Table 1. Calculation of currents in equivalent diagram in Figure 1

Resistance of the balanced line phase	Resistance of phase A loading	Resistance of phase B loading	Resistance of phase C loading	
$r_w = 0.1$ $L_w = 0.001$ $z_w = r_w + \omega \cdot L_w \cdot j$ $z_w = 0.1 + 0.314j$	$r_{na} = 0.5$ $L_{na} = 0.025$ $z_{na} = r_{na} + \omega \cdot L_{na} \cdot j$ $z_{na} = 1.5 + 7.85j$ $y_{na} = 1/z_{na}$ $z_a = z_{na} + z_w$ $y_a = 1/z_a$	$r_{nb} = 1$ $L_{nb} = 0.01$ $z_{nb} = r_{nb} + \omega \cdot L_{nb} \cdot j$ $z_{nb} = 1 + 3.14j$ $y_{nb} = 1/z_{nb}$ $z_b = z_{nb} + z_w$ $y_b = 1/z_b$	$r_{nc} = 0.5$; $L_{nc} = 0.025$ $z_{nc} = r_{nc} + \omega \cdot L_{nc} \cdot j$ $z_{nc} = 1.5 + 7.85j$ $y_{nc} = 1/z_{nc}$ $z_c = z_{nc} + z_w$ $y_c = 1/z_c$	$\omega = 314$ c^{-1} $j = \sqrt{-1}$ $a = e^{2\cdot(\pi/3)\cdot j}$
Source of power	$E = 100$	$E_A = E$	$E_B = E_A \cdot a^2$	$E_C = E_A \cdot a$

Determining current of branches in the diagram

Finding potential of load neutral shift $U_{Nn} = \dfrac{E_A \cdot y_a + E_B \cdot y_b + E_C \cdot y_c}{y_a + y_b + y_c} = -7.859 - 38.92j$	$\lvert U_{Nn} \rvert = 39.705$	$\arg(U_{Nn}) = -101.416°$
Determining current of the branches $I_{na} = y_a \cdot (E_A - U_{Nn}) = 7.084 - 11.823j$ $I_{nb} = y_b \cdot (E_B - U_{Nn}) = -16.062 + 7.085j$ $I_{nc} = y_c \cdot (E_C - U_{Nn}) = 8.977 + 4.738j$	$\lvert I_{na} \rvert = 13.783$ $\lvert I_{nb} \rvert = 17.555$ $\lvert I_{nc} \rvert = 10.151$	$\arg(I_{na}) = -59.07°$ $\arg(I_{nb}) = 156.196°$ $\arg(I_{nc}) = 27.823°$
Calculating balanced components of the current of branches $I_{n1} = 1/3 \cdot (I_{na} + aI_{nb} + a^2 I_{nc}) = 2.864 - 13.14j$ $I_{n2} = 1/3 \cdot (I_{na} + a^2 I_{nb} + aI_{nc}) = 4.22 + 1.317j$ $I_{n0} = 1/3 \cdot (I_{na} + I_{nb} + I_{nc}) = 0$	$\lvert I_{n1} \rvert = 13.448$ $\lvert I_{n2} \rvert = 4.421$	$\arg(I_{n1}) = -77.702°$ $\arg(I_{n2}) = 17.327°$
Calculation of the value of unbalance coefficient, % by reverse sequence	$K_{nc} = \dfrac{\lvert I_{n2} \rvert}{\lvert I_{n1} \rvert} \cdot 100 = 32.871$	

3. Formation of requirements for the parameters of the balancing device

In the diagram in Figure 1 the balancing device (BD) is connected to load clamps *A*, *B*, *C* according to Figure 2.

Requirements for the balancing device should be formulated as follows.

(i) If the purpose is to ensure only the balance of the currents of the power sources, then the BD must consume unbalanced currents that compensate for the component of the negative sequence of load currents. Then the currents of lines $\underline{I}_{wa}, \underline{I}_{wb}, \underline{I}_{wc}$ will contain only the balanced component of positive sequence $\underline{I}_{n1}$ (Table 1) from the currents of the loads considering the phase turn, meaning:

$$\underline{I}_{wa} = \underline{I}_{n1} = 13{,}448\, e^{j(-77{,}702°)}\,\text{A} \tag{1}$$

$$\underline{I}_{wb} = \underline{I}_{n1} \cdot a^2 = 13{,}448\, e^{j(162{,}298°)}\,\text{A} \tag{2}$$

$$\underline{I}_{wc} = \underline{I}_{n1} \cdot a = 13{,}448\, e^{j(42{,}298°)}\,\text{A} \tag{3}$$

