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Effects of Memory Dependent Derivative of Bio-heat Model in Skin Tissue exposed to Laser Radiation

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Abstract

INTRODUCTION: Thermal processes are the essence of living organisms and are necessary for understanding life. The study of Transfer of heat in tissues is known as Bioheat transfer. Many techniques are developed for the thermal treatment of skin and other disease such as skin cancer, skin burns and injured skin tissue with laser. The tissue is inhomogeneous and at times anisotropic with complex thermal properties. Moreover, there may be skin tissue damage when irradiated with a laser beam.

OBJECTIVES: In this research a novel one-dimensional (1-D) bioheat model has been used with memory-dependent derivative (MDD) in Pennes' bioheat transfer equation due to laser radiation and the thermal damage in tissue caused due to laser heating has been examined.

METHODS: Bioheat transfer model has been used with memory-dependent derivative (MDD) in Pennes' bioheat transfer equation. The problem is solved using Laplace transform technique.

RESULTS: The temperature and thermal damage in the skin exposed to heating with laser radiation is calculated and obtained in physical form. The thermal reaction of skin tissues during laser radiation is studied under memory-dependent derivative (MDD) in Pennes' bioheat transfer equation.

CONCLUSION: Analyzed a novel bioheat mathematical model on the basis of MDD involving time-delay parameter χ for the Pennes' bioheat transfer equation and applied to examine the thermal properties of the skin tissue for burns caused due to laser radiations. The thermal damages can be measured in a better way with the MDD model. The blood perfusion prevent the tissue damage by developing the cooling function. Effect of memory dependent derivatives and time delay parameter are represented graphically and analysed.

Keywords: Memory dependent derivative, Bioheat transfer, Time delay, Kernel function.

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Nomenclature

T_b	Blood temperature
ω_b	Rate of blood perfusion
t	Time
I_0	Intensity of the laser
c	Specific heat of the tissue
c_b	Specific heat of the blood
K(t−ξ)	Kernel function
ρ	Tissue mass density
C_1, C_2, k_1, k_2	Functions of diffuse reflectance R_d
k	thermal conductivity of the tissue
U(t)	unit step function
B	factor of frequency
μ_a	coefficient of absorption
Q_{ext}	heat generated per unit volume of tissues

ρ_b	Blood mass density
δ	Penetration depth
χ	Time delay parameter
μ_s	Scattering coefficient
τ_p	exposure time of the laser
g	factor of anisotropy
R	Universal gas constant
Q_m	Metabolic heat generations in living tissues
Ea	Activation energy
θ_0	Incident heat flux intensity

1. Introduction

Thermal processes are the essence of living organisms and are necessary for understanding life. Transfer of heat in tissues is complicated as it is affected by the flow of blood and asymmetrical structure of vascular tissues. The tissue is inhomogeneous and at times anisotropic with complex thermal properties. It also produces heat as part of the active metabolism. Maintenance of a fairly constant body temperature in a range of thermal environments implies that there is a continuous exchange of energy between deep, surface tissues and the environment.

Lipkin & Hardy [1] developed the method for measuring the temperature of human tissues. Giering[2] described the numerous thermal characteristics of biological tissues. Gardner *et al.*[3] proposed the 1-D light transfer case in tissue for fluence rate and escape function using Monte Carlo method. Cheng & Plewes [4] developed a technique to determine the thermal properties precise to the patient and its organ or tumor. Shrivastava and Vaughan [5] proposed a generic bioheat transfer model which include protection of energy, rate of transfer of heat from blood vessels to tissue and applicable to any type of tissue. Othman et al. [6] used the Penne's bio-heat transfer equation for characterization of temperature variation in tissues.

Youn and Lee [7] evaluates the light dispersal and penetration depth in skin tissue using high-intensity light sources. Kengne*et al.*[8] predicts the distribution of temperature in a finite biological tissue with spatial heating and oscillatory surface. Deng & Liu [9] described the theoretical approaches to solve 3-D bioheat transfer problems in presence and absence of phase change. Kengne *et al.*[10]discussed the bio-heat transfer model for thermal traveling-wave dispersal in biological tissues. Kujawska *et al.* [11]explored the ultrasonic method to determine the thermal conductivity of animal tissues using pulsed focused ultrasound. Kujawska*et al.*[11]developed a novel ultrasonic method measure the thermal conductivity of certain animal tissues. Kumar *et al.*[12, 13] described the transfer of heat in a skin tissue of finite domain using metabolic heat. Sarkar [14] defined "*the memory-dependent derivative in an integral form of a common derivative with a Kernel function on a slipping interval and insisted that this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Sarkar[14] pleaded the definition to be more intuitionistic for understanding the physical meaning and the corresponding memory dependent differential equation had more expressive force*". Youssef and Alghamdi[15] proposed a mathematical model one dimensional thermoelastic skin tissue of small thickness using dual-phase-lag heat law. Hobiny *et al.*[16] explored the bioheat model to thermal damage of living tissue caused by laser irradiation using the fractional order derivative. Ezzat et al. [17, 18] discussed the thermal responses of skin tissue using a fractional model of Bioheat equation with sinusoidal heat flux applied on the skin surface. Despite of this several researchers as Mahmoud *et al.* [19], Marin *et al.*[20, 21], Zhang and Fu [22], Abbas & Marin [23], Bhatti *et al.*[24], Marin [25, 26], Lata and Kaur [27–30], Riaz*et al.*[31]worked on different theory of thermoelasticity.

