# Performance Analysis of Opportunistic Relay Multi-antenna Selection Scheme in Nakagami-m Fading Channels Based RAN for 5G

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*Abstract*—**The paper study the capability of Opportunistic Relaying (OR) in multi-antenna selection schemes on independent, Nakagami-*m* fading channels in Decode-and-Forward (DF) mode. Based on signal-to-noise ratio (SNR), we deduce the representation of error and outage probability for binary phase shift keying (BPSK) with the moment generating function (MGF). The simulation shows the theoretical study validity. Compared with single-antenna OR scheme, the proposed multi-antenna selection schemes can improve the network performance. Using beamforming and maximum ratio combining (MRC), we can prove multiple antennas at OR can obtain remarkable diversity order and system gain.**

*Keywords*—**Opportunistic Relaying (OR), Maximum Ratio Combining (MRC), Moment Generating Function (MGF).**

## I. INTRODUCTION

Cooperative diversity has attracted a lot attentions in the 5G networks [1] and in dense networks, the application of relay nodes will be more and more widespread. Although Previous study with Opportunistic Relaying (OR) focusing multiple relays has studied in [2], choosing single relay network brings system capacity loss comparing multiple relay network [3]-[4]. [3] analyzes low complexity OR and studied the performance of outage probability based on both Decode-and-Forward (DF) mode and Amplify-and-Forward (AF) mode under an total power constraint. Based on three different channel state information (CSI) scenes, [4] analyzes the outage probability with OR strategy. Previous studies have focused on single relay networks, but in practical systems, multi antenna systems have become more and more widely used.

Paper [5]-[8] had studied the multiple antennas over relay transmission. [5] analyzing the tradeoff performance of diversity multiplexing technology in multi antenna relay network when receiving MRC, and adopting beamforming technology in relay. In addition, [6] analyzes outage

antenna multi relay T-SC is analyzed [8]. In these papers, the research on multi relay network system, the multi relay only one antenna and use the decoding success forwarding mode, which brings great synchronization signaling overhead, but the performance of multi antenna relay selection research, compared to the single antenna relay systems, multi antenna relay system more complex.

Motivated by this, OR over multiple antenna relay selection is proposed. Based on instantaneous Signal-to-Noise Ratio (SNR), we can select the best relay. This paper further deduces the system performance of the relay network, including the representation of error and outage probability for binary phase shift keying (BPSK) with the moment generating function (MGF) over independent, Nakagami-*m* fading channels in Decode-and-Forward (DF) mode in multi-antenna multi-relay cooperative relay network in 5G network.

In our relay chosen scheme, different from OR scheme [4], the received signal source is decoded and the beamforming destination is forwarded with the MRC decoding signal with multiple antennas. By comparing the representation of error and outage probability, our proposed scheme with OR scheme, we show that opportunistic relay with multiple antennas can achieve significant performance improvement.

## II. SYSTEM MODEL

In the system, the source S transmits data to the destination D with $N$ relays adopting DF. The source and destination are equipped with only one antenna respectively, while each relay has multiple antennas, e.g. $n_i$ for relay $i$.

In the first hop, S broadcasts transmission. Both D and relays receive the signal. As each relay has multiple antennas, it combines the signal received from each antenna with MRC. Thus the relay $i$ of instantaneous SNR and D can be derived as follows respectively

$$\eta_{s,i} = \alpha_1 \sum_{j=1}^{n_i} \left| c_{ij} \right|^2 \tag{1}$$

$$\eta_{s,d} = \alpha_1 \left| c_{sd} \right|^2 \tag{2}$$

where $\alpha_1$, $c_{ij}$ and $c_{sd}$ represent the transmitted SNR at S, S to relay $i$'s $j$th antenna of the channel coefficient from and the S to D of the channel coefficient respectively. $\left| c_{ij} \right|$ and $\left| c_{sd} \right|$ are the Nakagami-*m* distribution with $m_{si}$, $\Omega_{si}$ and $m_{sd}$ $\Omega_{sd}$ respectively.

If a relay decodes the signal received from S successfully, it sends training sequence to D to estimate the instantaneous

channel state between itself to D in terms of SNR and CSI. Then D selects the best relay according to SNR and feedbacks the required CSI to the selected relay.