By the known phase currents of the lines $\underline{I}_{wa}, \underline{I}_{wb}, \underline{I}_{wc}$ and phase currents of the load $\underline{I}_{na}, \underline{I}_{nb}, \underline{I}_{nc}$, we calculate the phase currents $\underline{I}_{ka}, \underline{I}_{kb}, \underline{I}_{kc}$ of balancing device according the Kirchhoff first law:

$$\underline{I}_{ka} = \underline{I}_{wa} - \underline{I}_{na} \tag{4}$$

$$\underline{I}_{kb} = \underline{I}_{wb} - \underline{I}_{nb} \tag{5}$$

$$\underline{I}_{kc} = \underline{I}_{wc} - \underline{I}_{nc} \tag{6}$$

(ii) If reactive power compensation is to be performed alongside with the balancing, the BD must compensate for both the reverse component of the load current and the reactive component of the positive sequence of load currents. Under this condition, the line current will contain only the active component of the positive sequence of load currents (Table 1):

$$\underline{I}_{wa} = \text{Re}(\underline{I}_{n1}) = 2.864\, e^{j0°}\,\text{A} \tag{7}$$

$$\underline{I}_{wb} = \mathrm{Re}(\underline{I}_{n1}) \cdot a^2 = 2.864\, e^{\,j240°}\ \mathrm{A} \qquad (8)$$

$$\underline{I}_{wc} = \mathrm{Re}(\underline{I}_{n1}) \cdot a = 2.864\, e^{\,j120°}\ \mathrm{A} \qquad (9)$$

The balancing device currents are calculated by the formulae (4) – (6).

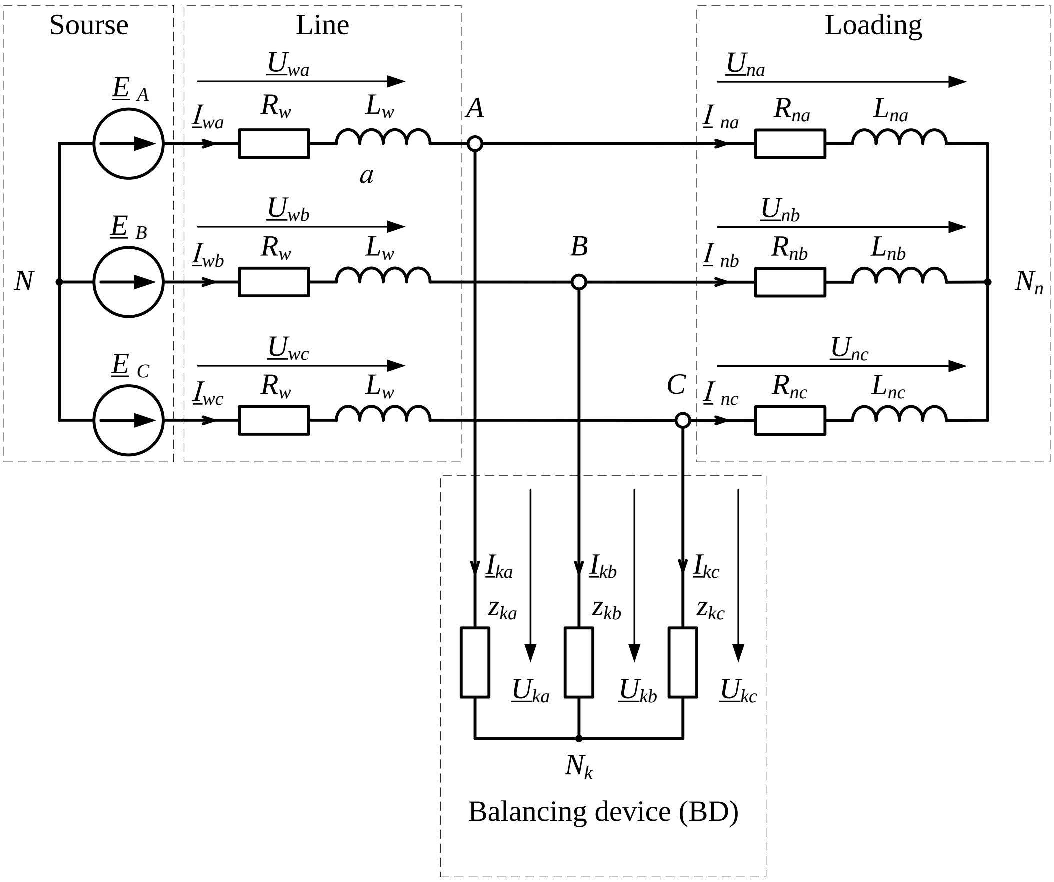

Figure 2. Equivalent diagram of three-phase system of power supply with abalanced load and connected balancing device [17]

4. Determination the scheme of the balancing device and the parameters of its elements

Concerning the scheme of the balancing device: we will assume that the supports are connected like a "star". If it is necessary to use the "triangle" scheme, the known resistance of the "star" can be easily calculated in the usual way.

The decisions about the type of elements are made on the basis of the following considerations: since it is impractical to increase the consumed active power, the elements of the BD must be reactive. Since there is a need to compensate for the inductive nature of the loads, the nature of the reactivity must be capacitive.

When deciding on a balancing with full reactive power compensation, the load currents should be specified for further determination of BD currents.

