

# Effects of Memory Dependent Derivative of Bio-heat Model in Skin Tissue exposed to Laser Radiation

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## Abstract

**INTRODUCTION:** Thermal processes are the essence of living organisms and are necessary for understanding life. The study of Transfer of heat in tissues is known as Bioheat transfer. Many techniques are developed for the thermal treatment of skin and other disease such as skin cancer, skin burns and injured skin tissue with laser. The tissue is inhomogeneous and at times anisotropic with complex thermal properties. Moreover, there may be skin tissue damage when irradiated with a laser beam.

**OBJECTIVES:** In this research a novel one-dimensional (1-D) bioheat model has been used with memory-dependent derivative (MDD) in Pennes' bioheat transfer equation due to laser radiation and the thermal damage in tissue caused due to laser heating has been examined.

**METHODS:** Bioheat transfer model has been used with memory-dependent derivative (MDD) in Pennes' bioheat transfer equation. The problem is solved using Laplace transform technique.

**RESULTS:** The temperature and thermal damage in the skin exposed to heating with laser radiation is calculated and obtained in physical form. The thermal reaction of skin tissues during laser radiation is studied under memory-dependent derivative (MDD) in Pennes' bioheat transfer equation.

**CONCLUSION:** Analyzed a novel bioheat mathematical model on the basis of MDD involving time-delay parameter  $\chi$  for the Pennes' bioheat transfer equation and applied to examine the thermal properties of the skin tissue for burns caused due to laser radiations. The thermal damages can be measured in a better way with the MDD model. The blood perfusion prevent the tissue damage by developing the cooling function. Effect of memory dependent derivatives and time delay parameter are represented graphically and analysed.

**Keywords:** Memory dependent derivative, Bioheat transfer, Time delay, Kernel function.

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## Nomenclature

$T_b$	Blood temperature
$\omega_b$	Rate of blood perfusion
$t$	Time
$I_0$	Intensity of the laser
$c$	Specific heat of the tissue
$c_b$	Specific heat of the blood

$K(t-\xi)$	Kernel function
$\rho$	Tissue mass density
$C_1, C_2,$ $k_1, k_2$	Functions of diffuse reflectance $R_d$
$k$	thermal conductivity of the tissue
$U(t)$	unit step function
$B$	factor of frequency
$\mu_a$	coefficient of absorption
$Q_{ext}$	heat generated per unit volume of tissues





$$\eta^2 = (1 + G)(\rho C_s + R_b) \quad (25)$$

$$\zeta_1 = -(1 + G) \left( \frac{R_b T_b}{s} + \frac{R_m}{s} \right) \quad (26)$$

$$\zeta_2 = -(1 + G) \left( \frac{R_r}{s} [1 - e^{-\tau_p s}] C_1 \right) \quad (27)$$

$$\zeta_3 = -(1 + G) \frac{R_r}{s} [1 - e^{-\tau_p s}] C_2 \quad (28)$$

And the boundary conditions after the application of Laplace transform (17) takes the form

$$\bar{\theta}(0, s) = \frac{\theta_0}{s}, \left. \frac{d\theta}{dx} \right|_{x=d} = 0, 0 \leq x \leq d, \text{Re}(s) > 0. \quad (29)$$

By using the boundary conditions defined in Eq. (29) in Eq. (24), the exact solution is obtained as:

$$\bar{\theta}(x, s) = \frac{\theta_0}{s} \frac{\cosh \eta(x-d)}{\cosh \eta d} + \zeta_4 \frac{\cosh \eta x}{\cosh \eta d} + \quad (30)$$

$$\zeta_5 \frac{\sinh \eta x}{\eta \cosh \eta d} - \frac{\zeta_1}{\eta^2} + \frac{\zeta_2 e^{-\frac{k_1}{\delta} x}}{k_1^2 - \eta^2} + \frac{\zeta_3 e^{-\frac{k_2}{\delta} x}}{k_2^2 - \eta^2},$$