In the second hop, the best relay $i$ beamforms the signal to D. As each relay has knowledge of instantaneous CSI, the instantaneous SNR of D is represented as follows.

$$\eta_{i,d} = a_2 \sum_{j=1}^{n_i} \left| e_{ij} \right|^2 \qquad (3)$$

where $a_2$ is the sent SNR at relay, $e_{ij}$ denotes the channel coefficient between relay $i$'s antenna $j$ to D. Assuming that the channel follows Nakagami-$m$ distribution with parameters $m_{id}$ and $\Omega_{id}$.

## III. PERFORMANCE ANALYSIS

In this section, we defined $Rr=\{R_1, R_2,\ldots, R_N\}$. C is on behalf of the decoding set, and all subset can be constituted $R$. Then C is expressed as

$$O_n \overset{\Delta}{=} \left\{ R_{r_j} \in Rr : \frac{1}{2}\log_2(1+\eta_{s,R_{r_j}}) \ge R_d, \right.$$
$$\left. \sum_{j=1}^{l} 2^{r_j-1} = n-1, r_1 \ne r_2 \ne \cdots \ne r_l \right\}$$

and $n = 1,2,\cdots,2^N$, $l$ represents the number of relays $O_n$. $R_{r_j}$ as relay node is expressed successfully decoding received signal from S if $\frac{1}{2}\log_2(1+\eta_{s,R_{r_j}}) \ge R_d$. $R_d$ can be defined as spectral efficiency. Then we define $\eta_{th} = 2^{2R_d}-1$ as the threshold value of SNR.

Because of each $c_{ij}$ is random variable (RV) with Nakagami-m channel, and $\left|c_{ij}\right|^2$ is gamma distributed RV, the the moment generating function (MGF) as $Na_X(k) = E\left(e^{-kX}\right)$ (where $E$ is on the behalf of the statistical average operator) is able to express as

$$Na_{\left|c_{ij}\right|^2}(k) = \left(1+\Omega_{si}/m_{si} \, k\right)^{-m_{si}} \qquad (4)$$

Because of the independence of $\left|c_{ij}\right|^2$, $j=1,2,\ldots,n_i$, the MGF of $\eta_{s,i}$ is able to express as

$$Na_{\eta_{s,i}}(k) = \prod_{j=1}^{n_i} Na_{\left|c_{ij}\right|^2}(a_1 k) = \left(1+a_1\Omega_{si}/m_{si} \, k\right)^{-n_i m_{si}} \qquad (5)$$

Assuming that $m_{si}$, $m_{sd}$ and $m_{sd}$ are considered to be integer, we can get $\eta_{s,i}$, which is on the behalf of the cumulative distribution function (CDF) as follow

$$F_{\eta_{s,i}}(\eta) = L^{-1}\left(k^{-1}Na_{\eta_{s,i}}(k)\right)\big|_\eta = 1 - \sum_{f=0}^{n_i m_{ki}-1}\left(\frac{m_{fi}}{\Omega_{fi}a_1}\right)^f \frac{\eta^f}{f!} e^{-\frac{m_{si}}{\Omega_{si}a_1}\eta}$$

$$(6)$$

where $L^{-1}(\cdot)$ can be expressed the inverse Laplace Transform, and we can obtain the outage probability as follow

$$\Pr\{O_n\} = \left(\prod_{i\in O_n}\left(1-F_{\eta_{k,i}}(\eta_{th})\right)\right)\left(\prod_{j\in O_n}F_{\eta_{k,j}}(\eta_{th})\right) \qquad (7)$$

where $O_n$ and $\overline{O}_n$ are on the behalf of the sets composed of decoded successfully relay and decoded non-successfully relay respectively.

### A. The Performance of Outage

In our proposed scheme in DF mode, the most optimal relay $r_{sc}$ constitutes the subset of decode $O_n$, which can be selected by D with maximizing the SNR in the 2nd hop for $r_{sc} \in \Theta_n$ :

$$r_{sc} = \arg\max_{i\in O_n}\left\{\eta_{i,d}\right\} \qquad (8)$$

The CDF of $\gamma_{i,d}$ can be obtained as

$$F_{\eta_{i,d}}(\eta) = 1 - \sum_{k=0}^{n_i m_{id}-1}\left(\frac{m_{id}}{a_2\Omega_{id}}\right)^k \frac{\eta^k}{k!} e^{-\frac{m_{id}}{a_2\Omega_{id}}\eta} \qquad (9)$$

