Impulsive Noise Suppression for Single-Carrier Power Line Communication based on Turbo Equalisation

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ABSTRACT

The impulsive noise suppression problem for single-carrier power line communication based on turbo equalisation is investigated. In this work, the sparse property of the impulsive noise in time domain is utilized to actively suppress the noise. Based on this idea, an optimization framework is developed to estimate the impulsive noise, which is effectively solved by the alternating direction method of multipliers (ADMM). For the power line communication, due to the low complexity and low peak-to-average power ratio (PAPR), the single-carrier frequency-domain equalization is used in this work. Numerical studies demonstrate the superior performance of the proposed approach.

KEYWORDS

Impulsive noise, sparsity, ADMM, power line communication.

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1 INTRODUCTION

The power line communication (PLC) [4] as a means to transmit signals has been widely studied due to the universal existence of power lines. However, power line is originally designed to deliver electrical power, and therefore, signal transmitted through PLC channel suffers from strong impulsive noise (IN) interference. The IN usually is time-varying with random occurrences and duration lasting from microseconds to milliseconds. In the time domain, its amplitude is relatively large that may exceed the background noise by more than 50 dB [11]. Therefore, to successfully receive the intended information, the IN must be effectively suppressed.

In order to overcome the detrimental impact of IN and to improve the reliability of data communication under practical PLC systems, IN mitigation step is urgently needed. For orthogonal frequency division multiplexing OFDM [1] based PLC system, the receiver front-end IN processor [10] and the equaliser back-end IN processor [9] are developed. The IN mitigation schemes based on receiver front-end nonlinearity pre-processors have simple system structures and mathematical representations, which make them easy to be realized. However, signal symbols with high amplitude causes incorrect triggering of the clipping or blanking processor leading to significant performance deterioration. For the equaliser back-end IN mitigation approach, by first subtracting data signal from the received signal, it alleviates the problem of incorrect thresholding triggers. When the IN is strong, the estimate of signal obtained via the frequency-domain equaliser is not accurate anymore because of the spreading of frequency-domain IN symbols. It seems that the single-carrier based scheme is a more favorable choice to cope with the IN in the PLC systems.

In this work, the IN suppression problem is discussed for single-carrier power line communication based on turbo equalisation. Due to the impulsive nature of the noise, its sparse property is observed in the time domain [7]. Based on this observation, an optimization framework is devised to suppress the impulsive noise. To efficiently obtain the solution, alternating direction method of multipliers (ADMM) based solver is developed. Compared with the traditional approaches, the threshold decision is not required and that information is usually hard to obtain in practice.

2 PROBLEM FORMULATION

In power line communication system, the received signal at receiver is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{i} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ denotes the channel matrix, $\mathbf{x}$ is the frequency-domain transmitted signal, $\mathbf{n}$ and $\mathbf{i}$ respectively represent background noise and IN. The goal of the IN suppression is to remove the IN, i.e., $\mathbf{i}$, from the received signal $\mathbf{y}$. In this work, shown in Figure 1, there are two important parts constituting the novel receiver for PLC, namely the IN suppression module and the single carrier modulation with frequency domain turbo equalisation (SC-FDTE). Seeing the Figure 1, those two parts feed information to each other, which means the estimate of the transmitted signal via the turbo equalisation is input to the IN suppression module, and in turn, the IN suppression module supplies its output to the turbo equaliser. Therefore, with the increase of iterations, both the turbo equalisation and the IN mitigation are improved, thereby IN can be mitigated iteratively. For more information on the SC-FDTE, the interested readers are referred to [8]. In what follows, the IN mitigation step is developed.

To obtain the accurate estimation of impulsive noise, the following optimization problem is devised by utilizing signal sparse property

$$\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}} - \mathbf{i}\|_2 + \lambda\|\mathbf{i}\|_0, \tag{2}$$

where $\|\cdot\|_0$ is ℓ_0-norm that is known to promote sparse solutions [3, 6]. To efficiently obtain the solution in (2), the convex relaxation is usually performed since ℓ_0-norm is NP hard. To utilize convex relaxation, ℓ_1-norm is used to replace the ℓ_0-norm. That is,

$$\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}} - \mathbf{i}\|_2 + \lambda\|\mathbf{i}\|_1, \tag{3}$$

By solving (3), one achieves the objective of impulsive noise suppression. In what follows, an approach based on ADMM is developed to solve the optimization problem.

