

Robust controller design for speed regulation of a flexible wind turbine

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Abstract

The complexity of large wind turbines increases due to fatigue, aerodynamics, structural flexibility and wind turbulence which lead to uncertainties in the wind turbine model. Hence, a robust controller is necessary to deal with these uncertainties. The uncertainties are represented in the wind turbine model by variations in the elements of the state space matrix comprising of mass, stiffness and damping elements. The generator speed is regulated with changes in the rotor collective blade pitch angle and disturbances in wind speeds. In this paper, we propose to design an H_∞ controller which qualifies to regulate the generator speed of an uncertain wind turbine. The proposed H_∞ controller demonstrates robust performance and stability on system with $\pm 20\%$ uncertainty. The obtained 14th order H_∞ controller is reduced to 7th order by using a balanced truncation model order reduction method. The performances of the designed H_∞ controller and the reduced controller are compared.

Keywords: H_∞ controller, model order reduction, robust stability, wind turbine.

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1. Introduction

Recent times, many governments across the globe understands the benefits of wind power and hence are further strengthening their measures to rapidly improve the existing growth-rate of wind power capture every year. Statistical data [1] clearly indicates that the overall installed capacity of wind power worldwide increased from 296,581 MW in the year 2013 to 539,291 MW by the end of year 2017. This scenario in wind power capture led to the increase in the size of the wind turbine ranging from 1 kW to several MWs. These modern flexible wind turbines are very large structures and are available as vertical-axis and horizontal-axis wind turbines (VAWTs

and HAWTs). The latter type has commendable advantages when compared with the former, wherein, 1) the entire rotor can be placed on top of a tall tower to ensure greater proximity to larger wind speeds for increased power capture 2) blades with provision for a maximum pitch angle of 18 degrees, 3) no need for tensioned cables viz., guy wires that are used to add structural stability. [2] The components of the up-wind HAWT comprises of the generator, gear box, low and high speed shafts, rotor, blades, tower, etc. Modelling of a wind turbine is complex and involves several degrees of freedom to encompass the effects of aerodynamics, rigid bodies (earth, base plate, nacelle, generator and hub) and flexible bodies (blades, low speed shaft and tower) [3]. The wind speeds are categorized into three regions: (a)

below 6 m/s (b) 6 to 11.6 m/s and (c) above 6 m/s as regions I, II and III respectively. [4] In region III, the power capture should be limited in order to ensure mechanical and electrical safety. The control of the rotor blade pitch angle is of utmost necessity in region III.

Classical proportional integral differential (PID) controllers combined with fuzzy logic [5] are used for blade pitch regulation in region 3. The uncertainties in the wind turbine model cannot be considered under the effect of PID and Fuzzy logic controllers. In [6], the authors dealt with individual pitch control to limit power in high winds, particularly for large turbines. Classical PID algorithms may design good closed-loop controllers for fixed and variable speed turbines, but owing to the large structure and higher flexibility of present day wind turbines, the component loads must be considered upon which the effect of the classical PID controllers may not perform as desired [7]. Henceforth, it must be understood that there is a definite uncertainty in the wind turbine dynamics, due to the wind speeds that affect the component loads, the moment of inertias acting on the rotor and generator, the stiffness coefficients, the mass coefficients, etc. dealt with the robust model reference adaptive controller design for wind turbine speed regulation simulated by using fatigue, aerodynamics, structural flexibility and wind turbulence (FAST) code [8] of a three-state model of the wind turbine where the uncertainties were not considered. Similarly uncertainties are a concern in [9] wherein a DSOGI-PLL based power control method is used to mitigate control errors under the various disturbances of the grid connected systems.

Since, wind turbines are prone to external factors like wind disturbances and measurement noise, it is necessary to consider the differences between the actual model and the mathematical models used for design. It is also desirable to stabilize a plant which is originally unstable and satisfy certain performance objectives in the proximity of disturbance signals, noise interference, unmodelled plant dynamics and plant-parameter variations. The standard H_∞ problem is solved by deriving simple state space formulas for all controllers by the given value of a positive real number $\gamma (>0)$, the H_∞ norm should be strictly maintained less than γ . It is said that a controller exists if and only if the unique stabilizing solutions to two algebraic Riccati equations are positive definite. This parameterization of all controllers solving the problem is given as linear fractional transformation (LFT) [10], [11].

The potential threat to the stability and performance under the influence of uncertainties on the wind turbines gained prominence and is hence considered for analysis in this paper. Improper control of wind turbines subjected to uncertainties owing to its large size and cost may lead to loss in reliability and economic aspects. Therefore, the focus lays generally on the active control of larger flexible wind turbines subjected to uncertainties, that lead to reduction in the losses incurred economically and structurally which sums up to costly compensation. In the robust stability analysis, the designed controller sums up

to higher order, at times an order which is much greater than the order of the nominal turbine model. Hence, it is necessary to apply the model order reduction techniques to preserve/retain the most important properties of the original controller. In such a way the complexity of the system reduces. [12]-[16]. The balanced truncation model order reduction method [17],[18] is used to reduce the original higher order controller in order to reduce the complexity in realization of the physical higher order controller.