We assume $O_n$ is the CDF of $\eta_{r_{sc},d} = \max\left\{\eta_{i,d}\right\}$ can be obtained as

$$F_{\eta_{r_{sc},d}}(x) = \Pr\left\{\eta_{r_{sc},d} < \eta \mid O_n\right\} = \prod_{i\in O_n}F_{\eta_{i,d}}(\eta) \qquad (10)$$

With formula (10), the MGF of $\eta_{r_{sc},d}$ can be deduced based on [9, Eq. 3.381.4] as

$$Na_{\eta_{r_{sc},d}|O_n}(k) = k\int_0^\infty F_{\eta_{r_{sc},d}}(x)e^{-kx}dx = 1+k\sum_{t=1}^{l}\sum_{\lambda_1=1}^{l-t+1}\sum_{\lambda_2=\lambda_1+1}^{l-t+2}\cdots\sum_{\lambda_t=\lambda_{t-1}+1}^{l}(-1)^t$$
$$\times\sum_{k_1=0}^{m_{\lambda_1}-1}\sum_{k_2=0}^{m_{\lambda_2}-1}\cdots\sum_{k_t=0}^{m_{\lambda_t}-1}\Gamma(H_t+1)\left(\frac{Z_t}{\alpha_2}+s\right)^{-H_t-1}\left(\prod_{n=1}^{t}\frac{m_{\lambda_n}^{k_n}}{k_n!\alpha_2^{k_n}\Omega_{\lambda_n}^{k_n}}\right)$$
$$(11)$$

Then $t=1,2,\ldots,$ $l$ can be indicated by relay $i$ in $O_n$, $Z_t = \sum_{n=1}^{t}\frac{m_{\lambda_n}}{\alpha_2\Omega_{\lambda_n}}$, $H_t = \sum_{n=1}^{t}k_n$, $\Gamma(\cdot)$ can be defined by Gamma function in [9, Eq. 8.310].

Based on formula (11), we can be calculated from the multinomial expansion as follow

$$\prod_{t=1}^{l}(1-B_t) = 1 + \sum_{t=1}^{l}\sum_{\lambda_1=1}^{l-t+1}\sum_{\lambda_2=\lambda_1+1}^{l-t+2}\cdots\sum_{\lambda_t=\lambda_{t-1}+1}^{l}\prod_{n=1}^{t}\left(-B_{\lambda_n}\right)$$

Based on MRC, we combine the relay $i$ and received signals from S node, and at D node we can calculate the total SNR as follow

$$\eta_{tl} = \eta_{s,d} + \eta_{r_{sc},d} \qquad (12)$$

Because of $\eta_{s,d}$ can be expressed by gamma distributed RV, PDF and MGF are able to calculate as follow

$$Na_{\eta_{s,d}}(k) = \left(1+\alpha_1\Omega_{sd}/m_{sd} \, k\right)^{-m_{sd}} \qquad (13)$$

$$f_{\eta_{s,d}}(x) = \frac{1}{\Gamma(m_{sd})}\left(\frac{m_{sd}}{\Omega_{sd}a_1}\right)^{m_{sd}} x^{m_{sd}-1} e^{-\left(\frac{m_{sd}}{\Omega_{sd}\alpha_1}\right)x} \qquad (14)$$

Furthermore, $\eta_{s,d}$ and $\eta_{r_{sc},d}$ are independent, so the MGF of $\eta_{tl}$ can be written as

$$Na_{\eta_{tl}|O_n}(k) = Na_{\eta_{s,d}}(k)Na_{\eta_{r_{sc},d}|O_n}(k) \qquad (15)$$

Bring formula (11) and formula (13) into formula (15), a closed form expression can be expressed as $Na_{\eta_{tl}|O_n}(k)$. Using formula (10) and formula (14), we can obtain the conditional outage probability based on [9, Eq. 3.383.1] as formula (16), where $B(\cdot,\cdot)$ is Bata function (BF) based on [9, Eq. 8.38], $_1F_1(\cdot;\cdot;\cdot)$ is Degenerate hypergeometric function (DHF) based on [9, Eq. 9.210].

With formula (7) and formula (16), the outage probability over DF mode is able to calculate as follow

$$P_D^O(outage) = \sum_{O_n} \Pr\{outage \mid O_n\} \Pr\{O_n\} \qquad (17)$$

## B. The Performance of Error

At D node, based on formula (15), we can calculate the conditional error rate. Then, we can deduced the closed form based on [9, Eq. 3.211], [9, Eq. 9.182] and [10, Eq. 8.23].

$$P_{e,d|O_n}^B = \frac{1}{\pi}\int_0^{\pi/2} M_{\eta_d|O_n}\left(\frac{g_P}{\sin^2(\theta)}\right)d\theta \qquad (18)$$

and $g_P = \sin^2\left(\dfrac{\pi}{M}\right)$, $_2F_1(\cdot,\cdot;\cdot;\cdot)$ is GHF based on [9, Eq. 9.10].

