Products, Coproducts and Universal Properties of Autonomic Systems

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Abstract

Self-* is widely considered as a foundation for autonomic computing. The notion of autonomic systems (ASs) and self-* serves as a basis on which to build our intuition about category of ASs in general. In this paper we will specify ASs and self-* and then move on to consider products, coproducts and some universal properties of ASs. All of this material is taken as an investigation of our category, the category of ASs, which we call **AS**.

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1. Introduction

Autonomic computing (AC) imitates and simulates the natural intelligence possessed by the human autonomic nervous system using generic computers. This indicates that the nature of software in AC is the simulation and embodiment of human behaviors, and the extension of human capability, reachability, persistency, memory, and information processing speed. AC was first proposed by IBM in 2001 where it is defined as

"Autonomic computing is an approach to selfmanaged computing systems with a minimum of human interference. The term derives from the body's autonomic nervous system, which controls key functions without conscious awareness or involvement" [1].

AC in our recent investigations [2–8] is generally described as self-*. Formally, let self-* be the set of self-__'s. Each self-__ to be an element in self-* is called a *self-* facet*. That is,

$$self-* = \{self-_ | self-_ is a self-* facet\}$$
(1)

We see that self-CHOP is composed of four self-* facets of self-configuration, self-healing, self-optimization and self-protection. Hence, self-CHOP is a subset of self-*. That is, self-CHOP = {self-configuration, self-healing, selfoptimization, self-protection} \subset self-*. Every self-* facet must satisfy some certain criteria, so-called *self-* properties*.

In its AC manifesto, IBM proposed eight facets setting forth an AS known as *self-awareness*, *self-configuration*, *self-optimization*, *self-maintenance*, *self-protection* (*security and integrity*), *self-adaptation*, *self-resource- allocation* and *open-standard-based* [1]. In other words, consciousness (selfawareness) and non-imperative (goal-driven) behaviors are the main features of autonomic systems (ASs).

In this paper we will specify ASs and self-* and then move on to consider products and coproducts of ASs. All of this material is taken as an investigation of our category, the category of ASs, which we call **AS**.

2. Outline

In the paper, we attempt to make the presentation as selfcontained as possible, although familiarity with the notion of self-* in ASs is assumed. Acquaintance with the associated notion of algebraic language is useful for recognizing the results, but is almost everywhere not strictly necessary.

The rest of this paper is organized as follows: Section 3 presents the notion of autonomic systems (ASs). In section 4, self-* actions in ASs are specified. In section 5, products and coproducts of ASs are considered. Some universal properties of ASa are investigated in section 6. Finally, a short summary is given in section 7.

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3. Autonomic Systems (ASs)

We can think of an *AS* as a collection of states $x \in AS$, each of which is recognizable as being in *AS* and such that for each pair of named states $x, y \in AS$ we can tell if x = y or not. The symbol \oslash denotes the *AS* with no states.

If AS_1 and AS_2 are ASs, we say that AS_1 is a sub-system of AS_2 , and write $AS_1 \subseteq AS_2$, if every state of AS_1 is a state of AS_2 . Checking the definition, we see that for any system AS, we have sub-systems $\oslash \subseteq AS$ and $AS \subseteq AS$.

We can use system-builder notation to denote sub-systems. For example the autonomic system can be written $\{x \in AS \mid x \text{ is a state of AS}\}$.

The symbol \exists means "there exists". So we can write the autonomic system as $\{x \in AS \mid \exists y \text{ is a final state such that } self-*action(x) = y\}$

The symbol \exists ! means "there exists a unique". So the statement " \exists ! $x \in AS$ is an initial state" means that there is one and only one state to be a start one, that is, the state of the autonomic system before any self-* action is processed.

Finally, the symbol \forall means "for all". So the statement " $\forall x \in AS \exists y \in AS$ such that *self-* action*(*x*) = *y*" means that for every state of autonomic system there is the next one.

In the paper, we use the $\stackrel{def}{=}$ notation " $AS_1 \stackrel{def}{=} AS_2$ " to mean something like "define AS_1 to be AS_2 ". That is, a $\stackrel{def}{=}$ declaration is not denoting a fact of nature (like 1+2=3), but our formal notation. It just so happens that the notation above, such as Self-CHOP $\stackrel{def}{=}$ {self-configuration, self-healing, selfoptimization, self-protection}, is a widely-held choice.