The point of interest lies in the design of controllers to regulate the generator speed via the control of blade pitch. However, the task meted out is not easy owing to the rapid turbulence in the wind, as well as complexities and flexibilities of the wind turbine system. The following shows how this paper deals with the problem and the solution: In section 2, the mathematical modelling of the wind turbine is discussed, section 3 covers the implementation of the robust stability concepts on the wind turbine, section 4 shows how the original controller is reduced using the model order reduction technique. The robust stability and robust performances are discussed in section 5 and section 6 shows the simulation results for various scenarios encountered by the uncertain wind turbine model. Finally the conclusions are made from the obtained results for the reduced order robust H_∞ controller designed for an uncertain wind turbine speed regulation.

2. Wind turbine model

The wind turbine dynamics are represented in the state space form with blade pitch as the control input and wind as disturbance input. This paper lays focus on the design of robust controllers for a 600 kW 2-bladed Controls Advanced Research Turbine (CART2), [3],[4] machine operated by National renewable Energy Laboratory (NREL), Colorado. The robust control of the uncertain wind turbine is shown in figure 1.

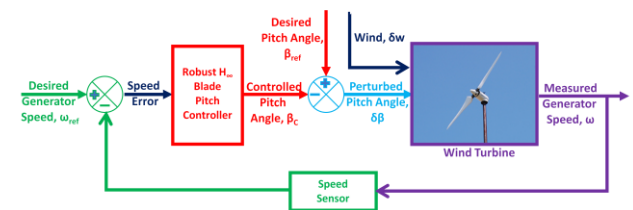


Figure 1. Block diagram of the wind turbine with control feedback loop

2.1. CART2 wind turbine linearized model

The model under considerations is CART2 two-bladed HAWT machine, it is developed using the high-fidelity turbine simulator FAST code [3]. The reliable simulation results obtained from FAST provides models of wind turbines by considering the flexible bodies (tower, low speed shaft and blades) and the rigid bodies (earth, base

plate, nacelle, generator and hub). The CART2 is developed in the National Wind Technology Centre (NWTC) a sub-centre of NREL, Colorado. It is a 600 kW 2-bladed wind turbine operated by NREL. A seven state linearized model of the upwind HAWT machine is considered for this study since the controllability and observability of this model are still preserved when subjected to $\pm 20\%$ uncertainty.

2.2. A seven-state linear wind turbine model

The general state space description of the linearized model of a wind turbine is expressed as

$$\begin{aligned} M\dot{x} &= Ax + Bu + \Gamma u_D \\ y &= Cx \end{aligned} \quad (1)$$

Where $x \in R^N$ is the state vector, \dot{x} denotes the time derivative of x , $u \in R^M$ is the control input, $u_D \in R^Z$ is the disturbance input, $y \in R^P$ is the output vector to be measured, $M \in R^{N \times N}$ is the mass matrix, $A \in R^{N \times N}$ is the system matrix, $B \in R^{N \times M}$ is the control input gain matrix, $\Gamma \in R^{N \times Z}$ is the disturbance input gain matrix and $C \in R^{P \times N}$ provides the relationship between measured output y and the turbine states x . The rotor collective blade pitch is considered as the principal control input and the wind velocity as the external disturbance input, while the seven states of the system considered are: x_1 is the perturbed rotor first symmetric flap mode displacement, x_2 is the perturbed rotor first symmetric flap mode velocity, x_3 is the perturbed rotor rotational speed, x_4 is the perturbed drive train torsional spring force, x_5 is the perturbed generator rotational speed, x_6 is the perturbed tower first fore-aft mode displacement and x_7 is the perturbed tower first fore-aft mode velocity. In addition, $\delta\beta$ represents the perturbations in rotor collective pitch and is the basic control input considered in this paper, while δw represents the perturbations in the turbulent wind speed and is assumed uniform across the rotor disk, $M'_{ij}, K'_{ij}, C'_{ij}$ represents the mass, stiffness and damping elements of the respective matrices ($i, j = 1, 2, \dots, 7$), ξ' represents the partial derivative of the rotor aerodynamic torque with respect to the perturbed rotor collective pitch angle, α' represents the partial derivative of the rotor aerodynamic torque with respect to the perturbed wind velocity and γ' represents the partial derivative of the rotor aerodynamic torque with respect to the generator speed. I_{rot} and I_{gen} represent the moment of inertia of rotor and generator respectively, K_d and C_d represents the stiffness and damping factors. [3]

The seven-state linearized wind turbine model developed by FAST [3] is given by

$$\begin{aligned} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{11} & M_{14} & 0 & 0 & 0 & M_{17} \\ 0 & 2M_{14} & I_{rot} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{gen} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2M_{71} & 0 & 0 & 0 & 0 & M_{77} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \\ & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -K_{11} & -C_{11} & -C_{14} & 0 & 0 & -K_{17} & -C_{17} \\ -2K_{41} & -2C_{41} & \gamma - C_d & -1 & C_d & -K_{47} & -C_{47} \\ 0 & 0 & K_d & 0 & -K_d & 0 & 0 \\ 0 & 0 & C_d & 1 & -C_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -2K_{71} & -2C_{71} & -C_{74} & 0 & 0 & -K_{77} & -C_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 \\ \xi_b \\ \xi \\ 0 \\ 0 \\ 0 \\ \xi_t \end{bmatrix} [\delta\beta] + \begin{bmatrix} 0 \\ \alpha_b \\ \alpha \\ 0 \\ 0 \\ 0 \\ \alpha_t \end{bmatrix} [\delta w] \\ & y = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0] [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]^T \quad (2) \end{aligned}$$

2.3. Open loop performance of wind turbine

The nominal values of the CART's state matrices A, B and Γ are taken from [3], the general specifications of the wind turbine are given in appendix A. The open loop poles obtained from the eigen analysis of matrix A are: $-0.039888 \pm 22.574j$, $-4.4422 \pm 13.508j$, $-0.11715 \pm 5.8673j$ and -0.12094 . There are three pole-pairs and an individual pole: the first pole-pair provides very light damping in the drive-train torsion mode, the second pole-pair monitors high damping in the rotor first symmetry flap mode, the lightly damped tower first fore-aft mode is represented by the third pole-pair and the generator speed is represented by the last pole. Speed regulation improves when the generator pole moves farther away to the left from its own open loop value and when damping is increased to the lightly damped pole-pairs.