In this research a novel one-dimensional (1-D) bioheat model has been used with memory-dependent derivative (MDD) in Pennes' bioheat transfer equation due to laser radiation and the thermal damage in tissue caused due to laser heating has been examined. Laplace transform technique is used to solve the given problem. The temperature and thermal damage in the skin exposed to heating with laser radiation is calculated and obtained in physical form. Effect of memory dependent derivatives and time delay parameter are represented graphically and analyzed.

2. Basic Equations

For the differentiable function f(t), Wang and Li [32] introduced the first-order MDD with respect to the time delay $\chi > 0$ for a fixed time t:

$$D_\chi f(t) = \frac{1}{\chi}\int_{t-\chi}^{t} K(t-\xi)\, f'(\xi)d\xi. \tag{1}$$

The K(t−ξ) and the time delay parameter χ depends on the material properties. The kernel function K(t − ξ) is differentiable with respect to the variables t and ξ. Following Ezzat *et al.*[33–35] the kernel function K(t − ξ) is taken here in the form

$$K(t-\xi) = 1 - \frac{2b}{\chi}(t-\xi)\frac{a^2}{\chi^2}(t-\xi)^2 = \begin{cases} 1 & a=0, b=0 \\ 1+(\xi-t)/\chi & a=0, b=1/2 \\ \xi - t + 1 & a=0, b=1/\chi \\ [1+(\xi-t)/\chi]^2 & a=1, b=1 \end{cases}, \tag{2}$$

where a and b are constants.

Following Sarkar [14] and Hobiny *et al.*[16]the memory dependent bio heat transfer equation with instant surface heating due to laser irradiation is given by

$$k\nabla^2 T = \left(1+\chi D_\chi\right)\left(\rho C\dot{T} - Q - Q_{ext}\right), \tag{3}$$

Where,

$$\theta(x,t) = T(x,t) - T_0, \quad (4)$$

$$Q = \omega_b \rho_b c_b (T_b - T) + Q_m, \quad (5)$$

$$Q_{ext}(x,t) = I_0 \mu_a [U(t) - U(t - \tau_p)] \left[C_1 e^{-\frac{k_1}{\delta}x} - C_2 e^{-\frac{k_2}{\delta}x}\right]. \quad (6)$$

Following Jacques [36]C_1, C_2, k_1 and k_2 are given by

$$C_1 = 3.09 + 5.44R_d - 2.12\exp(-21.5R_d), \quad (7)$$

$$C_2 = 2.09 + 1.45R_d - 2.09\exp(-21.5R_d), \quad (8)$$

$$k_1 = 1 - 0.423\exp(-20.1R_d), \quad (9)$$

$$k_2 = 1.53\exp(3.4R_d). \quad (10)$$

and penetration depth following[16]is defined as

$$\delta = \frac{1}{\sqrt{3\mu_a(\mu_a + \mu_s(1-g))}}.. \quad (11)$$

Eq. (3) represents the novel bioheat transfer equation with MDD. We get the basic Pennes' bioheat equation when $\chi \to 0$.

3. Method and solution of the problem

The temperature distribution in a semi-infinite biological tissue with instantaneous surface heating and with the laser thermal source of 1-D model of Memory-Dependent Derivative Pennes bioheat transfer equation (MDDPBE) in a finite medium is considered. The 1-D form of Eq. (3) by taking Q_m as constant, is written as:

$$k\frac{\partial^2\theta}{\partial x^2} = (1 + \chi D_\chi)\left(\rho C\frac{\partial\theta}{\partial t} - \omega_b \rho_b c_b (T_b - \theta + T_0) - Q_m - I_0\mu_a[U(t) - U(t-\tau_p)]\left[C_1 e^{-\frac{k_1}{\delta}x} - C_2 e^{-\frac{k_2}{\delta}x}\right]\right). \quad (12)$$