4. Self-* Actions of Autonomic Systems

If AS and AS' are sets of autonomic system states, then a self-* action self-*action from AS to AS', denoted self-*action: $AS \rightarrow AS'$, is a mapping that sends each state $x \in AS$ to a state of AS', denoted self-*action(x) $\in AS'$. We call AS the domain of self-*action and we call AS' the codomain of self-*action.

Note that the symbol AS', read "AS-prime", has nothing to do with calculus or derivatives. It is simply notation that we use to name a symbol that is suggested as being somehow like AS. This suggestion of consanguinity between AS and AS' is meant only as an aid for human cognition, and not as part of the mathematics. For every state $x \in AS$, there is exactly one arrow emanating from x, but for a state $y \in AS'$, there can be several arrows pointing to y, or there can be no arrows pointing to y.

Suppose that $AS' \subseteq AS$ is a sub-system. Then we can consider the self-* action $AS' \to AS$ given by sending every state of AS' to "itself" as a state of AS. For example if $AS = \{a, b, c, d, e, f\}$ and $AS' = \{b, d, e\}$ then $AS' \subseteq AS$ and we turn that into the self-* action $AS' \to AS$ given by $b \mapsto$ $b, d \mapsto d, e \mapsto e$. This kind of arrow, \mapsto , is read aloud as "maps to". A self-* action *self-*action*: $AS \to AS'$ means a rule for assigning to each state $x \in AS$ a state *self-*action*(x) $\in AS'$. We say that "x maps to *self-*action*(x)" and write $x \mapsto$ *self-*action*(x). As a matter of notation, we can sometimes say something like the following: Let *self-*action*: $AS' \subseteq AS$ be a sub-system. Here we are making clear that AS' is a sub-system of AS, but that *self-*action* is the name of the associated self-* action.

Given a self-* action self-*action: $AS \rightarrow AS'$, the states of AS' that have at least one arrow pointing to them are said to be in the image of self-*action; that is we have $\operatorname{im}(self-*action) \stackrel{def}{=} \{y \in AS' \mid \exists x \in AS \text{ such that self-*action}(x) = y\}$. Given self-*action: $AS \rightarrow AS'$ and self-*action': $AS' \rightarrow AS''$, where the codomain of self-*action is the same set of autonomic system states as the domain of self-*action' (namely AS'), we say that self-*action and self-*action' are composable

$$AS \xrightarrow{self-*action} AS' \xrightarrow{self-*action'} AS''$$

The composition of *self-*action* and *self-*action'* is denoted by *self-*action'* \circ *self-*action*: $AS \rightarrow AS''$.

We write Hom_{AS}(AS, AS') to denote the set of *self-*actions* $AS \rightarrow AS'$. Two self-* actions *self-*action*, *self-*action'* : $AS \rightarrow AS'$ are equal if and only if for every state $x \in AS$ we have *self-*action*(x) = *self-*action'*(x).

We define the identity *self-*action* on *AS*, denoted id_{AS} : $AS \rightarrow AS$, to be the self-* action such that for all $x \in AS$ we have $id_{AS}(x) = x$.

A self-*action: $AS \rightarrow AS'$ is called an *isomorphism*, denoted self-*action: $AS \xrightarrow{\cong} AS'$, if there exists a self-* action self-*action' : $AS' \rightarrow AS$ such that self-*action' \circ self-*action= id_{AS} and self-*action \circ self-*action' = $id_{AS'}$. We also say that self-*action is invertible and we say that self-*action' is the inverse of self-*action. If there exists an isomorphism $AS \xrightarrow{\cong} AS'$ we say that AS and AS' are isomorphic autonomic systems and may write $AS \cong AS'$.

Proposition 1. The following facts hold about isomorphism.

- 1. Any autonomic system AS is isomorphic to itself; i.e. there exists an isomorphism $AS \xrightarrow{\cong} AS$.
- 2. For any autonomic systems AS and AS', if AS is isomorphic to AS' then AS' is isomorphic to AS.
- 3. For any autonomic systems AS, AS' and AS", if AS is isomorphic to AS' and AS' is isomorphic to AS" then AS is isomorphic to AS".

Proof:

1. The identity self-* action $id_{AS} : AS \to AS$ is invertible; its inverse is id_{AS} because $id_{AS} \circ id_{AS} = id_{AS}$.

2. If *self-*action*: $AS \rightarrow AS'$ is invertible with inverse *self-*action'* : $AS' \rightarrow AS$ then *self-*action'* is an isomorphism with inverse *self-*action*.