The real part of the first pole is very close to the origin and the slightest of uncertainties in the system parameters will further deteriorate the robust properties which are further discussed in section 3.

3. Robust stability of wind turbine

The variation/change in wind speed that is treated as external disturbance along with the noise due to measurement sensors used to measure generator speed causes uncertainties in the nominal model. The design of controller requires considering the external disturbances and measurement noise. These factors render differences between the mathematically modelled system and the actual system. The prominent criteria one must consider in the design are (i) to design a controller so that it stabilizes the originally unstable plant (ii) to drive the system towards internal stability and (iii) to design stabilizing controllers. Summarily, if the robust stability, robust performance and the nominal performances are

obtained satisfactorily, then the designed controller may be called as a robust controller. The various factors leading to uncertainties in the wind turbine model are fatigue, aerodynamics, structural flexibility and wind turbulence. Finally, the weighting functions were so chosen to adequately counter these uncertainties in achieving the robust properties of the wind turbine in the presence of $\pm 20\%$ uncertainties as considered in this paper.

3.1. Modelling of uncertainties

Modelling of uncertainties are done in such a way that the moments, stiffness and damping factors are affected by real uncertainties. It is assumed to consider $\pm 20\%$ relative errors around the nominal values of I_{gen} , I_{rot} , M_{ij} 's, K_{ij} 's and C_{ij} 's of the wind turbine. By rewriting equation (1) into the standard state space form we have,

$$\dot{x} = (M^{-1}A)x + (M^{-1}B)u + (M^{-1}\Gamma)u_D \quad (3)$$

That can be written as

$$\dot{x} = \bar{A}x + \bar{B}u + \bar{\Gamma}u_D \quad (4)$$

Where, $M^{-1}A = \bar{A}$, $M^{-1}B = \bar{B}$, $M^{-1}\Gamma = \bar{\Gamma}$

The block diagram shown in figure 2 represents the nominal wind turbine model given by equation (3).

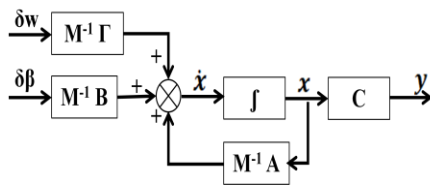


Figure 2. Block diagram of a nominal wind turbine model

The range of uncertainties that were introduced in the system matrix \bar{A} of equation (4) are $\pm 20\%$. This formulates the wind turbine model as an LFT of the real uncertainties of the respective parameters. The uncertainties in moment of inertias in rotor and generator are represented by $I_{rot} = \bar{I}_{rot}(1 + p\delta_{I_{rot}})$ and $I_{gen} = \bar{I}_{gen}(1 + p\delta_{I_{gen}})$ respectively. The uncertainties in stiffness factors, damping factors and mass elements are represented by $K_{ij} = \bar{K}_{ij}(1 + p\delta_{K_{ij}})$, $C_{ij} = \bar{C}_{ij}(1 + p\delta_{C_{ij}})$ and $M_{ij} = \bar{M}_{ij}(1 + p\delta_{M_{ij}})$ respectively, where i and j represents the i^{th} row and j^{th} column of the system matrix \bar{A} . The values of p are considered as ± 0.2 that represent the maximum relative uncertainty in each of the factors and all δ 's range from $[-1,1]$. As a result, the uncertainty matrix Δ has the form represented by equation (5). The order of the uncertainty matrix Δ is considered as 15 after thorough simulations during the course of this work. In

figure 3 the uncertain matrix Δ is included to the nominal wind turbine model.

$$\Delta = \begin{bmatrix} \delta_{I_{rot}} & 0 & 0 & 0 & 0 \\ 0 & \delta_{I_{gen}} & 0 & 0 & 0 \\ 0 & 0 & \delta_{K_{ij}} & 0 & 0 \\ 0 & 0 & 0 & \delta_{C_{ij}} & 0 \\ 0 & 0 & 0 & 0 & \delta_{M_{ij}} \end{bmatrix} \quad (5)$$

The uncertain wind turbine model is depicted in figure 4, which represents the LFT of the real uncertain parameters δI_{rot} , δI_{gen} , δK_{ij} , δC_{ij} and δM_{ij} . The input vector to Δ is represented as y_A and the output matrix from Δ as u_A . The dynamic equations of motion of the wind turbine with uncertainties are rearranged as

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + \bar{B}u(t) + \bar{\Gamma}u_D(t) \\ y(t) &= Cx(t) \\ u_A &= \Delta y_A \end{aligned} \quad (6)$$

The output equation $y(t) = Cx(t)$ with C as the output matrix given by $C = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ in equation (6) provides the generator speed as the output which is the 5th state of the wind turbine model.