Let us assume that in the balancing mode with full reactive power compensation in the line phases, a balanced active current flows I_{1w}, whose active value in the first approximation is determined by calculations in Table 1:

$$I_{1w} = \mathrm{Re}(\underline{I}_{n1}) = 2.864\ \mathrm{A} \qquad (10)$$

We determine the phase drops of voltage in the lines $\underline{U}_{wa}, \underline{U}_{wb}, \underline{U}_{wc}$ when current I_{1w} is flowing through them as:

$$\underline{U}_{wa} = I_{1w} \cdot z_w \qquad (11)$$

$$\underline{U}_{wb} = a^2 \cdot I_{1w} \cdot z_w \qquad (12)$$

$$\underline{U}_{wc} = a \cdot I_{1w} \cdot z_w \qquad (13)$$

We specify potentials $\underline{U}_{na}, \underline{U}_{nb}, \underline{U}_{nc}$ of the points *A*, *B*, *C* (see Table 1) of load connection

$$\underline{U}_{na} = \underline{E}_A - \underline{U}_{wa} \quad (14)$$
$$\underline{U}_{nb} = \underline{E}_B - \underline{U}_{wb} \quad (15)$$
$$\underline{U}_{nc} = \underline{E}_C - \underline{U}_{wc} \quad (16)$$

Then we calculate the voltage of the load neutral shift

$$\underline{U}_{Nnn} = \frac{\underline{U}_{na} \cdot y_{na} + \underline{U}_{nb} \cdot y_{nb} + \underline{U}_{nc} \cdot y_{nc}}{y_{na} + y_{nb} + y_{nc}} \quad (17)$$

Then we specify the currents in load phases:

$$\underline{I}_{nna} = (\underline{U}_{na} - \underline{U}_{Nnn}) \cdot y_{na} \quad (18)$$
$$\underline{I}_{nnb} = (\underline{U}_{nb} - \underline{U}_{Nnn}) \cdot y_{nb} \quad (19)$$
$$\underline{I}_{nnc} = (\underline{U}_{nc} - \underline{U}_{Nnn}) \cdot y_{nc} \quad (20)$$

We calculate the phase currents of the compensator according to (4) – (6) considering condition (10) and equations (18) – (20):

$$\underline{I}_{ka} = I_{1w} - \underline{I}_{nna} \quad (21)$$
$$\underline{I}_{kb} = a^2 \cdot I_{1w} - \underline{I}_{nb} \quad (22)$$
$$\underline{I}_{kc} = a \cdot I_{1w} - \underline{I}_{nc} \quad (23)$$

The results of calculations by formulae (10) – (23) with the necessary explanations are given in Table 2. All calculations were carried out in the Mathcad program.

Table 2 – Calculation of the balancing device currents by the diagram of Figure 2 according to formulae (10)–(23)

$I_{1w} = \text{Re}(I_{n1})$		
Calculating voltage decrease on the lines resistances when compensated current is flowing in them		
$U_{wA} = I_{1w} \cdot z_w = 0.286 + 0.899j$	$\lvert U_{wA} \rvert = 0.944$	$\arg(U_{wA}) = 72.335°$
$U_{wB} = a^2 \cdot I_{1w} \cdot z_w = 0.636 - 0.698j$	$\lvert U_{wB} \rvert = 0.944$	$\arg(U_{wB}) = -47.665°$
$U_{wC} = a \cdot I_{1w} \cdot z_w = -0.922 - 0.202j$	$\lvert U_{wC} \rvert = 0.944$	$\arg(U_{wC}) = -167.665°$
Elaborated calculation of currents in loadings considering voltage decrease in lines phases due to the flow of compensated current		
$U_{na} = E_A - U_{wA}$	$U_{nb} = E_B - U_{wB}$	$U_{nc} = E_C - U_{wC}$
$\text{U}_{\text{Nnn}} = \frac{\text{E}_{\text{na}} \cdot \text{y}_{\text{na}} + \text{E}_{\text{nd}} \cdot \text{y}_{\text{nb}} + \text{E}_{\text{nc}} \cdot \text{y}_{\text{nc}}}{\text{y}_{\text{na}} + \text{y}_{\text{nb}} + \text{y}_{\text{nc}}} = -9.292 - 40.736\text{j}$		
$I_{nna} = (U_{na} - U_{Nnn}) \cdot y_{na}$ $I_{nna} = 7.456 - 12.461j$	$I_{nnb} = (U_{nb} - U_{Nnn}) \cdot y_{nb}$ $I_{nnb} = -16.867 + 7.795j$	$I_{nnc} = (U_{nc} - U_{Nnn}) \cdot y_{nc}$ $I_{nnc} = 9.411 + 4.666j$
Phase currents of compensator with the "star" connection circuit		
$I_{ka} = I_{1w} - I_{nna} = -4.591 + 12.461j$	$\lvert I_{ka} \rvert = 13.28$	$\arg(I_{ka}) = 110.227°$
$I_{kb} = a^2 \cdot I_{1w} - I_{nnb} = 15.435 - 10.276j$	$\lvert I_{kb} \rvert = 18.543$	$\arg(I_{kb}) = -33.653°$
$I_{kc} = a \cdot I_{1w} - I_{nnc} = -10.844 - 2.186j$	$\lvert I_{kc} \rvert = 11.062$	$\arg(I_{kc}) = -168.604°$

To determine the phase resistances z_{ka}, z_{kb}, z_{kc} of the BD it is necessary to know the corresponding vectors of phase voltages $\underline{U}_{ka}, \underline{U}_{kb}, \underline{U}_{kc}$. We know that phase voltage vectors' origin has the potential of joining points of load phases *A*, *B*, *C* (see Figure 2) according to (14) - (16), and their ends have the potential of a common point $\underline{U}_{Nk}$ (Figure 2).