3. If self-*action: $AS \to AS'$ and self-*action : $AS' \to AS''$ are each invertible with inverses self-*action' : $AS' \to AS$ and self-*action' : $AS'' \to AS'$ then the following calculations show that self-*action \circ self-*action is invertible with inverse self-*action' \circ self-*action': $(self-*action \circ self-*action) \circ (self-*action' \circ self-*action') = self-*action \circ (self-*action \circ self-*action') \circ self-*action' = self-*action \circ id_{AS'} \circ self-*action' = self-*action \circ self-*action' = id_{AS''}$

and

 $(self-*action' \circ self-*action') \circ (self-*action \circ self-*action) =$ $self-*action' \circ (self-*action' \circ self-*action) \circ self-*action =$ $self-*action' \circ id_{AS'} \circ self-*action =$ $self-*action' \circ self-*action = id_{AS}$

For any natural number $n \in \mathbb{N}$, define a set $\underline{n} = \{1, 2, ..., n\}$. So, in particular, $\underline{0} = \emptyset$. A function $f : \underline{n} \to AS$ can be written as a sequence f = (f(1), f(2), ..., f(n)). We say that *AS* has cardinality *n*, denoted |AS| = n if there exists an isomorphism $AS \cong \underline{n}$. If there exists some $n \in \mathbb{N}$ such that *AS* has cardinality *n* then we say that *AS* is finite. Otherwise, we say that *AS* is infinite and write $|AS| \ge \infty$.

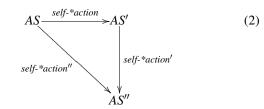
Proposition 2. Suppose that AS and AS' are finite. If there is an isomorphism of autonomic systems $f: AS \rightarrow AS'$ then the two autonomic systems have the same cardinality, |AS| = |AS'|.

Proof: Suppose that $f : AS \to AS'$ is an isomorphism. If there exists natural numbers $m, n \in \mathbb{N}$ and isomorphisms $\alpha : \underline{m} \xrightarrow{\cong} AS$ and $\beta : \underline{n} \xrightarrow{\cong} AS'$ then

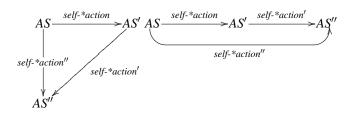
$$\underline{m} \stackrel{\alpha}{\to} AS \stackrel{f}{\to} AS' \stackrel{\beta^{-1}}{\to} \underline{n}$$

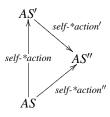
is an isomorphism. We can prove by induction that the sets \underline{m} and \underline{n} are isomorphic if and only if m = n. Q.E.D.

Consider the following diagram:

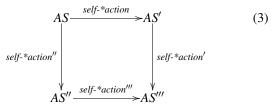


We say this is a diagram of autonomic systems if each of AS, AS', AS'' is an autonomic system and each of *self-***action*, *self-***action*', *self-***action*'' is a self-* action. We say this diagram commutes if *self-***action*' \circ *self-***action* = *self-***action*''. In this case we refer to it as a commutative triangle of autonomic systems. Diagram (2) is considered to be the same diagram as each of the following:





Consider the following picture:



We say this is a diagram of autonomic systems if each of AS, AS', AS'', AS''' is an autonomic system and each of *self*-**action*, *self*-**action''*, *self*-**action'''* is a self-**action*. We say this diagram commutes if *self*-**action'* \circ *self*-**action* = *self*-**action'''* \circ *self*-**action'''*. In this case we refer to it as a commutative square of autonomic systems.

5. Products and Coproducts of Autonomic Systems

Let *AS* and *AS'* be autonomic systems. The product of *AS* and *AS'*, denoted $AS \times AS'$, is defined as the autonomic system of ordered pairs (x, y) where states of $x \in AS$ and $y \in AS'$. Symbolically, $AS \times AS' = \{(x, y) | x \in AS, y \in AS'\}$. There are two natural projection actions of self-* to be *self-*action*₁ : $AS \times AS' \rightarrow AS$ and *self-*action*₂ : $AS \times AS' \rightarrow AS'$

$$AS \times AS'$$
(4)
self-*action₁ self-*action₂
S^t AS'

For illustration, suppose that $\{a,b,c\}$ are states in AS and $\{d,e\}$ in AS', the states are happening in such autonomic systems. Thus, AS and AS', which are running concurrently, can be specified by $AS|AS' \stackrel{def}{=} \{(a|d), (a|e), (b|d), (b|e), (c|d), (c|e)\}$. Note that the symbol "|" is used to denote concurrency of states existing at the same time. We define self-* actions as disable(d,e) and disable(a,b,c) to be able to drop out relevant states.