The initial conditions are:

$$\theta(x,0) = \dot{\theta}(x,0) = 0, 0 \le x \le d. \quad (13)$$

The skin tissue is exposed to instantaneous surface heating. We consider that heat flux →0 deep inside the tissue. Therefore, applicable boundary conditions are:

$$\theta(0,t) = \theta_0, \left.\frac{\partial\theta}{\partial x}\right|_{x=d} = 0, 0 \le x \le d, t > 0. \quad (14)$$

For simplify the solution, following non-dimensional quantities are given by

$$T' = \frac{T-T_0}{T_0}, T_b' = \frac{T_b - T_0}{T_0}, t' = \frac{k}{\rho c L^2}t, \xi' = \frac{k}{\rho c L^2}\xi, \chi' = \frac{k}{\rho c L^2}\chi, \tau_0' = \frac{k}{\rho c L^2}\tau_0, \tau_p' = \frac{k}{\rho c L^2}\tau_p, x' = \frac{x}{L}, k_1' = Lk_1, k_2' = Lk_2, R_b = \frac{\rho_b \omega_b c_b L^2}{k}, R_m = \frac{L^2 Q_m}{kT_0}, R_r = \frac{L^2 I_0 \mu_a}{kT_0}. \quad (15)$$

Using these non-dimensional quantities defined in (15), the governing Equation (12) and initial and boundary conditions (13) and (14) can be written as (by ignoring dashes)

$$\frac{\partial^2\theta}{\partial x^2} = (1 + \chi D_\chi)\left(\rho C\frac{\partial\theta}{\partial t} - R_b(T_b - \theta) - R_m - R_r[U(t) - U(t-\tau_p)]\left[C_1 e^{-\frac{k_1}{\delta}x} - C_2 e^{-\frac{k_2}{\delta}x}\right]\right). \quad (16)$$

Laplace transforms is given by

$$\mathcal{L}[f(x,t)] = \int_0^\infty e^{-st} f(x,t)dt = \bar{f}(x,s). \quad (17)$$

with basic properties

$$L\left(\frac{\partial f}{\partial t}\right) = s\bar{f}(x,s) - f(x,0), \quad (18)$$

$$L\left(\frac{\partial^2 f}{\partial t^2}\right) = s^2\bar{f}(x,s) - sf(x,0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}. \quad (19)$$

$$L(\chi D_\chi) = G = \frac{\tau_0}{\chi}\left[(1 - e^{-s\chi})\left(1 - \frac{2b}{\chi s} + \frac{2a^2}{\chi^2 s^2}\right) - \left(a^2 - 2b + \frac{2a^2}{\chi s}\right)e^{-s\chi}\right]$$

$$= \begin{cases} (1 - e^{-s\chi}), a = b = 0, \\ (1 - e^{-s\chi})\left(1 - \frac{1}{s\chi}\right), a = 0, b = \frac{1}{2}, \\ (1 - e^{-s\chi})\left(1 - \frac{1}{s}\right) + \chi e^{-s\chi}, a = 0, b = \frac{\chi}{2}, \\ (1 - e^{-s\chi})\left(\frac{2}{\chi^2 s^2}\right) + \left(1 - \frac{2}{\chi s}\right), a = 1, b = 1. \end{cases} \quad (20)$$

$$L(U(t-a)) = \frac{e^{-as}}{s}, L(U(t)) = \frac{1}{s}. \quad (21)$$

where

$$U(t-a) = \begin{cases} 0, if\ 0 \le t < a \\ 1, if\ t > a \end{cases} \quad (22)$$

Thus we get

$$D^2\bar{\theta} = (1 + G)\left(\rho C s\bar{\theta} - \frac{R_b T_b}{s} + R_b\bar{\theta} - \frac{R_m}{s} - \frac{R_r}{s}[1 - e^{-\tau_p s}]\left[C_1 e^{-\frac{k_1}{\delta}x} - C_2 e^{-\frac{k_2}{\delta}x}\right]\right), \quad (23)$$

where

$$D = \frac{d}{dx}.$$

which can be further simplified as

$$(D^2 - \eta^2)\bar{\theta} = \zeta_1 + \zeta_2 e^{-\frac{k_1}{\delta}x} + \zeta_3 e^{-\frac{k_2}{\delta}x} \quad (24)$$

Where

$$\eta^2 = (1+G)(\rho C s + R_b) \quad (25)$$
$$\zeta_1 = -(1+G)\left(\frac{R_b T_b}{s} + \frac{R_m}{s}\right) \quad (26)$$
$$\zeta_2 = -(1+G)\left(\frac{R_r}{s}[1-e^{-\tau_p s}]C_1\right) \quad (27)$$
$$\zeta_3 = -(1+G)\frac{R_r}{s}[1-e^{-\tau_p s}]C_2 \quad (28)$$