Where

$$\zeta_4 = \frac{\zeta_1}{\eta^2} - \frac{\zeta_2}{k_1^2 - \eta^2} - \frac{\zeta_3}{k_2^2 - \eta^2}, \quad (31)$$

$$\zeta_5 = \frac{\zeta_1}{\eta^2} - \frac{\zeta_2 e^{-\frac{k_1}{\delta} d}}{k_1^2 - \eta^2} - \frac{\zeta_3 e^{-\frac{k_2}{\delta} d}}{k_2^2 - \eta^2},$$

The thermal damage i.e. evaluation of burn caused by laser radiation, following Jasiński[37], Askarizadeh & Ahmadikia[38] is given by,

$$\Omega = \int_0^t B e^{-\frac{E_a}{RT}} dt, \quad (32)$$

## 4. Results and discussion

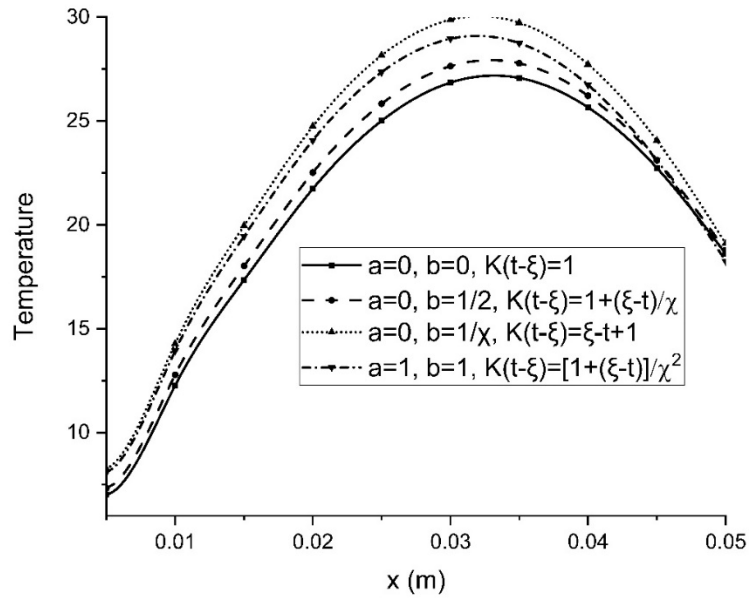
Following Askarizadeh & Ahmadikia[38] for numerical results, the specific values of different parameters are

$$\begin{aligned} \rho_b &= 1060 \text{kgm}^{-3}, \\ c_b &= 3860 \text{Jkg}^{-1} \text{K}^{-1}, \\ \omega_b &= 1.87 \times 10^{-3} \text{s}^{-1}, \\ T_b &= 37^\circ \text{C}, \\ Q_m &= 1.19 \times 10^3 \text{Wm}^{-3}, \\ g &= 0.9, \end{aligned}$$

$$\begin{aligned} \mu_s &= 12000 \text{m}^{-1}, \\ R &= 8.313 \text{J/mol} \cdot \text{K} \\ Ea &= 6.28 \times 10^5 \text{J/mol} \\ c &= 4187 \text{Jkg}^{-1} \text{K}^{-1}, \\ \rho &= 1000 \text{kgm}^{-3}, \\ k &= 0.628 \text{Wm}^{-1} \text{K}^{-1}, \\ \tau_p &= 10 \text{s}, \\ L &= 0.03 \text{m}, \\ \mu_a &= 40 \text{m}^{-1}, \\ T_0 &= 37^\circ \text{C}, \\ B &= 3.1 \times 10^9 \text{s}^{-1}. \end{aligned}$$

The results are simulated using MATLAB software and illustrated graphically. The impact of laser source on the skin surface was incorporated. The proposed mathematical models depends on the bio-heat transfer found and suitable boundary conditions. The conducting heat source, metabolic and perfusions are used in the formulations of the mathematical model. Numerical results are presented graphically in Figures 1–4 to study the influence of memory dependent derivative, kernel function  $K(t-\xi)$ , the laser exposure time  $\tau_p$ , the thermal relaxation time  $\tau_0$  and the time delay parameter  $\chi$  on the temperature and the thermal damages. The skin tissue is considered as .03 m thick and the reference temperature is taken equal to skin normal temperature, that is,  $T_0 = T_b = 37^\circ \text{C}$ .