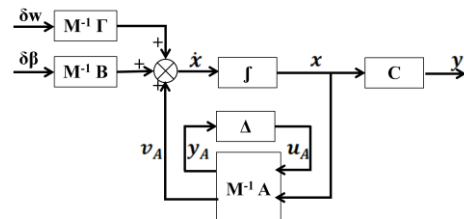


Figure 3. System block diagram with uncertain parameters represented in uncertain matrix Δ

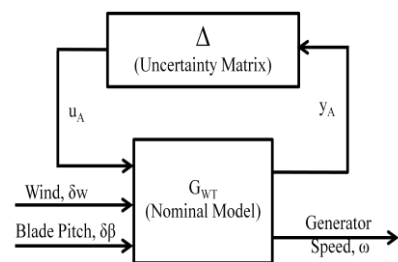


Figure 4. The LFT representation of the perturbed wind turbine

3.2. Weighting functions

For this design, four weighting functions were considered, which are $W_p(s)$, $W_u(s)$, $W_m(s)$ and $W_n(s)$. These weighting functions are chosen as scalar functions. $W_p(s)$ is chosen to represent the frequency characteristics of the wind turbine outputs (generator speed and tower deflection) and $W_u(s)$ represents the performance

requirement of controller-effort. Similarly $W_m(s)$ is chosen to represent the frequency characteristics of the model transfer function of the wind turbine while $W_n(s)$ is chosen to represent the noise generated randomly in the generator speed output which is taken as input to the controller. The considered scalar values of these four weighting functions from simulations are:

$$w_p(s) = 0.1581 * \frac{s+1}{200s+1}; \text{ and}$$

$$W_p(s) = \begin{bmatrix} w_p(s) & 0 \\ 0 & w_p(s) \end{bmatrix};$$

$$w_u(s) = 10^{-7} * \frac{3.162s+948.7}{s+10}; \text{ and } W_u(s) = w_u(s);$$

$$w_m(s) = 1; \text{ and } W_m(s) = w_m(s);$$

$$w_n(s) = 2 * 10^{-8} * \frac{s+1}{s+10}; \text{ and } W_n(s) = w_n(s)$$

The interconnection structure of the closed loop wind turbine comprises of the nominal model G_{WT} , the uncertain matrix Δ , the controller K and the weighting functions $W_p(s)$, $W_u(s)$, $W_m(s)$ and $W_n(s)$ as depicted in figure 5. The wind turbine model represents an upper LFT which takes the form $G=F_U(G_{WT},\Delta)$ and is depicted inside the dashed rectangular box. The H_∞ norm inequality bound $\|\Delta\|_\infty < 1$ helps the controller in achieving the performance objectives. The output of the designed controller must ensure certain robust properties of the closed loop wind turbine – they are (i) nominal stability and performance (ii) robust stability and (iii) robust performance. The input-output relationship of the designed controller is given by

$$u(s) = K(s).y(s) \quad (7)$$

where, $K(s)$ is the H_∞ controller's transfer function, $y(s)$ is the 5th state of the wind turbine model, i.e., the generator speed state, that is fed back as input signal to the controller $K(s)$ and $u(s)$ is the controlled output signal from the controller.

3.3. Sub-optimal H_∞ controller design

The complexity of the controller design increases with the number of measurements used for control, hence, the main objective in the design of an optimal controller should be to utilize the minimum number of required measurements, else the complexity of the controller increases along with the operation and maintenance costs. Therefore, the generator speed is the only main feedback control signal which is considered in order to obtain a minimal state and optimal system. The open loop structure of the uncertain system is shown in figure 6 with inputs (disturbance signal, control signal and u_A) and outputs (generator speed, e_p and e_u , and y_A). This system is termed as “*perturbed_sys*”, which has interconnections between weighting functions, controller, perturbations and the nominal model. The controller that is designed first

for this “*perturbed_sys*” is called an H_∞ sub-optimal controller, the infinite norm of $F_L(P,K)$ is minimized over all the set of stabilizing controllers K . Where, $F_L(P,K)$ is defined as the transfer function matrix of the nominal closed loop wind turbine system from the disturbance (wind velocity) to the error function e_p and e_u given by $e = [e_p \ e_u]^T$.