And the boundary conditions after the application of Laplace transform (17) takes the form

$$\bar{\theta}(0,s) = \frac{\theta_0}{s}, \left.\frac{d\theta}{dx}\right|_{x=d} = 0, 0 \le x \le d, Re(s) > 0. \quad (29)$$

By using the boundary conditions defined in Eq. (29) in Eq. (24), the exact solution is obtained as:

$$\bar{\theta}(x,s) = \frac{\theta_0}{s}\frac{\cosh\eta(x-d)}{\cosh\eta d} + \zeta_4\frac{\cosh\eta x}{\cosh\eta d} + \zeta_5\frac{\sinh\eta x}{\eta\cosh\eta d} - \frac{\zeta_1}{\eta^2} + \frac{\zeta_2 e^{-\frac{k_1}{\delta}x}}{k_1^2-\eta^2} + \frac{\zeta_3 e^{-\frac{k_2}{\delta}x}}{k_2^2-\eta^2}, \quad (30)$$

Where

$$\zeta_4 = \frac{\zeta_1}{\eta^2} - \frac{\zeta_2}{k_1^2-\eta^2} - \frac{\zeta_3}{k_2^2-\eta^2}, \quad (31)$$
$$\zeta_5 = \frac{\zeta_1}{\eta^2} - \frac{\zeta_2 e^{-\frac{k_1}{\delta}d}}{k_1^2-\eta^2} - \frac{\zeta_3 e^{-\frac{k_2}{\delta}d}}{k_2^2-\eta^2},$$

The thermal damage i.e. evaluation of burn caused by laser radiation, following Jasiński[37], Askarizadeh & Ahmadikia[38] is given by,

$$\Omega = \int_0^t B e^{-\frac{E_a}{RT}} dt, \quad (32)$$

4. Results and discussion

Following Askarizadeh & Ahmadikia[38] for numerical results, the specific values of different parameters are

$\rho_b = 1060 kgm^{-3}$,
$c_b = 3860 Jkg^{-1}K^{-1}$,
$\omega_b = 1.87 \times 10^{-3} s^{-1}$,
$T_b = 37^o C$,
$Q_m = 1.19 \times 10^3 Wm^{-3}$,
$g = 0.9$,
$\mu_s = 12000 m^{-1}$,
$R = 8.313\, J/mol \cdot K$
$Ea = 6.28 \times 105\, J/mol$
$c = 4187 Jkg^{-1}K^{-1}$,
$\rho = 1000 kgm^{-3}$,
$k = 0.628 Wm^{-1}K^{-1}$,
$\tau_p = 10s$,
$L = 0.03m$,
$\mu_a = 40m^{-1}$,
$T_0 = 37^o C$,
$B = 3.1 \times 1098\, s^{-1}$.

The results are simulated using MATLAB software and illustrated graphically. The impact of laser source on the skin surface was incorporated. The proposed mathematical models depends on the bio-heat transfer found and suitable boundary conditions. The conducting heat source, metabolic and perfusions are used in the formulations of the mathematical model. Numerical results are presented graphically in Figures 1–4 to study the influence of memory dependent derivative, kernel function K(t−ξ) , the laser exposure time τ_p , the thermal relaxation time τ_0 and the time delay parameter χ on the temperature and the thermal damages. The skin tissue is considered as .03 m thick and the reference temperature is taken equal to skin normal temperature, that is, $T_0 = T_b = 37^0 C$.

Figure 1 exhibits the deviation of temperature along the distance x by keeping the values of $\tau_0 = 5s$ and $\tau_p = 10s$ with different values of memory dependent derivative, kernel function K(t−ξ). It is seen that the temperature start rising with the distance and decrease as the blood perfusions in skin increases. Figure 2 illustrates the thermal damage w.r.t. time t keeping the values of $\tau_0 = 5s$ and $\tau_p = 10s$ with different values of memory dependent derivative, c The kernel function K(t−ξ) when a=0, b=0 has the highest effect on the thermal damages.