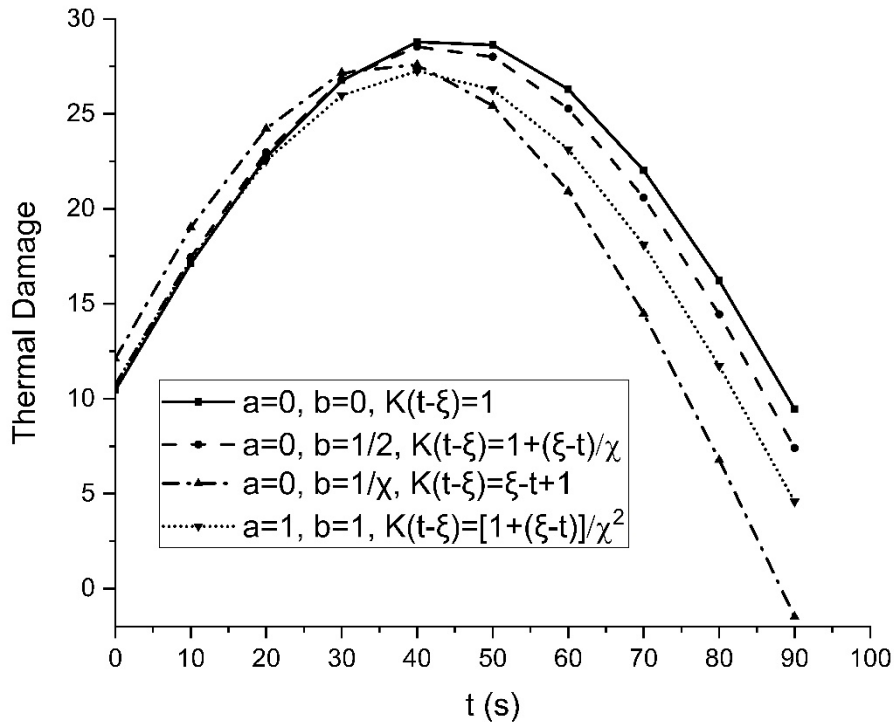
Figure 1 exhibits the deviation of temperature along the distance  $x$  by keeping the values of  $\tau_0 = 5 \text{s}$  and  $\tau_p = 10 \text{s}$  with different values of memory dependent derivative, kernel function  $K(t-\xi)$ . It is seen that the temperature start rising with the distance and decrease as the blood perfusions in skin increases. Figure 2 illustrates the thermal damage w.r.t. time  $t$  keeping the values of  $\tau_0 = 5 \text{s}$  and  $\tau_p = 10 \text{s}$  with different values of memory dependent derivative, c The kernel function  $K(t-\xi)$  when  $a=0, b=0$  has the highest effect on the thermal damages.



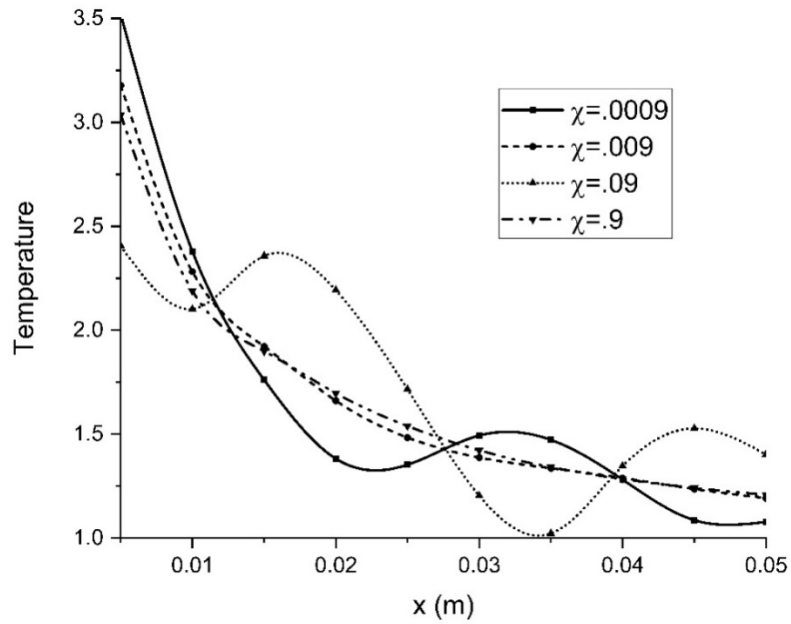
**Figure 1.** Temperature variations w.r.t. skin depth for different values of  $K(t - \xi)$ .