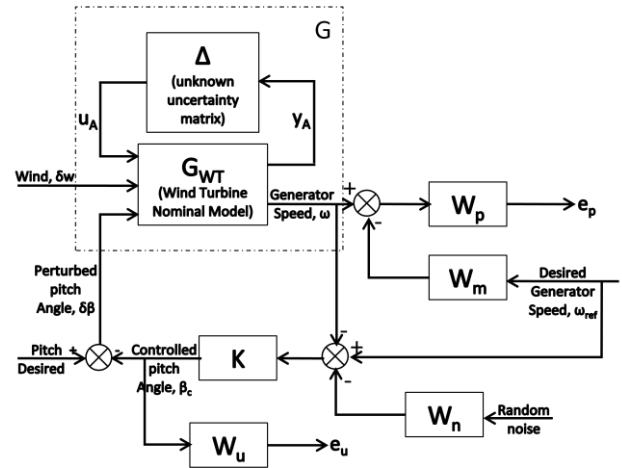


Figure 5. Interconnection structure of the closed-loop wind turbine system

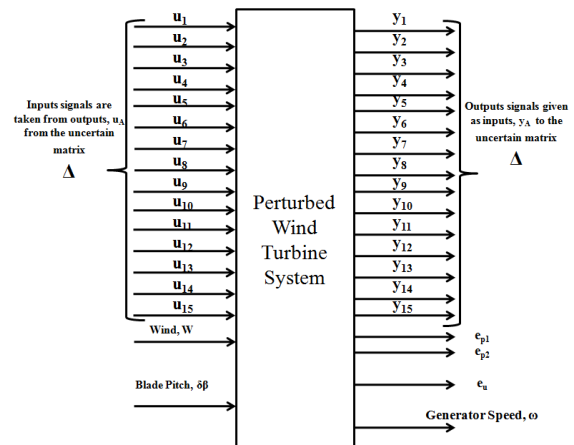


Figure 6. Generalised block diagram of open loop uncertain system

The Matlab command “*hinsyn*” computes a sub-optimal H_∞ controller based on the “*perturbed_sys*” – an open loop structure. The controller has one input and one output with the value of gamma to be ≤ 1 . From this proposed design the value of γ obtained is 0.4094 as shown in table.1 that represents the gamma calculations for the design of H_∞ controller. A 14th order controller is obtained along with the H_∞ norm ($\gamma = 0.4094$) of the closed-loop system. The number of poles obtained in the design is 14 and the values of poles of the H_∞ controller obtained from simulations lie in the stable region given by: -300.01; -13.545 ± 36.705i; -24.232 ± 17.004i;

0.67970 ± 6.1008i; -17.636; -10.028; -10.00; -0.0050013; -0.005; -8.00; -5.4535.

Table 1. Gamma calculations for H_∞ controller

gamma	hamx_eig	xinf_eig	hamy_eig	vinf_eig	nrho_xy	p/f
1.000	5.0e-03	0.0e+00	5.0e-03	-1.9e-22	0.000	p
0.700	5.0e-03	0.0e+00	5.0e-03	0.0e+00	0.000	p
0.550	5.0e-03	0.0e+00	5.0e-03	0.0e+00	0.000	p
0.475	5.0e-03	0.0e+00	5.0e-03	-1.3e-23	0.000	p
0.438	5.0e-03	0.0e+00	5.0e-03	0.0e+00	0.000	p
0.419	5.0e-03	0.0e+00	5.0e-03	0.0e+00	0.000	p
0.409	5.0e-03	0.0e+00	5.0e-03	0.0e+00	0.000	p

4. Model order reduction of H_∞ controller

Since a 14th order H_∞ controller is obtained, it is proposed to reduce it to a lower order 7th order which helps significantly in the physical realization of the controller. The complexity in the physical realizability of the original higher order controller creates the need to develop a systematic procedure to obtain a reduced order model in the state space or transfer function form which approximates effectively the higher order system. The key qualitative properties, viz. stability, realizability and good time and frequency response matching are retained in the simplified model [12]-[16]. The order reduction is done based on the balanced truncation method [17],[18]. The balanced truncation model reduction method designed for continuous stable/unstable plants in Matlab is used. The number of poles obtained in the design is 7 and the values of poles of the reduced H_∞ controller obtained from simulations lie in the stable region given by: -300.461; -13.078 ± 37.005i; -30.866 ± 20.528i; -0.00471; -0.01542.

5. Robust design specifications

A robust control system should remain stable and achieve certain performance criteria in the presence of possible uncertainties. Here, a controller is designed for a given plant, such that its closed-loop system is robust. The H_∞ optimisation approach is shown to be effective and efficient robust design method for a linear, time-invariant control systems [10],[11]. The stabilization and performance requirements for robust design are discussed in sections 5.1. and 5.2. respectively.

5.1. Robust stability

The nature of the turbulent wind velocity is invariably uncertain and hence the stiffness and damping factors that are associated with the blades, the nacelle joints and the cantilevered tower to the earth gets affected along with the moment of inertias of generator I_{gen} and rotor I_{rot} . The need for robust control design for a wind turbine arises due to such deviations. By assuming these uncertainties in the wind turbine, the robust properties are achieved by using the H_∞ controller. Considering G_{WT} , K and Δ from figure 5 it can be stated that, for stable uncertain, Δ the

closed loop system is robustly stable if controller, K stabilizes the nominal system G_{WT} and the following holds

$$\|\Delta K(I+G_{WT}K)^{-1}\| < 1 \text{ and } \|K(I+G_{WT}K)^{-1}\Delta\| < 1 \quad (8)$$

Hence, for all possible uncertain plant models $G=F_U(G_{WT},\Delta)$ obtained with ± 20% uncertainty in the system elements the closed loop system performance is achieved ($\mu < 1$) by the designed original (14th order) and the reduced (7th order) controllers as shown in figure 7. It means that the system retained its stability for all its values in the range from 80% to 120% of the nominal values.

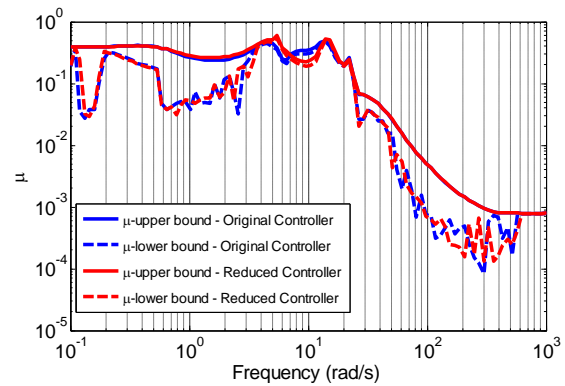


Figure 7. Robust stability analysis (upper and lower bounds) for original and reduced controllers

5.2. Robust performance

Once, all the possible internally stable plant models of the closed loop system $G = F_U(G_{WT}, \Delta)$ shows robust stability, they should eventually satisfy the performance criterion given by

$$\|W_p(I+G_{WT}K)^{-1}\| < 1 \text{ and } \|W_u K(I+G_{WT}K)^{-1}\| < 1 \quad (9)$$

The performance objectives usually are to ensure good tracking, good disturbance attenuation and good noise rejection such that for any reference, disturbance and noise inputs, the energy does not exceed 1 ($\mu < 1$), as obtained in figure 8.