To determine the potential $\underline{U}_{Nk}$ of the neutral point of the BD N_k, we note the following.

The known parameters of phase resistances of the BD are the vectors of the phase currents of the BD. At the same time, it was previously determined that capacitors would be used as phase resistors for the BD. It is known that the vectors of voltages and capacitance currents differ from each other by the direction at angle π/2. That is, the known parameters about each vector of the BD phase voltage are, firstly, the initial points with potentials $\underline{U}_{wa}, \underline{U}_{wb}, \underline{U}_{wc}$ according to (14) - (16) and, secondly, the directions which are perpendiculars to the current vectors $\underline{I}_{ka}, \underline{I}_{kb}, \underline{I}_{kc}$ of (18) - (20), which allows us to expressly form canonical equations of lines on a complex plane.

The potential of the intersection point of the straight lines, constructed according to the indicated signs for each phase resistance of BD corresponds to the potential of the common point $\underline{U}_{Nk}$.

By the rules of linear algebra we know that the canonical equation of the line in the plane in the orthogonal coordinates x, y has the form

$$A \cdot x + B \cdot y + C = 0 \qquad (24)$$

where *A*, *B*, *C* – are constant coefficients; x, y – are data points of coordinates by orthogonal axes.

For a complex plane, axis *x* corresponds to the true axis (horizontal axis Re), axis *y* corresponds to the imaginary axis (vertical axis Im).

The values of coefficients A_i, B_i, C_i of the canonical equations for the straight lines, on which the corresponding vectors of the stresses of the BD $\underline{U}_{ki}$ lie, where *i* is the designation of the phases *a*, *b*, *c* of the BD (Figure 2), are determined as follows:

$$A_i = \mathrm{Im}(j \cdot \underline{I}_{ki}) \qquad (25)$$

$$B_i = -\mathrm{Re}(j \cdot \underline{I}_{ki}) \qquad (26)$$

$$C_i = \mathrm{Re}(j \cdot \underline{I}_{ki}) \cdot \mathrm{Im}(\underline{U}_{ni}) - \mathrm{Im}(j \cdot \underline{I}_{ki}) \cdot \mathrm{Re}(\underline{U}_{ni}) \qquad (27)$$

To determine the coordinates of the intersection point of the phase voltage vectors directions, matrix equations for the coefficients of the corresponding canonical equations must be solved. For example, the coordinates of the intersection point of the directions of vectors $\underline{U}_{ka}$ and $\underline{U}_{kb}$ are determined by a column matrix NK_{ab} whose elements are the real Re_{ab} and imaginary Im_{ab} of the coordinates of the point of intersection:

$$NK_{ab} = (\mathrm{Re}_{ab} \quad \mathrm{Im}_{ab})^T \qquad (28)$$

Matrix equation to determine the coordinates of the intersection point of the directions of vectors $\underline{U}_{ka}$ and $\underline{U}_{kb}$ looks like this:

$$NK_{ab} = A_{ab}^{-1} C_{ab} \qquad (29)$$

where $A_{ab} = \begin{pmatrix} A_a & B_a \\ A_b & B_b \end{pmatrix}$, $C_{ab} = \begin{pmatrix} -C_a \\ -C_b \end{pmatrix}$

The elements of matrixes A_{ab} and C_{ab} are determined by formulae (25) – (27).

In the process of determining the coordinates of the potential $\underline{U}_{Nk}$ of the point N_k, a situation may occur where the coordinates of the intersection of the different pairs of vectors' directions will differ from each other.

In this case, it is advisable to take the coordinates $\mathrm{Re}(\underline{U}_{Nk})$ and $\mathrm{Im}(\underline{U}_{Nk})$ of the potentials of point N_k as the ones corresponding to the center of gravity of the triangle, whose vertices coordinates are determined at the intersection of the corresponding pairs of lines:

$$\mathrm{Re}(\underline{U}_{Nk}) = \frac{\mathrm{Re}(\underline{U}_{Nk\,ab}) + \mathrm{Re}(\underline{U}_{Nk\,bc}) + \mathrm{Re}(\underline{U}_{Nk\,ca})}{3} \qquad (30)$$

$$\mathrm{Im}(\underline{U}_{Nk}) = \frac{\mathrm{Im}(\underline{U}_{Nk\,ab}) + \mathrm{Im}(\underline{U}_{Nk\,bc}) + \mathrm{Im}(\underline{U}_{Nk\,ca})}{3} \qquad (31)$$