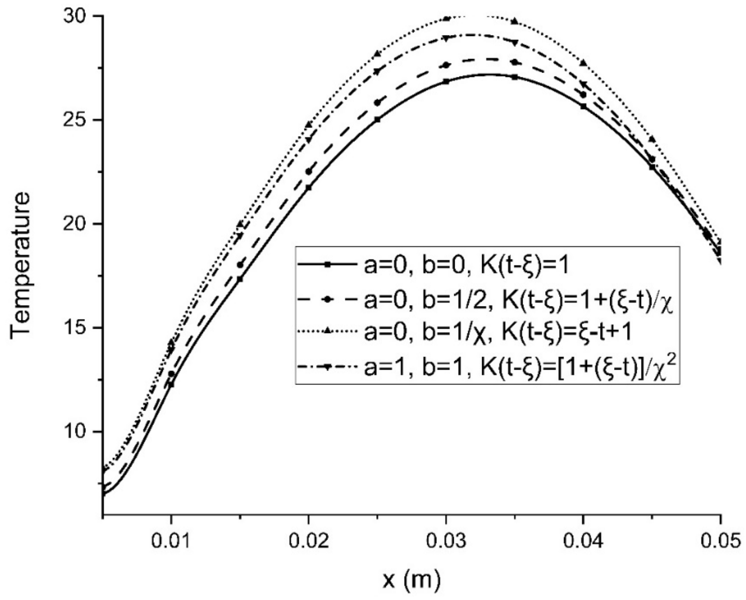

Figure 1. Temperature variations w.r.t. skin depth for different values of $K(t-\xi)$.

Figure 3 exhibits the variation of temperature along the distance x by keeping the values of $\tau_0 = 5s \; and \; \tau_p = 10s$ with different values time delay parameter χ for kernel function K(t−ξ) =$[1+(\xi-t)/\chi]^2$ and a=1, b=1. It is seen that the temperature start from the utmost value and decrease rapidly.

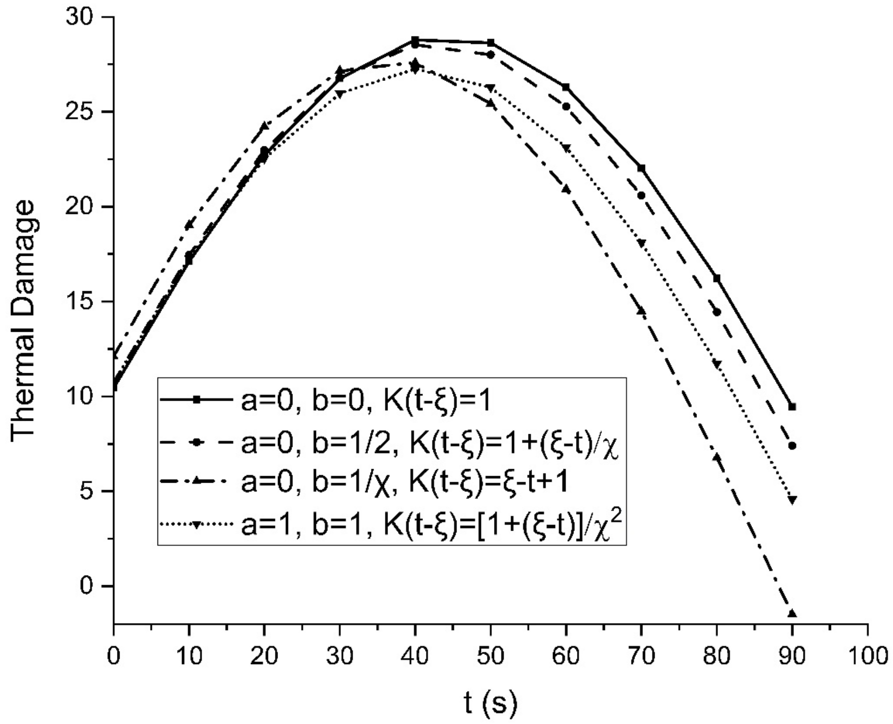

Figure 2. The variation of thermal damage w.r.t. different values of $K(t-\xi)$.

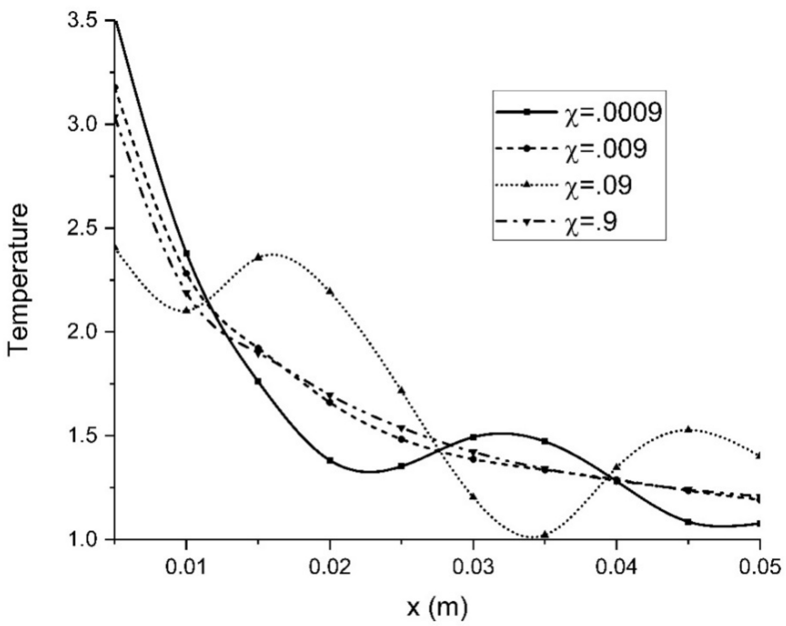

Figure 3. Temperature distributions w.r.t. skin depth for different values of time-delay χ at $K(t-\xi)=[1+(\xi-t)/\chi]^2$