3 APPROACH BASED ON ADMM

To efficiently solve the optimization problems of (3), in this paper, the ADMM approach [2] is utilized. The ADMM is devloped to solve the following optimization problem

$$\begin{aligned} &\text{minimize } F(x) + G(z) \\ &\text{subject to } \; Ax + Bz = c, \end{aligned} \tag{4}$$

with variables x and z. To solve (4), the ADMM is devised in which the dual ascent and method of multipliers are utilized recursively. For ease of references, the general ADMM steps are provided in Table 1.

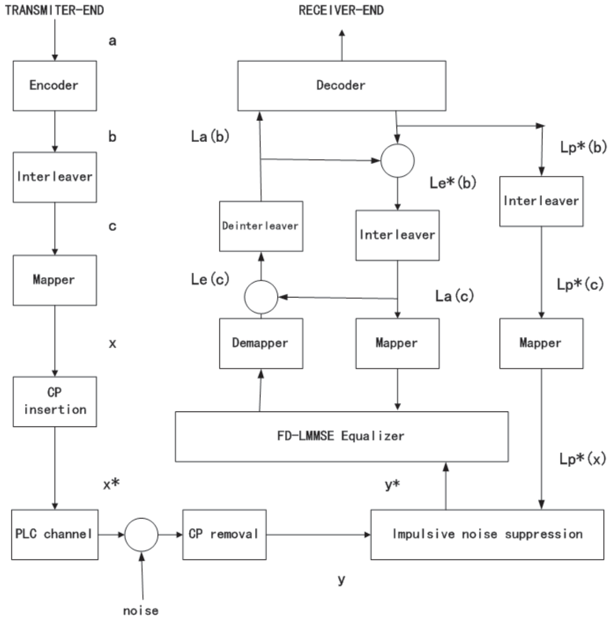

Figure 1: Block diagram of single carrier modulation which combines frequency domain turbo equalisation with the proposed iterative IN mitigation.

To apply ADMM approach to the problem at hand, the estimation of $\mathbf{i}$ is rewritten as

$$\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}} - \mathbf{i}\|_2 + \lambda\|\mathbf{i}\|_1, \tag{5}$$

with variable $\mathbf{i}$. For the problem in (5), the ADMM steps to estimate the $\mathbf{i}$ at lth iteration are

$$\begin{aligned} \mathbf{i}^{l+1} &= \text{minimize}_{\mathbf{n}}\big(\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}} - \mathbf{i}\|_2^2 + (\rho/2)\|\mathbf{i} - \mathbf{z}^l + \mathbf{s}^l\|_2^2\big) \\ \mathbf{z}^{l+1} &= \text{minimize}_{\mathbf{z}}\big(\lambda\|\mathbf{z}\|_1 + (\rho/2)\|\mathbf{i}^{l+1} - \mathbf{z} + \mathbf{s}^l\|_2^2\big) \\ \mathbf{s}^{l+1} &= \mathbf{s}^l + \mathbf{i}^{l+1} - \mathbf{z}^{l+1}. \end{aligned} \tag{6}$$

In the $\mathbf{i}$-step of (6), to obtain the solution, setting the derivative of the cost function with respect to $\mathbf{i}$ to zero yields

$$-2\mathbf{H}^{\mathrm{H}}\mathbf{y} + 2\mathbf{H}^{\mathrm{H}}\mathbf{H}\mathbf{i} + \rho(\mathbf{i} - \mathbf{z}^l + \mathbf{s}^l) = 0. \tag{7}$$

Rearranging the terms for variable $\mathbf{i}$ in (7), one obtains

$$(\rho\mathbf{I} + 2\mathbf{H}^{\mathrm{H}}\mathbf{H})\mathbf{i} = \rho(\mathbf{z}^l - \mathbf{s}^l) + 2\mathbf{H}^{\mathrm{H}}\mathbf{y}. \tag{8}$$

From (8), the estimate of $\mathbf{i}$ is written in a closed-form solution as

$$\mathbf{i} = (\rho\mathbf{I} + 2\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}(\rho(\mathbf{z}^l - \mathbf{s}^l) + 2\mathbf{H}^{\mathrm{H}}\mathbf{y}). \tag{9}$$