After determining the coordinates of potential $\underline{U}_{Nk}$ of the BD neutral by (30, 31) we can determine the vectors of phase voltages $\underline{U}_{ka}, \underline{U}_{kb}, \underline{U}_{kc}$:

$$\underline{U}_{ka} = \underline{U}_{na} - \underline{U}_{Nk} \qquad (32)$$

$$\underline{U}_{kb} = \underline{U}_{nb} - \underline{U}_{Nk} \qquad (33)$$

$$\underline{U}_{kc} = \underline{U}_{nc} - \underline{U}_{Nk} \qquad (34)$$

Let us determine values of z_{ka}, z_{kb}, z_{kc} of capacitive resistances of BD:

$$z_{ka} = \frac{|\underline{U}_{ka}|}{|\underline{I}_{ka}| \cdot j}; \quad z_{kb} = \frac{|\underline{U}_{kb}|}{|\underline{I}_{kb}| \cdot j} \underline{U}_{ka}; \quad z_{kc} = \frac{|\underline{U}_{kc}|}{|\underline{I}_{kc}| \cdot j} \qquad (35)$$

The results of calculations by formulae (24) – (35) with the necessary explanations are given in Table 3. All calculations were carried out in the Mathcad program.

Table 3. Diagram-analytical Mathcad-calculation of potential of a neutral point and phase resistances of the balancing device in Figure 2 according to formulae (24) – (35)

Coefficients of equation of voltage vector of phase A of a balancing device in canonic form ($A_A x + B_A y + C_A = 0$)		
$A_A = \mathrm{Im}(j \cdot I_{ka})$	$B_A = -\mathrm{Re}(j \cdot I_{ka})$	$C_A = \mathrm{Re}(j \cdot I_{ka}) \cdot \mathrm{Im}(U_{na}) - \mathrm{Im}(j \cdot I_{ka}) \cdot \mathrm{Re}(U_{na})$
$A_A = -4.591$	$B_A = 12.461$	$C_A = 469.039$
Coefficients of equation of voltage vector of phase B of a balancing device in canonic form ($A_B x + B_B y + C_B = 0$)		
$A_B = \mathrm{Im}(j \cdot I_{kb})$	$B_B = -\mathrm{Re}(j \cdot I_{kb})$	$C_B = \mathrm{Re}(j \cdot I_{kb}) \cdot \mathrm{Im}(U_{nb}) - \mathrm{Im}(j \cdot I_{kb}) \cdot \mathrm{Re}(U_{nb})$
$A_B = 15.435$	$B_B = -10.276$	$C_B = -101.165$

Coefficients of equation of voltage vector of phase C of a balancing device in canonic form ($A_C x+B_C y+C_C=0$)

$$A_C=\mathrm{Im}(j\cdot I_{kc}) \qquad B_C=-\mathrm{Re}(j\cdot I_{kc}) \qquad C_C=\mathrm{Re}(j\cdot I_{kc})\cdot\mathrm{Im}(U_{nc})-\mathrm{Im}(j\cdot I_{kc})\cdot\mathrm{Re}(U_{nc})$$

$$A_C=-10.844 \qquad B_C=-2.186 \qquad C_C=-342.463$$

Finding intersection points of voltages vectors of BD capacity separately for phases a&b, b&c, c&a

$$Aab=\begin{pmatrix}A_A & B_A\\ A_B & B_A\end{pmatrix};\quad Cab=\begin{pmatrix}-C_A\\ -C_B\end{pmatrix};\quad Abc=\begin{pmatrix}A_B & B_B\\ A_C & B_C\end{pmatrix};\quad Cbc=\begin{pmatrix}-C_B\\ -C_C\end{pmatrix}\quad Aca=\begin{pmatrix}A_C & B_C\\ A_A & B_A\end{pmatrix};\quad Cca=\begin{pmatrix}-C_C\\ -C_A\end{pmatrix}$$

$$NK_{ab}=Aab^{-1}\cdot Cab=\begin{pmatrix}-24.518\\ -46.673\end{pmatrix} \qquad NK_{bc}=Abc^{-1}\cdot Cbc=\begin{pmatrix}-22.719\\ -43.971\end{pmatrix} \qquad NK_{ca}=Aca^{-1}\cdot Cca=\begin{pmatrix}-22.336\\ -45.87\end{pmatrix}$$

The results of calculations show that coordinates of the intersection points do not coincide. Thus, we make the decision about the position of the BD neutral potential in the centre of gravity of a triangle with vertices in the points of intersection of the phase voltages pairs

$$N_K=\frac{NK_{ab0}+NK_{bc0}+NK_{ca0}}{3}+\frac{NK_{ab1}+NK_{bc1}+NK_{ca1}}{3}\cdot j=-23.191-45.505j$$

Making formulae for determination of vectors of the compensator phase voltages

$$U_{kaNk}=U_{na}-N_k=122.905+44.605j \qquad U_{kbBNk}=U_{nb}-N_k=-27.445-40.4j \qquad U_{kcBNk}=U_{nc}-N_k=-25.887+132.309j$$

Calculated values of phase resistances of the compensator

$$z_{ka}=\frac{|U_{kaNk}|}{|I_{ka}|\cdot j}=-9.845j \qquad z_{kb}=\frac{|U_{kbBNk}|}{|I_{kb}|\cdot j}=-2.634j \qquad z_{kc}=\frac{|U_{kcNk}|}{|I_{kc}|\cdot j}=-12.188j$$

5. Analyzing efficiency of balancing

To analyze the efficiency of balancing it is necessary to determine the currents in the branches of the diagram in Figure 2, taking into account the values of BD resistances determined by expression (35).

