Hypothetical Learning Trajectory: Whole Number Multiplication in Primary School

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Abstract. Piaget theory has been considerably influential in learning design. The learning paradigm has transformed from teacher-centred into student-centred learning. This implies that students become the main factor and must be considered in creating a learning design. Hypothetical learning trajectory is formulated as the guideline for learning implementation and an anticipatory action toward problems possibly experienced by the students in the learning process. This article presents exemplification of formulating hypothetical learning trajectory for the learning of whole number multiplication concept in primary school.

Keywords: hypothetical learning trajectory, whole number multiplication.

1 Introduction

In Indonesia, multiplication is a mathematical operation competency introduced to Grade 2 of elementary school students. In general, multiplication learning is carried out directly at the formal stage. In the learning activities, the teacher usually explains the concepts informatively, gives examples of questions, then gives practice questions. Such learning activities do not accommodate the development of students' abilities in problem-solving, reasoning, connection, and mathematical communication. As a result, high-level cognitive abilities of students are considered miserable because the usual learning activities only encourage students to think at a low level.

If the way of teaching is less varied as above, then learning becomes less meaningful and not even meaningful for students. Students will not understand the multiplication concept taught by the teacher. Learning activities that are only communicated by the teacher to students through one direction can be analogized as pouring water into a glass. Students only memorize concepts and are less able to use the concept if they encounter problems in real life related to the concepts they have. This can be seen from the results of research conducted in Grade 3 of elementary school. It turned out in working on the multiplication practice questions, there are some students who did it fast, some who did it slowly, some who found difficulties, and even some still misunderstood the concept of multiplication. For example, when students were instructed to solve the problem "There are seven boxes containing cakes. Each box contains three cakes. How many cakes are there? ... "

In solving the problem, some students used their best method by using a counting tool as a model. Students drew 7 boxes containing 3 small dots in each box, then counting it one by one (1, 2, 3, 4, 5, 6, ..., 21). There were also those who counted it three by three (3, 6, 9, 12,

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15, 18, 21) so that the answer was the number of small dots in the entire box, which was 21. It can be seen in the picture below:

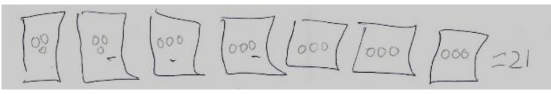

Fig.1. Student Worksheet 1

Based on the picture above, several students saw it as 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21. Furthermore, other students who were familiar with multiplication answered the question with 7 x 3 = 21. In other cases, when students were instructed to complete a multiplication sentence ... x ... = 28, students could generally answer the question correctly, which were 4x7 = 28, 7x4 = 28, 1x28 = 28, and 2x14 = 28. However, when instructed to describe a 4 x 7 multiplication operation = 28 there were still many students who did it incorrectly, as in the picture below:

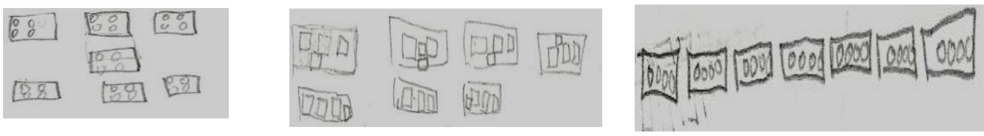

Fig.2. Student Worksheet 2

Based on the three images above, it can be seen that students were able to answer 4x7 = 28 because they memorized it and not because they understood (rote memorization), which was the process of memorizing information or rules without an understanding of principles or meaning contained in that information or rules. In this case, students had memorized 1-10 multiplication (*Raraban* 1-10) so that the answer could be immediately written.

Students' strategies for learning situations are very varied. There are various paths or thinking processes of students in understanding mathematics [1]. Clearly, each child has varied development of counting mentality. It is possible to introduce them to various representations that will help develop their strengths and accommodate their limitations. Every student has obtained their level of achievement through different routes in building the skills they have, the attitudes they cultivate, and modes of thinking that they make.