Based on [7, Eq. 7], $P_{e,E2E}^{BPSK}$ for BPSK with our proposed scheme is able to calculate as follow

$$P_{e,E2E}^B = \sum_{O_n} \Pr\{O_n\} P_{e,d|O_n}^B \qquad (20)$$

$$\Pr\{outage \mid O_n\} = \int_0^{\eta_{th}} \prod_{i\in O_n} \Pr\{\eta_{i,d} < \eta_{th} - x\} f_{\eta_{s,d}}(x) dx = 1 - \sum_{k=0}^{m_{sd}-1} \frac{m_{sd}^k \eta_{th}^k}{\alpha_1^k \Omega_{sd}^k k!} e^{-\frac{m_{sd}\eta_{th}}{\alpha_1 \Omega_{sd}}} + \frac{m_{sd}^{m_{sd}}}{\alpha_1^{m_{sd}} \Omega_{sd}^{m_{sd}} \Gamma(m_{sd})} e^{-\frac{m_{sd}\eta_{th}}{\alpha_1\Omega_{sd}}} \sum_{t=1}^{l}\sum_{\lambda_1=1}^{l-t+1}\sum_{\lambda_2=\lambda_1+1}^{l-t+2}\cdots\sum_{\lambda_t=\lambda_{t-1}+1}^{l}(-1)^t$$

$$\times \sum_{k_1=0}^{n_{\lambda_1}m_{\lambda_1}-1}\sum_{k_2=0}^{n_{\lambda_2}m_{\lambda_2}-1}\cdots\sum_{k_t=0}^{n_{\lambda_t}m_{\lambda_t}-1} \eta_{th}^{m_{sd}+H_t}\left(\prod_{n=1}^{t}\frac{m_{\lambda_n}^{k_n}}{k_n!\alpha_2^{k_n}\Omega_{\lambda_n}^{k_n}}\right) B(m_{sd},H_t+1)\, _1F_1\left(H_t+1;m_{sd}+H_t+1;\frac{m_{sd}\eta_{th}}{\alpha_1\Omega_{sd}}-\frac{Z_t\eta_{th}}{\alpha_2}\right) \qquad (16)$$

$$P_{e,d|O_n}^B = \frac{1}{2\pi}\left(\frac{m_{sd}}{\alpha_1\Omega_{sd}g_P}\right)^{m_{sd}} B\left(\frac{1}{2},m_{sd}+\frac{1}{2}\right)\, _2F_1\left(m_{sd},m_{sd}+\frac{1}{2};m_{sd}+1;-\frac{m_{sd}}{\alpha_1\Omega_{sd}g_P}\right) + \frac{1}{2\pi}\sum_{t=1}^{l}\sum_{\lambda_1=1}^{l-t+1}\sum_{\lambda_2=\lambda_1+1}^{l-t+2}\cdots\sum_{\lambda_t=\lambda_{t-1}+1}^{l}\sum_{k_1=0}^{m_{\lambda_1}-1}\sum_{k_2=0}^{m_{\lambda_2}-1}\cdots\sum_{k_t=0}^{m_{\lambda_t}-1}(-1)^t$$

$$\times \prod_{n=1}^{t}\frac{m_{\lambda_n}^{k_n}}{k_n!\alpha_2^{k_n}\Omega_{\lambda_n}^{k_n}}\frac{g_P^{\frac{1}{2}}m_{sd}^{m_{sd}}\Gamma(H_t+1)\alpha_2^{H_t+m_{sd}+\frac{1}{2}}}{(\alpha_1\Omega_{sd})^{m_{sd}}(\alpha_2 g_P+Z_t)^{H_t+m_{sd}+\frac{1}{2}}} B\left(\frac{1}{2},H_t+m_{sd}+\frac{1}{2}\right)\, _2F_1\left(H_t+m_{sd}+\frac{1}{2},m_{sd};H_t+m_{sd}+1;\frac{\alpha_1\Omega_{sd}Z_t-\alpha_2 m_{sd}}{(\alpha_2 g_P+Z_t)\alpha_1\Omega_{sd}}\right) (19)$$

## IV. NUMERICAL RESULTS

The DF mode relay network with 3 relays and $n$ antennas is considered. We assume the same index of Nakagami-$m$ channel for the two hops, i.e, $m_{si}=m_{id}=m$, $\Omega_{si}=\Omega_{id}=1$, $i=1,2,3$. In the direct link, we set $m_{sd}=1$ and $\Omega_{sd}=1$.