The scheme in Figure 2 has nine branches, six nodes, four independent circuits. The accepted positive directions of currents of branches are marked in Figure 2, the positive voltage directions of the diagram elements coincide with the positive current directions.

According to Kirchhoff's laws for the scheme in Figure 2 the system of equations will contain four equations under the second Kirchhoff law for circuits and five equations by the first Kirchhoff law for nodes *A*, *B*, *C*, N_n, N_k.

The elements of the first circuit and the direction of their circumvention are as follows: E_a, z_{wa}, z_{ka}, z_{kb}, z_{wb}, E_b.

The elements of the second circuit and the direction of their circumvention are as follows: E_b, z_{wb}, z_{kb}, z_{kc}, z_{wc}, E_c.

The elements of the third circuit and the direction of their circumvention are as follows: E_a, z_{wa}, z_{na}, z_{nb}, z_{wb}, E_b.

The elements of the fourth circuit and the direction of their circumvention are as follows: E_b, z_{wb}, z_{nb}, z_{nc}, z_{wc}, E_c.

We form a vector column of *IK* currents of branches:

$$IK=\begin{pmatrix}\underline{I}_{wa} & \underline{I}_{wb} & \underline{I}_{wc} & \underline{I}_{ka} & \underline{I}_{kb} & \underline{I}_{kc} & \underline{I}_{na} & \underline{I}_{nb} & \underline{I}_{nc}\end{pmatrix}^T \tag{36}$$

Considering (36) the own matrix F of the diagram in Figure 2 looks like:

$$F=\begin{pmatrix}z_w & -z_w & 0 & z_{ka} & -z_{kb} & 0 & 0 & 0 & 0\\ 0 & z_w & -z_w & 0 & z_{kb} & -z_{kc} & 0 & 0 & 0\\ z_w & -z_w & 0 & 0 & 0 & 0 & z_{na} & -z_{nb} & 0\\ 0 & z_w & -z_w & 0 & 0 & 0 & 0 & z_{nb} & -z_{nc}\\ 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1\\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1\end{pmatrix} \tag{37}$$

The column-vector of input impacts *E* relative to (36), (37) is as follows:

$$E=\begin{pmatrix}\underline{E}_{ab} & \underline{E}_{bc} & \underline{E}_{ab} & \underline{E}_{bc} & 0 & 0 & 0 & 0 & 0\end{pmatrix}^T \tag{38}$$

where $\underline{E}_{ab}=\underline{E}_a-\underline{E}_b$, $\underline{E}_{bc}=\underline{E}_b-\underline{E}_c$.

We obtain vector IK of complex values of currents in the result of solving the matrix equation:

$$IK=F^{-1}E \tag{39}$$

The result of the solution of the matrix equation (39) is given in Table 4.

In Table 4 we will pay attention to the values of I_{wa}, I_{wb}, I_{wc} phase currents of the lines: we can see that by the taken measures we were able to significantly symmetrize the currents of the lines in comparison with the initial results of I_{wa}, I_{wb}, I_{wc} (Table 1). Thus, the coefficient of unbalance in the negative sequence decreased from the initial value of 32.9 % (Table 1) to the level of 1.25 % (Table 4), the phase difference between the vectors of phase EMF voltages and currents does not exceed 0.7 electric degrees, which gives a reason to think that the measures taken for balancing and reactive power compensation are mostly effective.

Table 4. Calculation and assessment of coefficient of lines currents unbalance with the parameters of initial compensation

Calculating the vectors of currents of the branches	$IK := F^{-1} \cdot E$	$IK = \begin{pmatrix} 2.934+0.032j \\ -1.498-2.583j \\ -1.437+2.551j \\ -4.518+12.496j \\ 15.365-10.384j \\ 10.847-2.112j \\ 7.452-12.464j \\ -16.863+7.801j \\ 9.411+4.663j \end{pmatrix}$
Calculated values of currents in lines $I_{wa} = IK_0 = 2.934+0.032j$ $I_{wb} = IK_1 = -1.498-2.583j$ $I_{wc} = IK_2 = -1.437+2.551j$	$\lvert I_{wa}\rvert = 2.934$ $\lvert I_{wb}\rvert = 2.986$ $\lvert I_{wc}\rvert = 2.928$	$\arg(I_{wa}) = 0.625°$ $\arg(I_{wb}) = -120.104°$ $\arg(I_{wc}) = 119.384°$
Balanced components of current in the line $I_{w1} = 1/3 \cdot (I_{wa} + a \cdot I_{wb} + a^2 \cdot I_{wc}) = 2.949 - 1.638j \times 10^{-3}$	$I_{w2} = 1/3 \cdot (I_{wa} + a^2 \cdot I_{wb} + a \cdot I_{wc}) = -0.015 + 0.034j$	
Coefficient of unbalance (%) of compensated current in the line	$K_{nc2} = \frac{\lvert I_{w2}\rvert}{\lvert I_{w1}\rvert} \cdot 100 = 1.25$	

6. Elaboration of parameters of balancing device

The reason for the incomplete balancing is obviously the inaccurate calculation of the values of the compensating resistance of BD.