Figure 3 exhibits the variation of temperature along the distance  $x$  by keeping the values of  $\tau_0 = 5s$  and  $\tau_p = 10s$  with different values time delay parameter  $\chi$  for

kernel function  $K(t-\xi) = [1 + (\xi - t)/\chi]^2$  and  $a=1, b=1$ . It is seen that the temperature start from the utmost value and decrease rapidly.



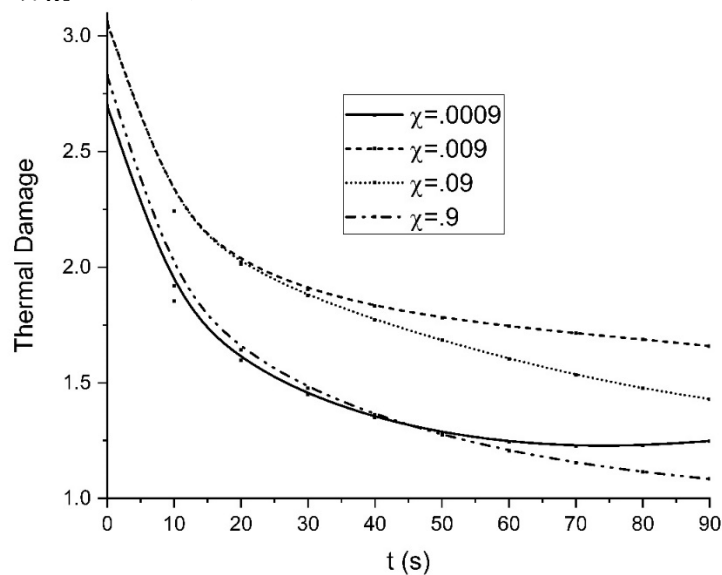
**Figure 2.** The variation of thermal damage w.r.t. different values of  $K(t - \xi)$ .



**Figure 3.** Temperature distributions w.r.t. skin depth for different values of time-delay  $\chi$  at  $K(t - \xi) = [1 + (\xi - t)/\chi]^2$

Figure 4 demonstrates the thermal damage w.r.t. time  $t$  keeping the values of  $\tau_0 = 5s$  and  $\tau_p = 10s$  with different values time delay parameter  $\chi$  for kernel function  $K(t-\xi) = [1 + (\xi - t)/\chi]^2$  and  $a=1, b=1$ . It is

seen that the thermal damage start from the utmost value and decrease rapidly. It is observed that  $\chi = .009$  has the highest impact on the thermal damage.



**Figure 4.** The variation of thermal damage at skin surface with different values of time-delay  $\chi$  and  $K(t - \xi) = [1 + (\xi - t)/\chi]^2$

## 4. Results and discussion

The main objective of this research work is to analyze a novel bioheat mathematical model on the basis of MDD involving time-delay parameter  $\chi$  for the Pennes' bioheat transfer equation and applied to examine the thermal properties of the skin tissue for burns caused due to laser radiations. The thermal damages can be measured in a better way with the MDD model. The blood perfusion prevent the tissue damage by developing the cooling function. In this research, the memory-dependent derivative involving time-delay parameter  $\chi$  becomes a new measure of efficiency for bioheat transfer in the skin tissues. These results may be beneficial in the study and further improvements in the applications of thermotherapy in skin tissues.

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