In the $\mathbf{z}$-step of (6), from the subdifferential calculus, the estimate of $\mathbf{z}$ is obtained by componentwise soft thresholding as

$$\mathbf{z}^{l+1} = \mathbf{T}_{\lambda/\rho}\big(\mathbf{i}^{l+1} + \mathbf{s}^l\big), \tag{10}$$

Table 1: ADMM Steps to solve (4).

```
**Objective function**: minimize $F(x) + G(z)$ subject to $Ax + Bz = c$
**Outputs**: Estimates of $x$ and $z$
**Initialization**: $l = 1$
**Repeat**
    $l = l + 1$
    $x^{l+1} = \text{argmin}_x L_\rho(x, z^l, y^l)$
    $z^{l+1} = \text{argmin}_z L_\rho(x^{l+1}, z, y^l)$
    $y^{l+1} = y^l + \rho(Ax^{l+1} + Bz^{l+1} - c)$,
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where the soft thresholding operator $\mathbf{T}$ is given by

$$\mathbf{T}_{\lambda/\rho}(a) = \begin{cases} a - \lambda/\rho, & a > \lambda/\rho \\ 0, & |a| < \lambda/\rho \\ a + \lambda/\rho, & a < -\lambda/\rho. \end{cases} \tag{11}$$

It is well documented that soft thresholding is the operator of the ℓ_1 norm [2].

4 NUMERICAL STUDIES

The results of numerical studies are presented in this section to demonstrate the performance of the proposed method. In this simulation, the encoder is a rate-1/2 convolutional encoder with generator (5, 7) that is initialized to an all-zero state, the interleaver applies random interleaving and the mapper utilizes the quadrature phase-shift keying Gray mapping. The Bernoulli-Gaussian model [5] is used as the noise model, which provides a straightforward physical characteristic of IN with a simple mathematical representation. Furthermore, in the Bernoulli Gaussian model, the fraction of the variances between the IN and the background noise is represented as $\mu = \sigma_i^2/\sigma_n^2$, where σ_i^2 and σ_n^2 respectively indicate the variances for the IN and the background noise. In this simulation, the IN has a form corresponding to a Gaussian process with an occurrence probability