When analyzing the reasons for such inaccuracy, the following is established. After compensation and balancing, the currents of the lines contain a symmetric component of the positive sequence with the value of the active component $\mathrm{Re}(\underline{I}_{w1})$ = 2.949 A (Table 4). This value is different from the value of I_{1w} of the first approximation (10) and is the second approximation of the symmetric current value in the lines during balancing and compensation.

To specify the parameters of the balancing device it is enough to replace in the first line of Table 4 the values of the first approximation of the symmetric current in the line I_{1w} = 2,864 A with the values of the second approximation $I_{1w} = \mathrm{Re}(\underline{I}_{w1})$ = 2,949 A specified by the results of the calculation (Table 4). After that the full recalculation of parameters of the diagram of balancing device and currents of the elements of the diagram by equations (11)-(39) will take place.

The results of the elaborated calculations of the parameters of the balancing device diagram by the second approximation of the symmetric current value in the lines (I_{1w} = $\mathrm{Re}(\underline{I}_{w1})$ = 2,949 A) are shown in the form of a fragment of the Mathcad-document in Table 5.

The results shown in Table 5 clearly illustrate how, due to elaboration of the parameters of the balancing device, the effect of accurate balancing of the currents in the lines with simultaneous compensation of reactive power under unbalanced loading was achieved. At the same time, the results of the current calculation completely coincide with the results obtained by the author [14].

It should be noted that Table 1 - Table 5 are the consecutive parts of a single Mathcad document, by which graphical and analytical determination of the balancing parameters and full reactive power compensation for a three-phase power supply system is practically implemented.

Table 5. The results of calculation and assessment of coefficient of unbalance of lines currents with elaborated compensation parameters

Calculating the vectors of currents of the branches $IK = F^{-1} \cdot E$	$IK = \begin{pmatrix} 2.949 + 5.969\,j \times 10^{-5} \\ -1.475 - 2.554\,j \\ -1.475 + 2.554\,j \\ -4.503 + 12.462\,j \\ 15.389 - 10.353\,j \\ 10.886 - 2.109\,j \\ 7.452 - 12.462\,j \\ -16.864 + 7.799\,j \\ 9.412 + 4.663\,j \end{pmatrix}$	
Calculated values of currents in lines		
$I_{wa} = IK_0 = 2.949 + 5.969\,j \times 10^5$ $I_{wb} = IK_1 = -1.475 - 2.554\,j$ $I_{wc} = IK_2 = -1.475 + 2.554\,j$	$\lvert I_{wa} \rvert = 2.949$ $\lvert I_{wb} \rvert = 2.949$ $\lvert I_{wc} \rvert = 2.949$	$\arg(I_{wa}) = (1.16 \times 10^{-3})^\circ$ $\arg(I_{wb}) = -120^\circ$ $\arg(I_{wc}) = 119.999^\circ$
Balanced components of current in the line		
$I_{w1} = 1/3 \cdot (I_{wa} + a \cdot I_{wb} + a^2 \cdot I_{wc}) = 2.949 - 3.593\,j \times 10^{-6}$	$I_{w2} = 1/3 \cdot (I_{wa} + a^2 \cdot I_{wb} + a \cdot I_{wc}) = -2.843 \times 10^{-5} + 6.328\,j \times 10^{-5}$	
Coefficient of unbalance (%) of compensated current in the line	$K_{nc2} = \frac{\lvert I_{w2} \rvert}{\lvert I_{w1} \rvert} \cdot 100 = 2.352 \times 10^{-3}$	

6. Conclusions

By the known parameters of asymmetric active-inductive load, source and power line, the proposed method allows obtaining graphically and analytically a numerical solution of the problem of balancing currents in the line and compensation of reactive power in a three-phase power supply system with unbalanced load, which, unlike the earlier methods, takes into account the impact of the power lines resistance.

It is established that the parameters of BD, determined according to the data of the initial analysis of the operation mode of the power supply system under unbalanced loading, do not always provide sufficient effect of balancing and compensation. The results of the calculations by the taken measures show that they lead to a significant balancing of the currents in the lines. Thus, the negative sequence unbalance coefficient decreased from 32.9 % to 1.25 %, the phase difference between the vectors of phase EMF voltages and currents does not exceed 0.7 electric degrees. Therefore, the result obtained shows that the measures taken for balancing and reactive power compensation are sufficiently effective.