A

$$\{(a|d), (a|e), (b|d), (b|e), (c|d), (c|e)\}$$

disable(d,e)
$$\{a,b,c\}$$

$$\{a,b,c\}$$

$$\{d,e\}$$

(5)

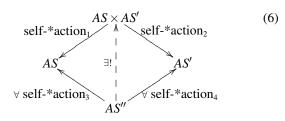
It is possible to take the product of more than two autonomic systems as well. For example, if AS_1 , AS_2 , and AS_3 are autonomic systems then $AS_1|AS_2|AS_3$ is the system of triples,

$$AS_1|AS_2|AS_3 \stackrel{def}{=} \{(a|b|c)|a \in AS_1, b \in AS_2, c \in AS_3\}$$



3

Proposition 3. Let *AS* and *AS'* be autonomic systems. For any autonomic system AS'' and actions self-*action₃ : $AS'' \rightarrow AS$ and self-*action₄ : $AS'' \rightarrow AS'$, there exists a unique action $AS'' \rightarrow AS \times AS'$ such that the following diagram commutes



We might write the unique action as

$$(\text{self-*action}_3, \text{self-*action}_4) : AS'' \to AS \times AS'$$

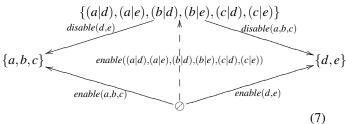
Proof: Suppose given self-* $action_3$ and self-* $action_4$ as above. To provide an action $z : AS'' \to AS \times AS'$ is equivalent to providing a state $z(a) \in AS \times AS'$ for each $a \in AS''$. We need such an action for which self-* $action_1 \circ z = self$ -* $action_3$ and self-* $action_2 \circ z = self$ -* $action_4$. A state of $AS \times AS'$ is an ordered pair (x, y), and we can use z(a) = (x, y) if and only if x = self-* $action_1(x, y) = self$ -* $action_3(a)$ and y = self-* $action_2(x, y) = self$ -* $action_4(a)$. So it is necessary and sufficient to define $\langle self$ -* $action_3, self$ -* $action_4 \rangle \stackrel{def}{=} (self$ -* $action_3(a), self$ -* $action_4(a)$) for all $a \in AS''$. Q.E.D.

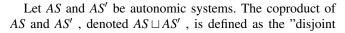
Given autonomic systems AS, AS', and AS'', and actions self-* $action_3 : AS'' \rightarrow AS$ and self-* $action_4 : AS'' \rightarrow AS'$, there is a unique action $AS'' \rightarrow AS \times AS'$ that commutes with self-* $action_3$ and self-* $action_4$. We call it the *induced action* $AS'' \rightarrow AS \times AS'$, meaning the one that arises in light of self-* $action_3$ and self-* $action_4$.

For example, as mentioned above autonomic systems $AS = \{a, b, c\}$, $AS' = \{d, e\}$ and $AS|AS' \stackrel{def}{=} \{(a|d), (a|e), (b|d), (b|e), (c|d), (c|e)\}$. For an autonomic system $AS'' = \emptyset$, which stops running, we define self-* actions as *enable*(*d*, *e*) and *enable*(*a, b, c*) to be able to add further relevant states. Then there exists a unique action

$$enable((a|d), (a|e), (b|d), (b|e), (c|d), (c|e))$$

such that the following diagram commutes





union" of AS and AS', i.e. the autonomic system for which a state is either a state of AS or a state of AS'. If something is a state of both AS and AS' then we include both copies, and distinguish between them, in $AS \sqcup AS'$. There are two natural inclusion actions self-* $action_1 : AS \rightarrow AS \sqcup AS'$ and self-* $action_2 : AS' \rightarrow AS \sqcup AS'$.

$$AS AS' (8)$$

$$self-*action_1 self-*action_2 AS \sqcup AS'$$

For illustration, suppose that $\{a,b,c\}$ are states in autonomic system AS and $\{d,e\}$ in AS'. Thus, $AS \sqcup AS'$, which is disjoint union, can be specified by $AS \sqcup AS' \stackrel{def}{=} \{a,b,c,d,e,\}$. We define self-* actions as ensable(d,e) and enable(a,b,c) to be able to add further relevant states.