of $\mathbf{p}$. In the simulations, the IN occurrence probability $\mathbf{p}$, the ratio between the IN and the background noise μ, and the optimization parameter λ are chosen as 0.05, 10^4 and 0.1 respectively. Note that the large μ indicates a severe impulsive noise situation. For comparison purposes, the results from LS-Thresholding and MMSE-Threshoding [8] are also provided, and the parameters of them are set to be the best. It is worthy mentioning that for LS-Thresholding and MMSE-Threshoding approaches, the $\mathbf{p}$ needs to known to perform threshoding, and its correctness plays an important factor in their final performances.

In Figure 2, we would like to showcase the ability of the proposed method that estimates the impulsive noise. It is observed that the estimated impulsive noise matches the true one well, which in turn affects positively the decoding performance. In Figures 3 and 4, BER performances of our proposed algorithm with the LS-Thresholding and MMSE-Thresholding IN-EC algorithms based on SC-FDTE PLC systems under different SNRs and different values of μ are respectively provided. From Figure 3, the proposed algorithm outperforms the LS-Thresholding and MMSE-Threshoding in different SNRs and has a faster decline rate in BER with the increase of SNRs, even without the prior knowledge of the $\mathbf{p}$. When the occurrence probability of impulsive noise $\mathbf{p}$ is unknown, the performances of the LS-Thresholding and MMSE-Threshoding degrade significantly. In Figure 4, when the noise level increases, the BERs of all the approaches grow as well. However, the proposed method exhibits the least increases and it outperforms others consistently. When the ratio μ is high, say 10^5, the performance gap is really noticeable because of the excellent noise cancelation performance provided by the proposed approach.

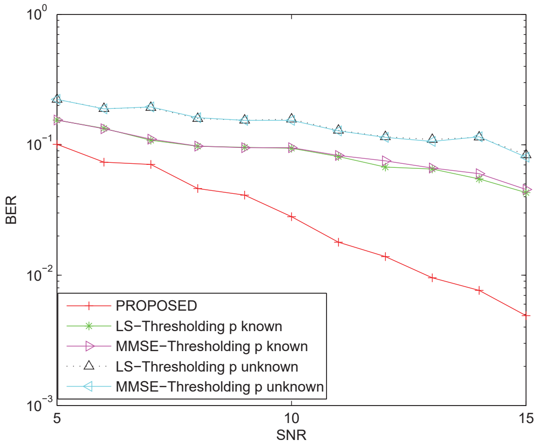