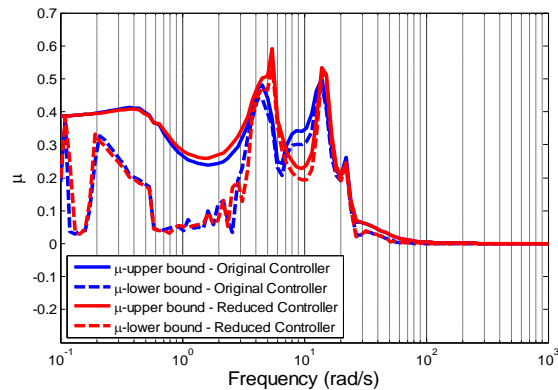


Figure 8. Robust performance analysis (upper and lower bounds) for original and reduced controllers

6. Simulation results

Simulation results in Matlab are produced in this section to justify the robust properties of the perturbed uncertain wind turbine system by considering blade pitch angle as the control input, the disturbance input is the perturbed wind speed and the generator speed as the only output considered for feedback. A $\pm 20\%$ uncertainty is considered throughout the simulation and to approve of the robust properties, viz., stability and performance of the wind turbine system, 10 samples of uncertain systems were considered. Moreover, to understand the overall performance of the wind turbine with the robust H_∞ controller (comparing both original and reduced controllers) in the closed-loop, it is sufficient to analyse the transient responses of three state variables out of the seven state variables of the wind turbine, they are x_1 (rotor first symmetric flap mode displacement in cm), x_5 (generator rotational speed in RPM) and x_6 (tower first fore-aft mode displacement in cm). Furthermore, the controlled rotor collective pitch angle in degrees is shown in various scenarios discussed in further 6.1. to 6.3. Eventually in sub-section 6.4, the worst case analysis is done for the prominent situations encountered by the wind turbine during its operation.

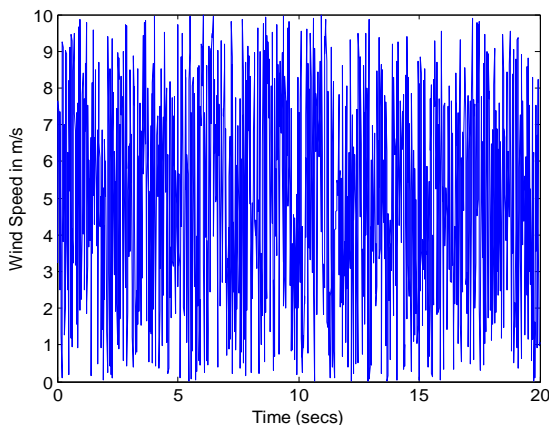


Figure 9. Wind profile (m/s) – disturbance input – random value with a maximum value of $10.0*(\text{rand}(1,\text{nstep}))\text{m/s}$

6.1. Effect of Wind Disturbance Input on Transient Responses of State Variables

In this scenario, random wind disturbance input of $10.0*(\text{rand}(1,\text{nstep}))\text{ m/s}$ is only considered by neglecting the blade pitch angle control input. figure 9 shows the wind profile curves applied as disturbance input on the uncertain wind turbine system. A wind profile with maximum wind speed of 10 m/s is applied on the nominal and uncertain wind turbine. The transient responses of the state variables pertaining to nominal and uncertain wind turbine are analysed. In figure 10, the difference of transient responses of state variables with original and reduced order H_∞ controllers pertaining to wind disturbance input of $10.0*(\text{rand}(1,\text{nstep}))\text{ m/s}$ are shown

in figure 10(a), 10(b) and 10(c). It is found that with the original and reduced order controllers in the closed loop operation of the nominal and uncertain wind turbine systems, the effect of disturbance input (wind speed) on the performance is small. As seen in figure 10(d), the error of controlled pitch angle in degrees with original and reduced order H_∞ controllers is very small. Hence, the performance of the reduced order controller in achieving the pitch angle is quite similar with that of the original controller.

