Design an Algebra Learning Sequence Based on Realistic Mathematics Education Theory: The Case of Linear Equation in One Variable using Balance-Scales Model

Ririn Sispiyati¹, Al Jupri², Dian Usdiyana³ {ririnsispiyati@upi.edu¹, aljupri@upi.edu², dianusdy@upi.edu³}

Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia¹²³

Abstract. This study aims to design teaching materials about linear equations in one variable with a model of balance-scales based on realistic mathematics theories intended for junior high school students. For this purpose, we used design research, particularly the preliminary design phase. First, we compiled the material based on curriculum and mathematics textbooks. This practical view was used as consideration in designing a learning sequence. Second, the practical learning sequence was revised based on the theory of realistic mathematics education. Third, we organized a focus group discussion to review and improve the design intend to produce a learning sequence that can further be used for the teacher for the teaching experiment. The results included learning sequence in the form of learning materials on the topic of linear equations in one variable with a balance-scale model. The designed learning sequence for this topic provides more meaningful than conventional learning for the students.

Keywords: Realistic mathematics education, balance-scale model, linear equation in one variable.

1 Introduction

Linear equation in one variable is the first learning for junior high school students of class VII in the form of abstract equations or early transition of mathematics learning from concrete to abstract [1]. The students' ability to complete the form of linear equation in one variable is also indispensable for students in understanding mathematical material in the next stage. So, we need a learning sequence to help students understand it [2].

Linear equation in one variable is a topic that students think is a difficult thing. This is seen from the student's ability to solve the problem of linear equations in one variable ([3], [4], [5]). Some difficulties in learning related to algebraic topics include applying arithmetic operations, understanding the notion of variable, understanding algebraic expressions, understanding the different meanings of the equal sign, and mathematization [3].

The problem is how to make a learning sequence to overcome students' difficulties on the topic of linear equations in one variable. Based on limited observations of the mathematics books used in Indonesia ([6], [7]), the use of the balance-scales model in learning on the topic of equality has often been used. However, the use of the model has not been systematic and is

not based on learning theory. Therefore, in this study, the learning sequence was made on this topic using a balance-scale model based on realistic mathematics education (RME).

In designing a learning sequence on the topic of linear equation in one variable, three didactic principles of RME theory are used, including the principle of reality, the principle of level, and the principle of intertwinement. Through the principle of reality, mathematics learning starts with a contextual problem and can also be solved by a mathematical process (mathematization). In this topic, this mathematical process leads to the activity of translating contextual problems into algebraic symbols or vice versa. The level principle emphasizes that in the process of learning mathematics through levels of understanding, mathematical, ranging from informal understanding to formal stages. In the case of algebra learning, mathematical models are needed in bridging formal and informal mathematics. Based on the principle of intertwinement, the mathematical content domains are seen as topics that are interrelated to one another. In the context of algebra learning, the relationship between topics such as algebra, numbers (arithmetics), also physic can facilitate students to solve algebraic problems.

2 Methodology

To design learning sequences on the topic of linear equation in one variable using the balance-scales model used design research methods. This method consists of three stages: developing an initial design, conducting teaching experiments, and conducting a retrospective analysis [8]. In this article, we specialize in reporting the results of the initial research, which is the initial design phase. This initial design phase is carried out in three steps.

First, compiling material for linear equations in one variable based on the curriculum (Kemdikbud, 2013) and books that are formally used in Indonesia [6]. In addition, for consideration, several mathematical books published by private publishers are used [7]. The results of this first step are introducing a linear equation in one variable with the balance-scale model using a contextual problem which is realistic as an initial step. Next, the learning sequence continued with the abstract problem of the equations. This problem is expected to be solved by students with the balance-scale model.