Figure 4 demonstrates the thermal damage w.r.t. time t keeping the values of $\tau_0 = 5s\ and\ \tau_p = 10s$ with different values time delay parameter χ for kernel function K(t−ξ) =$[1+(\xi-t)/\chi]^2$ and a=1, b=1. It is seen that the thermal damage start from the utmost value and decrease rapidly. It is observed that $\chi = .009$ has the highest impact on the thermal damage.

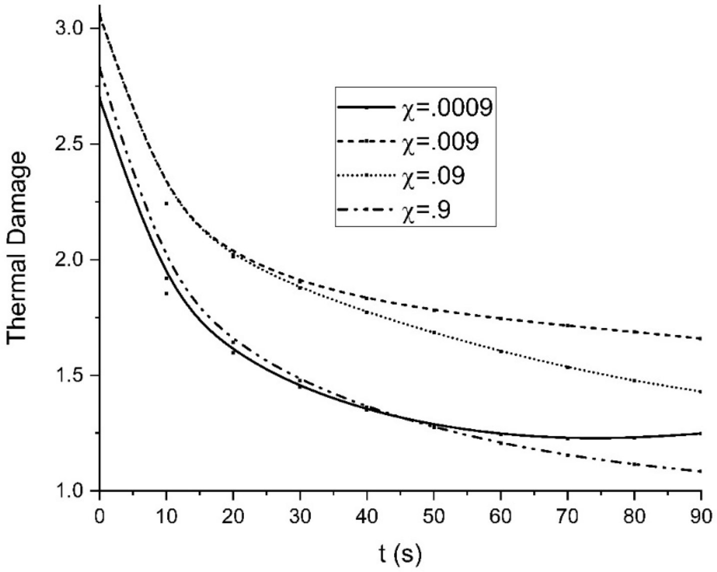

Figure 4. The variation of thermal damage at skin surface with different values of time-delay χ and $K(t-\xi)=[1+(\xi-t)/\chi]^2$

4. Results and discussion

The main objective of this research work is to analyze a novel bioheat mathematical model on the basis of MDD involving time-delay parameter χ for the Pennes' bioheat transfer equation and applied to examine the thermal properties of the skin tissue for burns caused due to laser radiations. The thermal damages can be measured in a better way with the MDD model. The blood perfusion prevent the tissue damage by developing the cooling function. In this research, the memory-dependent derivative involving time-delay parameter χ becomes a new measure of efficiency for bioheat transfer in the skin tissues. These results may be beneficial in the study and further improvements in the applications of thermotherapy in skin tissues.

References

[1] Lipkin, M., & Hardy, J. D. (1954). Measurement of Some Thermal Properties of Human Tissues. *Journal of Applied Physiology*, *7*(2), 212–217. https://doi.org/10.1152/jappl.1954.7.2.212

[2] Giering;, K., Minet;, O., Lamprecht;, I., & G. Muller. (1995). Review of thermal properties of biological tissues. *SPIE Opt. Eng. Press*, *044*(August 2018), 45–65. https://doi.org/10.1007/978-3-319-95432-5_4

[3] Gardner, C. M., Jacques, S. L., & Welch, A. J. (1996). Light transport in tissue: Accurate expressions for one-dimensional fluence rate and escape function based upon Monte Carlo simulation. *Lasers in Surgery and Medicine*, *18*(2), 129–138. https://doi.org/10.1002/(SICI)1096-9101(1996)18:2<129::AID-LSM2>3.0.CO;2-U

[4] Cheng, H.-L. M., & Plewes, D. B. (2002). Tissue thermal conductivity by magnetic resonance thermometry and focused ultrasound heating. *Journal of Magnetic Resonance Imaging*, *16*(5), 598–609. https://doi.org/10.1002/jmri.10199

[5] Shrivastava, D., &Vaughan, J. T. (2009). A generic bioheat transfer thermal model for a perfused tissue. *Journal of Biomechanical Engineering*, *131*(7), 1–5. https://doi.org/10.1115/1.3127260

[6] Othman, M. I. A., Ali, M. G. S., & Farouk, R. M. (2011). The Effect of Relaxation Time on the Heat Transfer and Temperature Distribution in Tissues. *World Journal of Mechanics*, *01*(06), 283–287. https://doi.org/10.4236/wjm.2011.16035

[7] Youn, J.-I., & Lee, S. (2012). Diffuse reflectance simulation in tissue for high-intensity laser therapy. *International Conference on Biological, Biomedical and Pharmaceutical Sciences (ICCEPS'2012)*, *1*(1), 90–93.

