.  $P_s$  and  $\rho$ . It is assumed that  $\lambda_s = 25/\text{km}^2$  and  $\lambda_p = 5/\text{km}^2$ .

Figure 2 shows the effects of transmitted power  $P_s$  and PS ratio  $\rho$  on spectral efficiency. Result shows that spectral efficiency is an increasing function with respect



**Figure 3.** Spectral Efficiency vs.  $\lambda_s$  and  $\lambda_p$ . It is assumed that  $P_s = 50$  dBm and  $P_p = 60$  dBm.

to  $P_s$  and  $\rho$ . This figure indicates that spectral efficiency is increased to 10.9% whereas  $\rho$  is varied from 0.5 to 0.6 at  $P_s = 30$  dBm. Spectral efficiency is increased to 20.7% whereas  $P_s$  is increased from 25 dBm to 26 dBm at  $\rho = 0.5$ . From Fig. 1, it is observed that spectral efficiency is notably low when  $\rho < 0.5$ . This is due to the fact that signal power of ID is not adequate.

Figure 3 shows the effects of densities  $\lambda_s$  and  $\lambda_p$  on spectral efficiency. Spectral efficiency increases with decreasing of  $\lambda_s$  and  $\lambda_p$ . This result attributes to the fact that high network density increases the interference and decreases spectral efficiency to 0.02% for increasing of  $\lambda_s = 5/\text{km}^2$  at fixed  $\lambda_p = 5/\text{km}^2$ .

## 5. Conclusion

The paper investigates the spectral efficiency of SR in CR network where the secondary system utilizes EH

and ID node. The numerical results have demonstrated that SR performs better at high  $\rho$  and  $P_s$ . Moreover, the performance of CR is decreased for high densities of  $\lambda_s$  and  $\lambda_p$ .

## References

- [1] N. Shinohara, *Wireless Power Transfer via Radiowaves*, Wiley, 2014.
- [2] X. Lu, P. Wang, D. Niyato, and E. Hossain, "Dynamic spectrum access in cognitive radio networks with RF energy harvesting," *IEEE Wireless Commun.*, vol. 21, no. 3, pp. 102-110, June 2014.
- [3] S. R. Sabuj and M. Hamamura, "Two-slope path-loss design of energy harvesting in random cognitive radio networks," *Computer Networks*, Elsevier, vol. 142, pp. 128-141, 2018.
- [4] S. R. Sabuj and M. Hamamura, "Outage and energy-efficiency analysis of cognitive radio networks: A stochastic approach to transmit antenna selection," *Pervasive and Mobile Computing*, Elsevier, vol. 42, pp. 444-469, 2017.
- [5] S. R. Sabuj and M. Hamamura, "Uplink modeling of cognitive radio network using stochastic geometry," *Performance Evaluation*, Elsevier, vol. 117, pp. 1-15, 2017.
- [6] 3GPP, "Technical specification group radio access network; evolved universal terrestrial radio access (E-UTRA); further advancements for E-UTRA physical layer aspects (Release 9). TR 36.814," 2010.
- [7] X. Zhang and J. G. Andrews, "Downlink cellular network analysis with multi-slope path loss models," *IEEE Trans. Commun.*, vol. 63, no. 5, pp. 1881-1894, Mar 2015.
- [8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Boston: Academic Press, Inc., 1980.