These strategies are one stage of the learning trajectory. Learning trajectory is a path or learning route that describes the learning transformation resulting from participation in mathematics learning activities. The activity of learning mathematics through the stages proposed by Jean Piaget, a famous psychologist from Switzerland, explains how people think, understand and learn. Piaget believes that intelligence is a cognitive or mental process used by children to acquire knowledge. He stated that every child has a cognitive structure called schemata, which is a concept system that exists in mind as a result of understanding objects in their environment. In this way, children can gradually develop knowledge through interaction with their environment. Based on this, the learning behaviour of children is strongly influenced by aspects of themselves and their environment. These two things cannot be separated because the learning process indeed takes place in the context of children's self-interaction with their environment.

Activities that do not limit the way students succeed on one route are key to planning learning experiences that stimulate student thinking. Sarama and Clements (2009) assert that "understanding the level of thinking of the class and individuals in that class is key in serving

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[1] Donovan, M. S., & John, D.B. (Eds). (2005). *How students learn: history, mathematics, and science in the classroom.* Washington, D. C: The National Academies Press.

the needs of all children". Effective learning requires teachers to meet student needs and help build students' knowledge. Hence, teachers should understand learning trajectories, how children think and learn mathematics, and how to help children learn better.

2 Discussion

a. Mathematical Learning Trajectory

The term learning trajectory is first used by Martin A. Simon who proposes the concept of a hypothetical learning trajectory. There are three main components in the hypothetical learning theory, which are learning goals, learning activities, and hypothetical learning processes. Learning goals as the first component indicate the need to formulate learning objectives as a form of results to be achieved after the learning process. Determination of learning objectives is very useful in determining the direction and learning strategies that will be used. Based on the learning objectives that have been formulated, learning activities as a 'way' to achieve learning goals can be designed. Learning activities are arranged into several sub-activities with sub-objectives of learning. The last component is a hypothetical learning process that is useful for designing alternative actions or strategies to overcome various problems that may be faced by students in the learning process. Learning trajectory is children's thinking when they learn to achieve specific goals in mathematical concepts through a series of instructional tasks designed to generate mental processes or hypothesized actions to lead children through the improvement of children's thinking development. Teachers should understand how students think and learn mathematics, and how to help them learn better.

b. Hypothetical Learning Trajectory in Multiplication Concept

The change in the learning process paradigm that was once teacher-oriented has now turned into student-oriented. This means that student factors become the main thing and must be considered in making a learning design. The ideal learning process cannot be separated from the planning and design processes of learning. Lesson plans are one of the concrete forms of planning and design processes. However, in reality, a lesson plan only contains formalities in the form of a 'standard package' of learning, namely a brief description of pre-activities, core activities, and post-activities. Teacher hardly ever prepares alternative hypotheses for problem-solving strategies used by students. The existence of alternative hypothesis strategies used by students will help the teacher in determining strategies for handling the possible difficulties faced by students.

The importance of the hypothetical learning trajectory can be analogized as planning travel routes. If the possible routes to the destination are known and understood, then good routes can be chosen. Also, the problems faced on the way can be solved if the routes are known or understood. For example, running out of fuel can be anticipated if the position of the gas station is known. On the other hand, the development of the model is very important to bring students informal knowledge (students' initial capital formed through experience-based activities) towards formal mathematical concepts (as the ultimate goal of mathematics learning). As stated by Simon (1995), there are three main components in the learning trajectory, namely learning goals, learning activities, and hypothetical learning processes.