[8] Kengne, E., Lakhssassi, A., & Vaillancourt, R. (2012). Temperature Distribution in Living Biological Tissue Simultaneously Subjected to Oscillatory Surface and Spatial Heating: Analytical and Numerical Analysis. *International Mathematical Forum*, *7*(48), 2373–2392. https://doi.org/10.4438/1988-599X-RE-2017-381-387

[9] Deng, Z.-S., & Liu, J. (2012). Analytical Solutions to 3-D Bioheat Transfer Problems with or without Phase Change. In *Heat Transfer Phenomena and Applications*. InTech. https://doi.org/10.5772/52963

[10] Kengne, E., Hamouda, F. Ben, & Lakhssassi, A. (2013). Extended Generalized Riccati Equation Mapping for Thermal Traveling-Wave Distribution in Biological Tissues through a Bio-Heat Transfer Model with Linear/Quadratic Temperature-Dependent Blood Perfusion. *Applied Mathematics*, *04*(10), 1471–1484. https://doi.org/10.4236/am.2013.410199

[11] Kujawska, T., Secomski, W., Kruglenko, E., Krawczyk, K., & Nowicki, A. (2014). Determination of Tissue Thermal Conductivity by Measuring and Modeling Temperature Rise Induced in Tissue by Pulsed Focused Ultrasound. *PLoS ONE*, *9*(4), e94929. https://doi.org/10.1371/journal.pone.0094929

[12] Kumar, R., Vashisth, A. K., & Ghangas, S. (2017). Analytical solution of bioheat transfer equation with variable thermal conductivity in skin. *International Journal of Engineering Research And Development*, *14*(1), 11–17.

[13] Kumar, R., Vashishth, A. K., & Ghangas, S. (2019). Phase-Lag Effects in Skin Tissue During Transient Heating. *International Journal of Applied Mechanics and Engineering*, *24*(3), 603–623. https://doi.org/10.2478/ijame-2019-0038

[14] Sarkar, N. (2016). A novel Pennes' bioheat transfer equation with memory-dependent derivative. *Mathematical Models in Engineering*, *2*(2), 151–157. https://doi.org/10.21595/mme.2016.18024

[15] Youssef, H. M., & Alghamdi, N. A. (2020). Modeling of One-Dimensional Thermoelastic Dual-Phase-Lag Skin Tissue Subjected to Different Types of Thermal Loading. *Scientific Reports*, *10*(1), 3399. https://doi.org/10.1038/s41598-020-60342-6

[16] Hobiny, A., Alzahrani, F., Abbas, I., & Marin, M. (2020). The Effect of Fractional Time Derivative of Bioheat Model in Skin Tissue Induced to Laser Irradiation. *Symmetry*, *12*(4), 602. https://doi.org/10.3390/sym12040602

[17] Ezzat, M. A., Alsowayan, N. S., Al-Muhiameed, Z. I. A., & Ezzat, S. M. (2014). Fractional modelling of Pennes' bioheat transfer equation. *Heat and Mass Transfer/Waerme- und Stoffuebertragung*, *50*(7), 907–914. https://doi.org/10.1007/s00231-014-1300-x

[18] Ezzat, M. A., El-bary, A. A., & Al-sowayan, N. S. (2016). Tissue responses to fractional transient heating with sinusoidal heat flux condition on skin surface. *Animal Science Journal*, *87*(10), 1304–1311. https://doi.org/10.1111/asj.12568

[19] Mahmoud, S. R., Marin, M., & Al-Basyouni, K. S. (2015). Effect of the initial stress and rotation on free vibrations in transversely isotropic human long dry bone. *Analele Universitatii "Ovidius" Constanta - Seria Matematica*, *23*(1), 171–184. https://doi.org/10.1515/auom-2015-0011

[20] Marin, M., Vlase, S., Ellahi, R., & Bhatti, M. M. (2019). On the Partition of Energies for the Backward in Time Problem of Thermoelastic Materials with a Dipolar Structure. *Symmetry*, *11*(7), 863. https://doi.org/10.3390/sym11070863

[21] Marin Marin, Eduard-Marius Craciun, Nicolae Pop. (2016). Considerations on mixed initial-boundary value problems for micropolar porous bodies. *Dynamic Systems and Applications*, *25*, 175–195.