In Fig. 1, the outage probability for our proposed scheme to SNR ($\rho=10\lg(\rho_1+\rho_2)$) with $\rho_1=\rho_2$, and we assume ($m=1,2,3$) and $n$ ($n=1,2,3$) are presented. The simulation results show that the performance analysis results agree well with the corresponding numerical results, and further verify the correctness of the outage probability performance analysis in this paper. In relay nodes, we set the number of antenna is 1 both in our proposed scheme and OR. The simulation results show that the performance of the proposed algorithm increases with the increase of the total signal to noise ratio (SNR). We use MRC and beamforming technology, and with the increase of SNR, for the same $m$ the algorithm proposed in this paper will bring greater performance gain than the OR scheme. Therefore, compared to OR scheme, our proposed scheme is able to get more and more both diversity and array gain.

In Fig. 2, for BPSK modulation BER performance was shown. In the same way, the derivation of the performance analysis agrees well with the results obtained by numerical simulation. The simulation results show that the performance of the proposed algorithm increases with the increase of SNR. We can draw similar conclusions from the perspective of BER, and the simulation results show that the performance of the BER has been significantly improved.
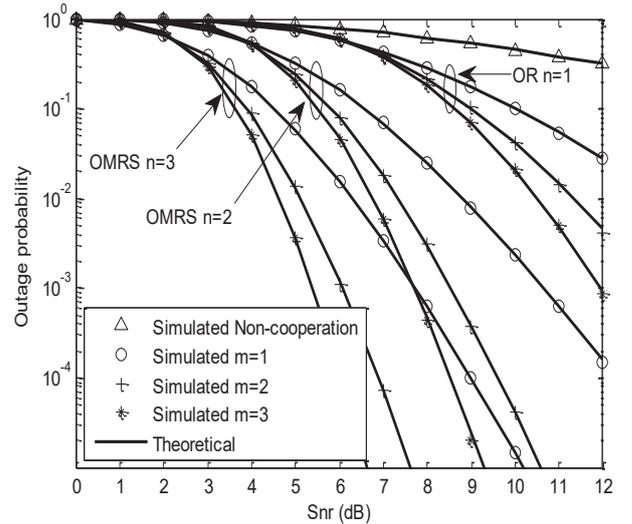


Fig. 1. Outage probability of SNR $\rho$.
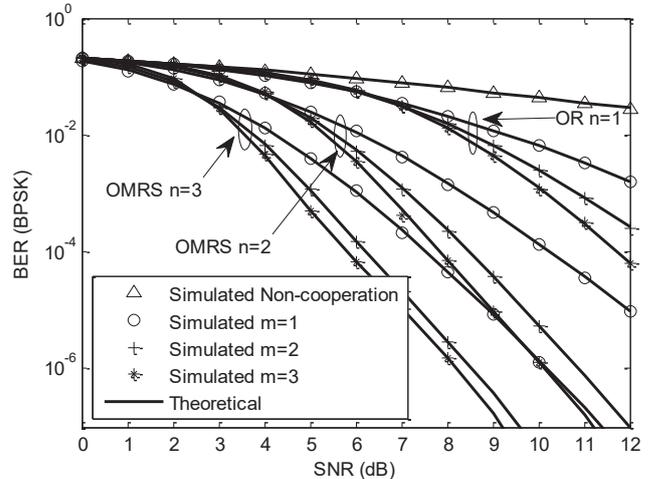


Fig. 2. BER performance of SNR $\rho$.

## V. Conclusion

In this paper, we derive the performance of relay systems over Nakagami-m fading channels, including expressions for outage probability, and BER in the presence of total power constraints in DF mode for BPSK modulation.The analytical derivations further demonstrate the correctness of the outage probability and BER performance. Finally, the proposed multiple antenna relay scheme has ramarkable improved outage probability and error performance compared with single antenna schemes. This illustrates that using beamforming and maximum ratio combining (MRC), we can prove multiple antennas at OR can obtain remarkable diversity order and system gain.

## VI. Acknowledgments

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