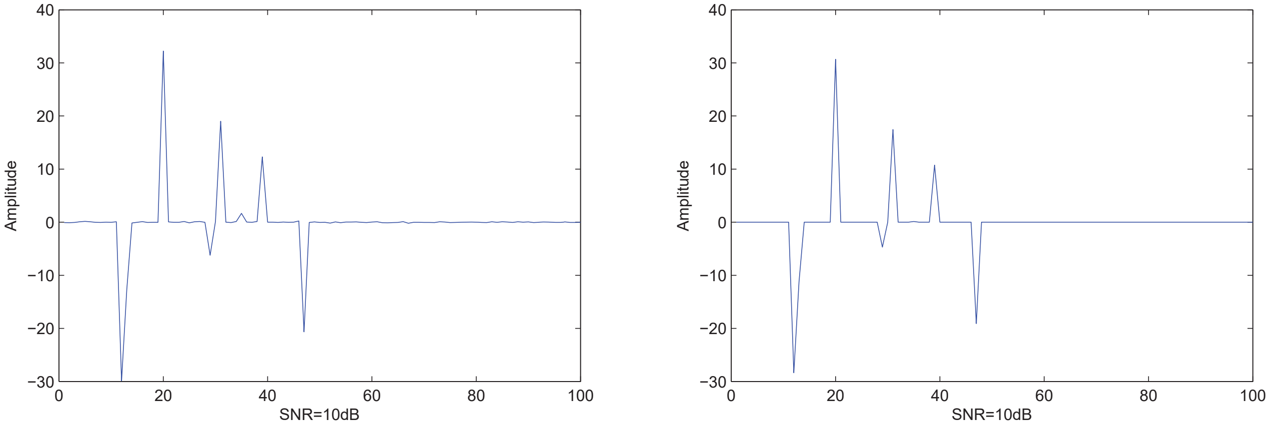


Figure 3: BER performance comparison of different algorithms versus different SNRs.

The convergence rates of different approaches under different levels of SNRs are presented in Figure 5. It is obvious that all the approaches converge fast when the SNR is increased. In the case of high SNR, say 12 dB, the proposed method only requires 4 iterations to converge and after convergence, it produces the lowest BER among the competitive approaches. When SNR is low, the proposed approach does require more iterations to converge. However, even before convergence, the proposed method still outperforms others in terms of BER performance.



(a) The original noise(impulsive noise plus background noise) **(b) The reconstruction of impulsive noise**

Figure 2: Noise reconstruction by the proposed method.

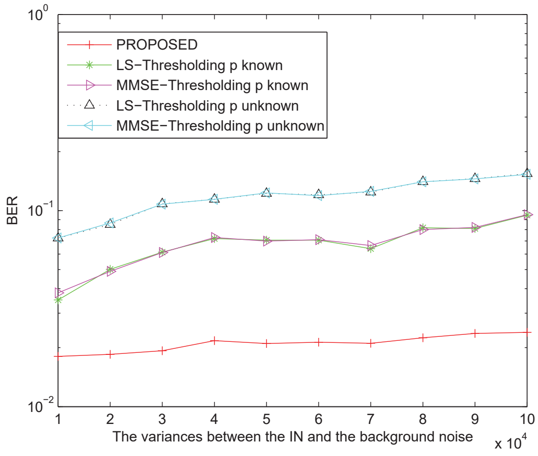


Figure 4: BER performance comparison of different algorithms versus different values of μ.

5 CONCLUSION

In this work, the problem of impulsive noise suppression in power line communication is studied. To effectively suppress the noise, its sparse property in time domain is utilized. Based on the spare property of the noise, an optimization problem is developed in order to reduce the noise. This optimization problem is solved efficiently by the ADMM approach. Numerical studies demonstrate that the proposed approach offers great performance improvements compared with other state-of-the-art algorithms.

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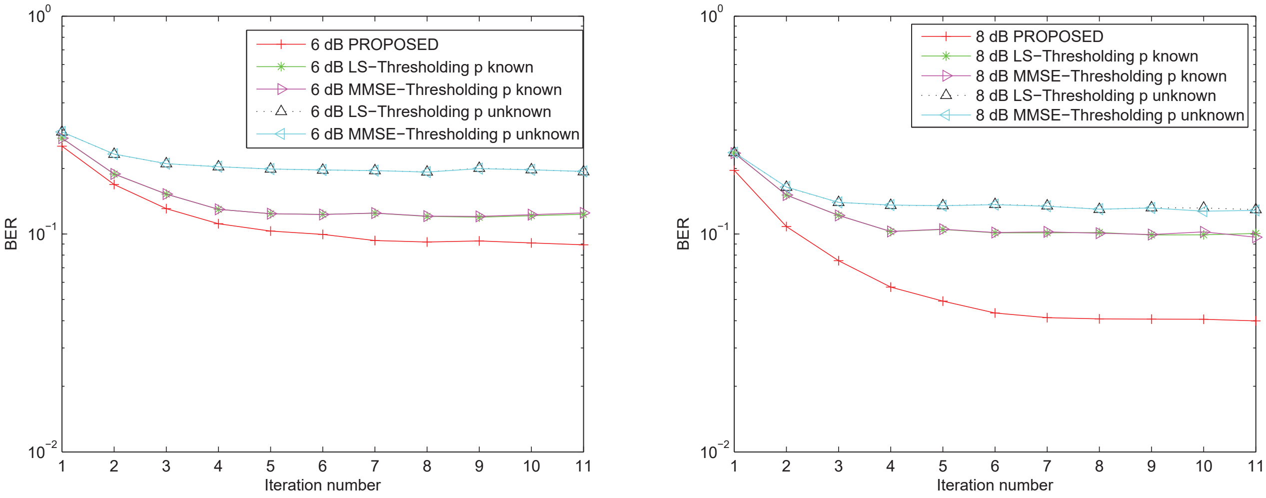
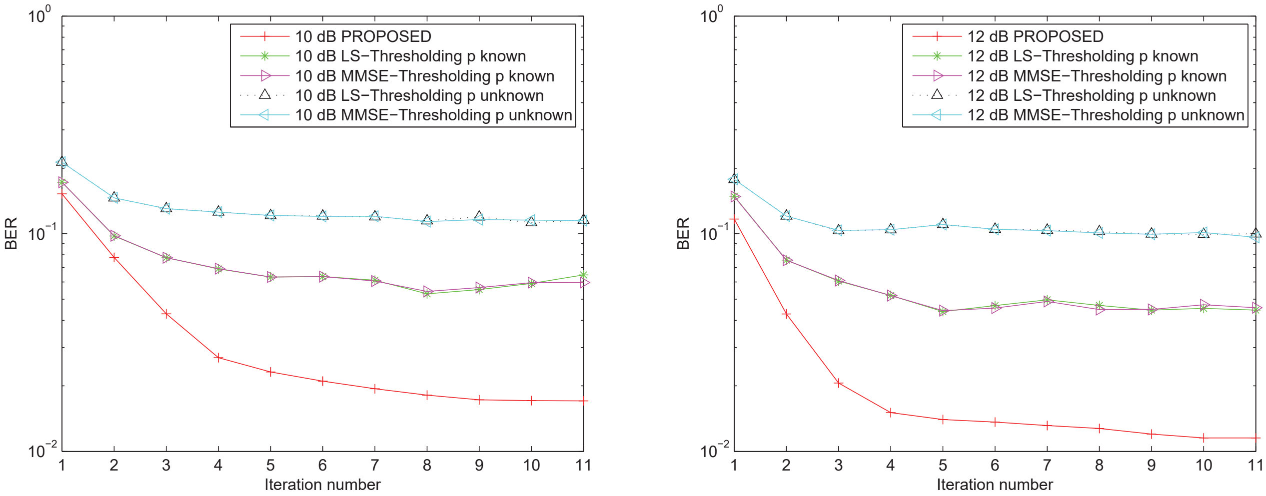


(a) Convergence rate at 6 dB **(b) Convergence rate at 8 dB**



(c) Convergence rate at 10 dB **(d) Convergence rate at 12 dB**

Figure 5: Convergence rates of the different algorithms versus different SNRs.

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