$$\{a,b,c\}$$

$$\{a,b,c\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

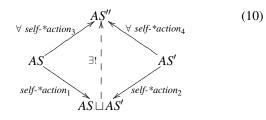
$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

$$\{a,b,c,d,e\}$$

Proposition 4. Let *AS* and *AS'* be autonomic systems. For any autonomic system AS'' and actions self-*action₃ : $AS \rightarrow AS''$ and self-*action₄ : $AS' \rightarrow AS''$, there exists a unique action $AS \sqcup AS' \rightarrow AS''$ such that the following diagram commutes

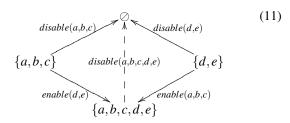


We might write the unique action as

self-*action₃, self-*action₄] :
$$AS \sqcup AS' \to AS''$$

Suppose given self-*action₃, self-*action₄ **Proof:** as above. To provide an action $z : AS \sqcup AS' \to AS''$ is equivalent to providing a state self-* $action_3(m) \in AS''$ is for each $m \in AS \sqcup AS'$. We need such an action such that $z \circ self$ -*action₁ = self-*action₃ and $z \circ self$ -*action₂ = *self-*action*₄. But each state $m \in AS \sqcup AS'$ is either of the form self-* $action_1x$ or self-* $action_2y$, and cannot be of both forms. So we assign $[self-*action_3, self-*action_4](m) =$ self-* $action_3(x)$ if $m = self - *action_1 x$ This $elf-*action_4(y)$ if $m = self - *action_2 y$ assignment is necessary and sufficient to make all relevant diagrams commute. O.E.D.

For example, as mentioned above autonomic systems $AS = \{a, b, c\}, AS' = \{d, e\}$ and $AS \sqcup AS' \stackrel{def}{=} \{a, b, c, d, e\}$. For an autonomic system $AS'' = \emptyset$, which stops running, we define self-* actions as disable(d, e) and disable(a, b, c) to drop out relevant states. Then there exists a unique action disable(a, b, c, d, e) such that the following diagram commutes



6. Universial Properties

We denote the coproduct of two autonomic systems AS and AS' by the notation AS + AS' rather than $AS \sqcup AS'$. It is a reasonable notation in general, and one that is often used.

The following isomorphisms exist for any autonomic systems AS, AS', and AS''

 $\begin{array}{l} AS + \underline{0} \cong AS \\ AS + AS' \cong AS' + AS \\ (AS + AS') + AS'' \cong AS + (AS' + AS'') \\ AS \times \underline{0} \cong \underline{0} \\ AS \times \underline{1} \cong AS \\ AS \times AS' \cong AS \\ AS \times AS' \cong AS' \times AS \\ (AS \times AS') \times AS'' \cong AS \times (AS' \times AS'') \\ AS \times (AS' + AS'') \cong (AS \times AS') + (AS \times AS'') \\ AS^{\underline{0}} \cong \underline{1} \\ AS^{\underline{1}} \cong AS \\ \underline{0}^{AS} \cong \underline{0} \\ \underline{1}^{AS} \cong \underline{1} \\ AS^{AS' + AS''} \cong AS^{AS'} \times AS^{AS''} \\ (AS^{AS'})^{AS''} \cong AS^{AS' \times AS''} \end{array}$

In the case of $\underline{0}^{\underline{0}}$, we get conflicting answers, because for any autonomic system AS, including $AS = \bigcirc = \underline{0}$, we have claimed both that $AS^{\underline{0}} \cong \underline{1}$ and that $\underline{0}^{AS} \cong \underline{0}$. Based on the definitions of 0, 1 and $AS^{\overline{AS'}}$ given in 4, the correct answer for $\underline{0}^{\underline{0}}$ is $\underline{0}^{\underline{0}} \cong \underline{1}$. The universal properties, which are considered in this section, are in some sense about isomorphisms. It says that understanding isomorphisms of autonomic systems reduces to understanding natural numbers. But note that there is much more going on in the category of **AS** than isomorphisms; in particular there are self-* actions that are not invertible.

7. Conclusions

The paper is a reference material for readers who already have a basic understanding of self-* in ASs and are now ready to consider products, coproducts and some universal properties of ASs using algebraic language. Algebraic specification is presented in a straightforward fashion by discussing in detail the necessary components and briefly touching on the more advanced components.

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