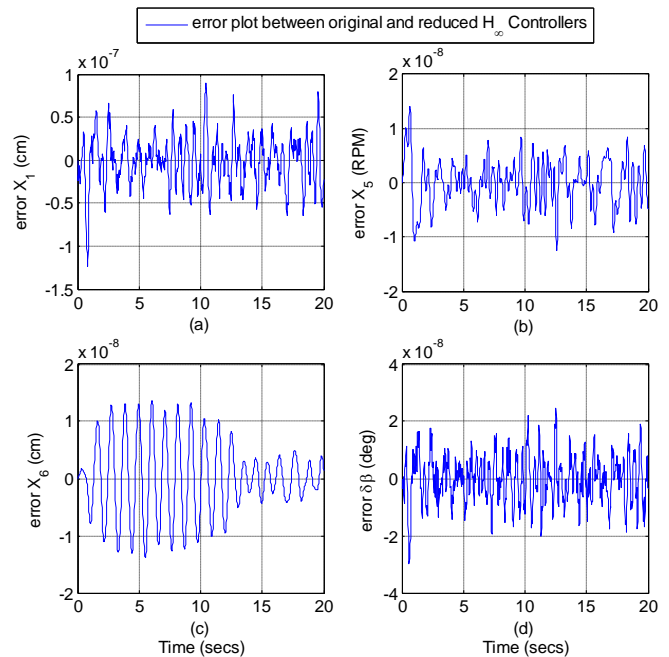


Figure 10. Error in transient responses of state variables (a-c) and controlled blade pitch angle (d) with original and reduced order H_∞ controllers pertaining to wind disturbance input of $10.0*(\text{rand}(1,\text{nstep}))\text{ m/s}$

6.2. Effect of Blade Pitch Angle Input and Random Wind Disturbance Input

In this scenario, both the inputs, i.e., the blade pitch angle control input of 18° and the random wind disturbance input of $10.0*(\text{rand}(1,\text{nstep}))\text{ m/s}$ are considered. The state variable's transient responses for the 10 samples of uncertain wind turbine are shown in figure 11(a), 11(b) and 11(c). The flap displacement, x_1 and the tower fore-aft deflection, x_6 show minimum deflection and reach the steady value close to zero with the minimum time. The generator's speed, x_5 is regulated to 42 rpm in a short duration of less than 5 seconds. figure 11(d) shows a comparison of the controlled pitch angle in degrees for original and reduced order controllers for 10 samples of uncertain closed loop wind turbine models. This is controlled below 1 second. The speed error plot between the generator and rotor is shown in figure 12. A comparison is made between the performance of the

original and reduced controllers of wind turbine uncertain (10 samples) model when the specified reference inputs i.e., pitch angle reference of 18° and speed reference of 42 rpm and the external wind disturbance input of $10.0 \cdot (\text{rand}(1, \text{nstep}))$ m/s are applied on the uncertain system. The error is less than 3 RPM and went to zero in 4 seconds.

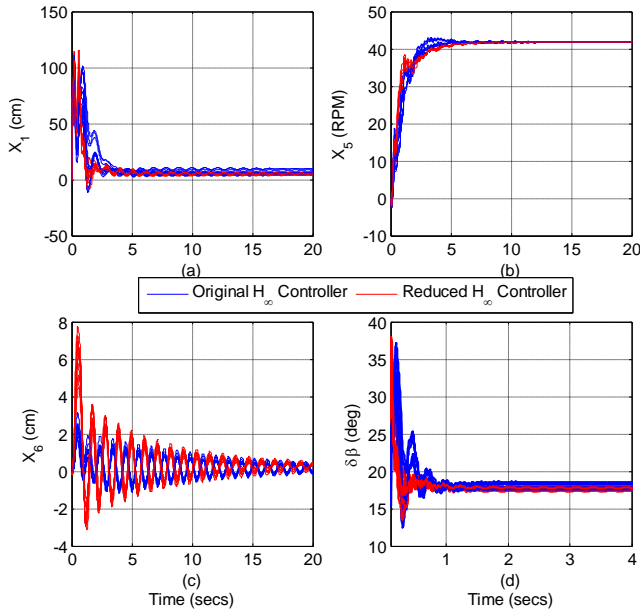


Figure 11. Transient responses of state variables (a-c) and controlled blade pitch angle (d) of wind turbine uncertain systems (10 samples) with 18° blade pitch angle, generator speed (maintained constant at 42 rpm) and wind speed disturbance of $10.0 \cdot (\text{rand}(1, \text{nstep}))$ m/s with original and reduced order H_∞ controllers

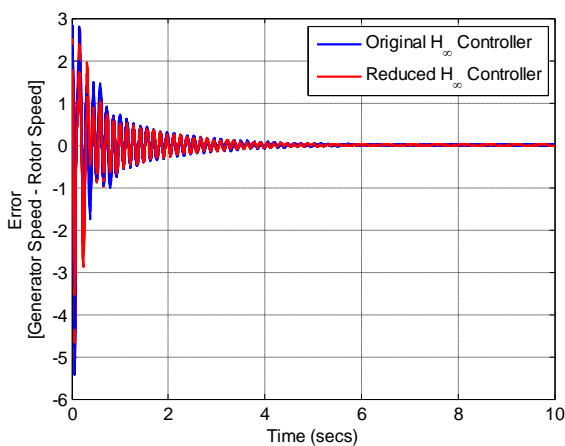


Figure 12. Speed error plot between the generator and rotor – a comparison between original and reduced controllers of wind turbine nominal model with response to reference inputs (pitch angle reference 18° and speed reference 42 rpm) and wind disturbance input of $10.0 \cdot (\text{rand}(1, \text{nstep}))$ m/s

6.3. Effect of Step Changed Blade Pitch Angle Input and Random Wind Disturbance Input

In this section the effect of step changed blade pitch angle input and random wind disturbance input on the state variable responses of uncertain system is analysed. A wind profile with maximum wind speed of 10 m/s is applied on the nominal and uncertain wind turbine along with the 18° blade pitch angle input. In figure 13, the controlled pitch angles ($14^\circ, 16^\circ, 18^\circ, 16^\circ, 14^\circ$) for uncertain models (10 samples) of wind turbine are obtained. In figure 14, the transient response of regulation of generator speed is maintained constant at 42 rpm for varied blade pitch angle in steps, i.e., $14^\circ, 16^\circ, 18^\circ, 16^\circ, 14^\circ$. Whereas figure 15(a), 15(b) and 15(c), shows the transient responses of state variables of wind turbine uncertain models (10 samples) for the step changes in the blade pitch angle inputs ($14^\circ, 16^\circ, 18^\circ, 16^\circ, 14^\circ$). The step changed variation of the blade pitch angle is shown in figure 15(d).