For further improvement of the balancing efficiency and reactive power compensation, the BD parameters should be elaborated based on the primary compensation data. After the elaboration of the parameters of the balancing device, the results were obtained, which clearly illustrate the achievement of complete balancing of currents in the lines with simultaneous compensation of the reactive power under unbalanced loading.

Acknowledgements.
This work was conducted under the Scientific research "Development of the system of energy saving and electric energy generation for vehicles", 0219U100696, funded by the Ministry of Education and Science of Ukraine.

Conflict of interests.
The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] Yang K, Gong Y, Zhang P, Liu Z (2015) A reactive power compensation method based on tracing the power flow and loss function of power system. In: *2015 5th International Conference on Electric Utility Deregulation and Restructuring and Power Technologies* (DRPT). IEEE, pp 615–619

[2] Arrillaga J, Watson NR, Chen S (2000) Power System Quality Assessment | Wiley

[3] Acha E, Agelidis V, Anaya-Lara O, Miller TJE (2002) Power electronic control in electrical systems. Newnes

[4] Mestriner D, Labella A, Bonfiglio A, Benfatto I, Li J, Ye Y, Song Z (2019) ITER Reactive Power Compensation

Systems: analysis on reactive power sharing strategies. In: *2019 IEEE International Conference on Environment and Electrical Engineering and 2019 IEEE Industrial and Commercial Power Systems Europe* (EEEIC/I&CPS Europe). IEEE, pp 1–6

[5] Hofmann W, Schlabbach J, Just W (2012) Reactive power compensation: a practical guide. John Wiley & Sons

[6] Liu Y-W, Rau S-H, Wu C-J, Lee W-J (2017) Improvement of power quality by using advanced reactive power compensation. *IEEE Transactions on Industry Applications* 54:18–24

[7] Wu Z (2018) A Novel Design Method of LCL Filters for Optimal Reactive Power Compensation in Microgrids. In: *2018 IEEE International Power Electronics and Application Conference and Exposition (PEAC)*. IEEE, pp 1–5

[8] Gnatov A, Argun S (2015) New Method of Car Body Panel External Straightening: Tools of Method. *International Journal of Vehicular Technology* 2015:1–7 . https://doi.org/10.1155/2015/192958

[9] Batygin Y, Hnatov A (2012) The features of the electrical magnetic forces excitation in the magnetic pulse sheet ferromagnetic metal working. *Technical Electrodynamics* 1:71–77

[10] Batygin YV, Chaplygin EA, Shinderuk SA, Strelnikova VA (2018) The main inventions for technologies of the magnetic-pulsed attraction of the sheet metals. A brief review. *Elektrotexnika i Elektromexanika* 3:43–52

[11] Huang X, Zhang Y, Huang H (2014) Automatic reactive power control in distribution network based on Feeder Power Factor Assessment. In: *2014 IEEE PES Asia-Pacific Power and Energy Engineering Conference* (APPEEC). IEEE, pp 1–4

[12] Yagup VG, Yagup EV (2016) Determination of reactive power compensation mode in four-wire three-phase electric power supply system using search engine optimization. *Tekhnichna Elektrodynamika* 1:60–66

[13] Yagup VG, Yagup EV (2016) Investigation of modes of full compensation of reactive power in a three-phase power supply system. *Elektrotexnika i Elektromexanika* 2:61–65

[14] Yagup EV (2016) Simulation of an unbalanced power supply system using optimization to determine the parameters of a balancing device. *Collection of scientific works of the Ukrainian State University of Railway Transport* 130–138

[15] Karki U, Gunasekaran D, Peng FZ (2015) Reactive compensation of overhead AC transmission lines using underground power cables. In: *2015 IEEE Power & Energy Society General Meeting*. IEEE, pp 1–5

[16] Gnilicz`ky`j VV, Polischuk OA (2015) Development of the model of voltage equation in three-phase grids. *Vostochno-Evropeyskiy zhurnal peredovyih tehnologiy* 11–14

[17] Shymuk D (2019) Graphoanalytic determination of symmetry parameters and total compensation of reactive power for a three-phase power supply system. *Vehicle and Electronics Innovative Technologies* 15:6–16

[18] Bird J (2017) Electrical circuit theory and technology. Routledge

[19] Gnatov A, Argun S, Ulyanets O (2017) Joint innovative double degree master program “energy-saving technologies in transport.” In: *2017 IEEE First Ukraine Conference on Electrical and Computer Engineering (UKRCON)*. Kiev, Ukraine, pp 1203–1207

[20] Patļins A, Hnatov A, Arhun S (2018) Safety of Pedestrian Crossings and Additional Lighting Using Green Energy. In: *Transport Means 2018: Proceedings of 22nd International Scientific Conference. Kaunas University of Technology*, Lietuva, Trakai, Kaunas, pp 527–531

[21] Arhun S, Hnatov A, Dziubenko O, Ponikarovska S (2019) A Device for Converting Kinetic Energy of Press Into Electric Power as a Means of Energy Saving. *Journal of the Korean Society for Precision Engineering* 36:105–110. https://doi.org/10.7736/KSPE.2019.36.1.105