a. Learning Objectives

If referring to the curriculum issued by the Indonesian Ministry of Education and Culture, multiplication is a mathematical counting operational competency for Grade 2 of elementary school students. These competencies as mandated in the Elementary School (*Sekolah Dasar*, SD)/Islamic Elementary School (*Madrasah Ibtidaiyah*, MI) Standards are "resolving everyday arithmetic problems as the application of understanding to the effects of addition, subtraction, multiplication, and division"[2]

b. Learning Activities

Based on the formulated learning objectives, learning activities can be designed. However, what must be done before designing a learning activity is to understand the unity of the multiplication concept as a whole so that the sequence or stages of learning activities are by the basic concept of multiplication.

c. The hypothesis of Student Learning Process

One very important element of the Hypothetical Learning Trajectory is the hypothesis of the student learning process. When making learning designs, the teacher should arrange hypothesis (expectations) or student reactions at each stage of learning. In the early stages of learning planning, the hypothesis is based on estimates of the initial knowledge (prior knowledge) that students already have and based on previous experience or practice of multiplication learning[3]. The following is one example of student learning hypothesis in learning multiplication by counting many objects.

When students count a lot of things such as the number of candies, marbles, or beads, students take the objects to say how many things in total and how to count them. Characteristics of marbles, corn kernels, or bead necklaces are concrete and easy to operate so that students do not experience problems in counting objects using candy or beads. For example, when operating, there are students who count one by one, two by two, or three by three. Some are irregular (one by one, two by two, one more, three, and so on). To overcome this, the teacher can ask the question: "What do you do to find out the number of candy in the plastic? How do you know that there are 30 sweets? "Or how to do a shorter/faster way to find 10 or 20 sweets in plastic? Furthermore, the teacher can invite students to take objects regularly (for example, if the first one takes one, then the next one must also be one. If two, then the next is also two, and so on). Then the teacher can ask several questions like "If you take two pieces of candy at once and you do it four times, then how many sweets do you take as the total? ..etc.

taking 2 candies once = 2; taking 2 candies twice= 2+2 = 4
taking 2 candies four times = 2+2+2+2=8 and so on.;
taking 4 candies once = 4; taking 4 candies twice = 4+4 = 8

Next, the students are directed into the multiplication concept, by removing the word **taking,** and the word **candy** and only use the word **times**, as follows.

1 **times** 2 = 2 2 **times** 2 = 4 = 2+2 = 4. And so on.
1 **times** 4 = 4 2 **times** 4 = 4+4 = 8

Explain to the students that the word in the mathematical operation has a symbol (sign) in the same as the words **plus** (+), **minus** (-), and **equal** (=). Meanwhile, the symbol for **times** is (x or .) So the child can replace the word "**times**" with a mathematical symbol like:

1x2 = 2	or 1.2 = 2	2x2 = 2+2 = 4	or 1.2 = 2+2=4 and so on.
1x4 = 4	or 1.4 = 4	2x4 = 4+4 = 8	or 2.4 = 4+4 = 8 and so on.

Based on the above activities, it was expected that students could understand the concept of multiplication and can distinguish 2 x 4 and 4 x 2 multiplication. If needed, to improve

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2 Indonesian Minister of Education and Culture's Regulation Number 21 of 2016 Concerning Standard Content for Primary and Secondary Education.

3 Gravemeijer, K. (2004). "Local Instruction Theories as Means of Support for Teachers in Reform Mathematics Education". *Mathematical Thinking and Learning*, 6(2), 105-128.

understanding, ask students to do it several times. After that, emphasize that 2x4 has the same RESULT but not the same MEANING as 4x2.

Conclusions

Based on a brief description of the hypothetical learning trajectory, the following conclusions have been drawn.

1. Hypothetical learning trajectory can provide understanding to the teacher about how important it is to pay attention to the students' initial knowledge and also the differences in students' abilities in composing learning designs.
2. Hypothetical learning trajectory can be used as a teacher's guide in dividing the stages of learning, namely by making several sub-learning goals to achieve the main learning goals.
3. Hypothetical learning trajectory is useful as a guide to the implementation of learning while providing various alternative strategies or scaffolding to help students overcome difficulties in understanding the concepts learned.

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