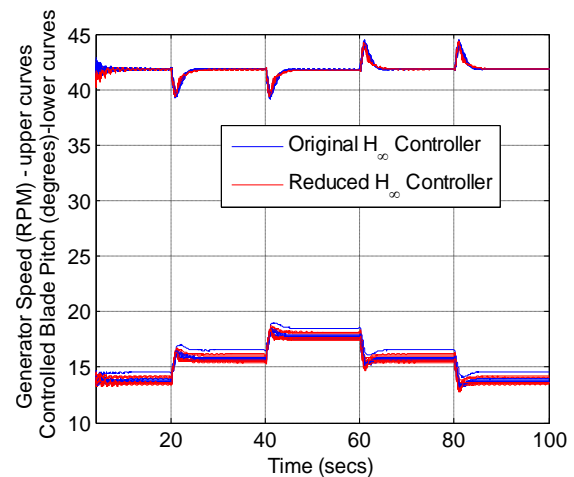


Figure 13. Controlled pitch angles ($14^\circ, 16^\circ, 18^\circ, 16^\circ, 14^\circ$) for uncertain models (10 samples) of wind turbine

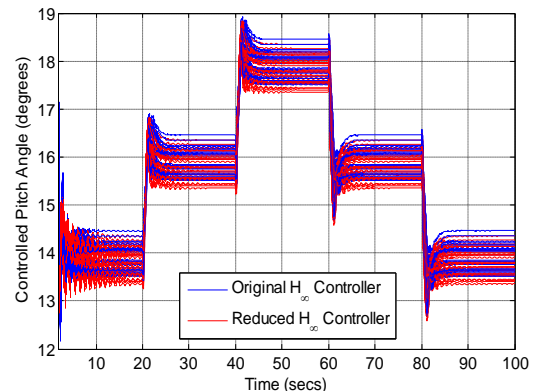


Figure 14. Transient response of regulation of generator speed (maintained constant at 42 rpm) for varied pitch angle in steps ($14^\circ, 16^\circ, 18^\circ, 16^\circ, 14^\circ$)

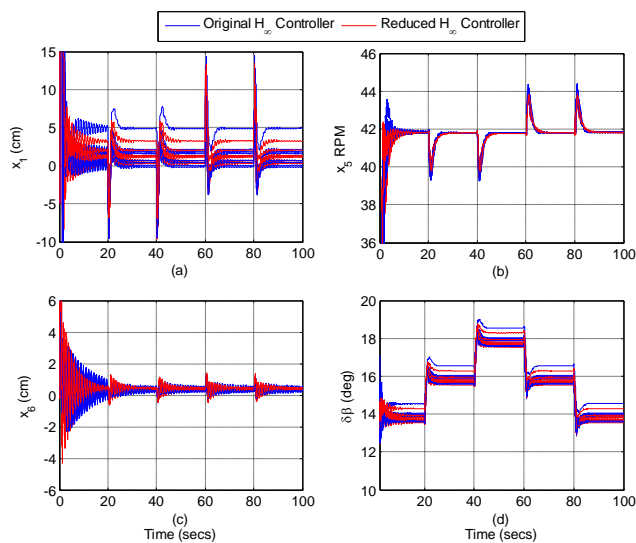


Figure 15. Transient responses of state variables (a-c) of wind turbine uncertain models (10 samples) with response to blade pitch angle (d) step changed inputs (14° , 16° , 18° , 16° , 14°)

Conclusions

An H_{∞} controller is designed for the seven state wind turbine model developed by FAST. A reduced controller is further obtained by the implementation of the balanced truncation model order reduction method on the originally designed H_{∞} controller. Upon implementation of both these controllers in the closed loop, their comparisons on most of the possible scenarios determined on any wind turbine model were satisfactorily achieved, i.e., the achieved robust performance and robust stability were found to be close enough for both the original and the reduced order controllers. In scenario 1, clear indication of the effects of random wind disturbance input of 10 m/s (a random value) on the transient responses of the state variables of the wind turbine are observed. The effects of these wind disturbances are relatively much smaller on all the considered state variables of the wind turbine. In scenario 2, the effects of a combination of the blade pitch angle input and the wind disturbance input on the desired generator speed were studied. The rated speed of 42 rpm is maintained for various pitch angles up to a maximum allowable value of 18° . Scenario 3 shows the effect of step changed blade pitch angle input and random wind disturbance input on all the state variables to obtain a satisfactory generator speed regulation of 42 rpm. The robustness of the designed H_{∞} controller and the reduced controller is evident with this measure.

Appendix A. General Specifications of the CART2 Machine [3]

- Turbine type: Horizontal axis, upwind rotor, teetering hub.

- Number of blades: 2.
- Rotor speed: 42 RPM.
- Power regulation: Full span blade pitch control.
- Rotor diameter: 43.3 m.
- Hub-Height: 36.3 m.

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