[22] Zhang, J., & Fu, Y. (2012). Pull-in analysis of electrically actuated viscoelastic microbeams based on a modified couple stress theory. *Meccanica*, *47*(7), 1649–1658. https://doi.org/10.1007/s11012-012-9545-2

[23] Abbas, I. A., & Marin, M. (2017). Analytical solution of thermoelastic interaction in a half-space by pulsed laser heating. *Physica E: Low-dimensional Systems and Nanostructures*, *87*, 254–260. https://doi.org/10.1016/j.physe.2016.10.048
[24] Bhatti, M. M., Ellahi, R., Zeeshan, A., Marin, M., & Ijaz, N. (2019). Numerical study of heat transfer and Hall current impact on peristaltic propulsion of particle-fluid suspension with compliant wall properties. *Modern Physics Letters B*, *33*(35), 1950439. https://doi.org/10.1142/S0217984919504396
[25] Marin, M. (2010). Lagrange identity method for microstretch thermoelastic materials. *Journal of Mathematical Analysis and Applications*, *363*(1), 275–286. https://doi.org/10.1016/j.jmaa.2009.08.045
[26] Marin, M. (1994). The Lagrange identity method in thermoelasticity of bodies with microstructure. *International Journal of Engineering Science*, *32*(8), 1229–1240. https://doi.org/10.1016/0020-7225(94)90034-5
[27] Kaur, I., & Lata, P. (2020). Stoneley wave propagation in transversely isotropic thermoelastic medium with two temperature and rotation. *GEM - International Journal on Geomathematics*, *11*(1), 1–17. https://doi.org/10.1007/s13137-020-0140-8
[28] Kaur, I., Lata, P., & Singh, K. (2020). Effect of Hall current in transversely isotropic magneto-thermoelastic rotating medium with fractional-order generalized heat transfer due to ramp-type heat. *Indian Journal of Physics*. https://doi.org/10.1007/s12648-020-01718-2
[29] Kaur, I., & Lata, P. (2019). Effect of hall current on propagation of plane wave in transversely isotropic thermoelastic medium with two temperature and fractional order heat transfer. *SN Applied Sciences*, *1*(8). https://doi.org/10.1007/s42452-019-0942-1
[30] Kaur, I., & Lata, P. (2019). Transversely isotropic thermoelastic thin circular plate with constant and periodically varying load and heat source. *International Journal of Mechanical and Materials Engineering*, *14*(1). https://doi.org/10.1186/s40712-019-0107-4
[31] Riaz, A., Ellahi, R., Bhatti, M. M., & Marin, M. (2019). Study Of Heat And Mass Transfer In The Eyring–Powell Model Of Fluid Propagating Peristaltically Through A Rectangular Compliant Channel. *Heat Transfer Research*, *50*(16), 1539–1560. https://doi.org/10.1615/HeatTransRes.2019025622
[32] Wang, J.-L., & Li, H.-F. (2011). Surpassing the fractional derivative: Concept of the memory-dependent derivative. *Computers & Mathematics with Applications*, *62*(3), 1562–1567. https://doi.org/10.1016/j.camwa.2011.04.028
[33] Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2014). Generalized thermo-viscoelasticity with memory-dependent derivatives. *International Journal of Mechanical Sciences*, *89*, 470–475. https://doi.org/10.1016/j.ijmecsci.2014.10.006
[34] Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2015). A novel magneto-thermoelasticity theory with memory-dependent derivative. *Journal of Electromagnetic Waves and Applications*, *29*(8), 1018–1031. https://doi.org/10.1080/09205071.2015.1027795
[35] Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2016). Modeling of memory-dependent derivative in generalized thermoelasticity. *European Physical Journal Plus*, *131*(10). https://doi.org/10.1140/epjp/i2016-16372-3
[36] Jacques, S. L. (1998). Light Distributions from Point, Line and Plane Sources for Photochemical Reactions and Fluorescence in Turbid Biological Tissues. *Photochemistry and Photobiology*, *67*(1), 23–32. https://doi.org/10.1111/j.1751-1097.1998.tb05161.x
[37] Jasiński, M. (2015). Modelling of thermal damage in laser irradiated tissue. *Journal of Applied Mathematics and Computational Mechanics*, *14*(4), 67–78. https://doi.org/10.17512/jamcm.2015.4.07
[38] Askarizadeh, H., & Ahmadikia, H. (2016). Skin Burn Evaluation under the Effects of the Dual-Phase-Lag Bio-heat Transfer Skin Burn Evaluation under the Effects of the Dual-Phase-Lag Bio-heat Transfer. In *24th Annual International Conference on Mechanical Engineering* (pp. 1–6). Yazd, Iran: azd University, Yazd, Iran.