Second, the learning sequences which were compiled practically in the first step was adjusted to the theory of realistic mathematics education. This adjustment, for example, is done by starting learning not directly to abstract equations but starting from realistic problems that are meaningful to students. Then, after recognizing the realistic use of the balance-scale model, then students are directed to use the principles of the scale model to solve equations that are more abstract in nature. With such an arrangement, we hope that the various difficulties students have in learning equations, such as mistakes in inverse addition or multiplication operations, and mathematical bias can be minimized ([3], [4], [5]).

Third, hold a focus group discussion with the research team to review and improve teaching materials that have been prepared in the second stage. The results of the activities in this third step are in the form of learning sequences that are ready to be used in the implementation of learning.

3 Results and Discussion

The following is the learning of the linear equation in one variable in the form of a balancescale model. As an initial activity, the balance-scale model is introduced in a balanced position which means that the weight on the right side is the same as the weight on the left side. On the right side, there are 5 marbles while the left side has 1 marble along with a box containing marbles which are presented in **Figure 1**.



Fig. 1. Tasks used in the activity of introducing linear equation in one variable with the balance-scale model.

According to the figure, answer this question sequentially.

- 1) If a marble on the right side is taken, what will happen?
- 2) Furthermore, if a marble on the left side is taken, what will happen?

3) Can you determine the number of marbles in the box?

For each question, students are expected to be able to answer 1) the balance-scale model becomes unbalanced, after taking 1 marble on the right side, there are 4 marbles left, while the left side is still fixed; 2) If the left side is taken 1 marble, the scales become back in balance, the right side is 4 marbles and the left side is left a box containing marbles; 3) from activity 2, leaving 1 box on the left side and 4 marbles on the right side in the balance position, then the number of marbles in the box is 4.

From the learning process, guiding students to solve linear equations in one variable in the form of a balance-scale model. By analogizing the state of the balance-scales model in the balance position can be written as an equation, we write variable x (for example) for a box containing marbles, and one marble is a number 1, then it can be written as the form x + 1 = 5. So to determine the number of marbles in the box, we take one marble from each side so that the state of the balance returns to the balance position, we write x + 1 - 1 = 5 - 1 and get x = 4. So, the number of marbles in the box are 4.

If students have been able to complete this section, then the same way is expected to complete tests :

- a. If on the balance-scale model, the right side contains 3 marbles and 1 box containing marbles, while on the left side there are 9 marbles. Draw in the form of a balance-scale model and determine the number of marbles in the box.
- b. If on the balance-scale model, there are 5 marbles and 2 boxes containing marbles on the right side, while on the left side there are 15 marbles. Draw in the form of a balance-scale model and determine the number of marbles in the box, if each box has the same number of marbles.
- c. If on the balance-scale model, the right side contains 4 marbles and 2 boxes containing marbles, while on the left side there are 1 box and 7 marbles. Draw in the form of a balance-scale and determine the number of marbles in the box, if each box has the same number of marbles.

d. If on the balance-scale model, the right side contains 4 marbles and 5 boxes containing marbles, while on the left side there are 2 boxes and 10 marbles. Draw in the form of a balance-scale and determine the number of marbles in the box, if each box has the same number of marbles.

Students are expected to be able to describe the problem in the form of a balance-scale model as an initial step. Next, by analogizing the balance position in the model as an equation, a box containing a marble with variable *x* for example, and a marble with a number 1, so they can make an equation in a mathematical model, then determine the value of *x*. For example, for test 2(a), can be written 7 = x + 3, then the two side minus 3 so that 7 - 3 = (x + 3) - 3, and by performing algebraic operations obtained 4 = x. For 2(b), can be written $15 = 2x + 5 \Leftrightarrow 15 - 5 = 2x + 5 - 5 \Leftrightarrow 10 = 2x \Leftrightarrow \frac{10}{2} = \frac{2x}{2} \Leftrightarrow 5 = x$. The same calculation for 2(c) and 2(d) each can be written $x + 7 = 2x + 4 \Leftrightarrow x - x + 7 = 2x - x + 4 \Leftrightarrow 7 - 4 = x + 4 - 4 \Leftrightarrow 3 = x$ and $2x + 10 = 5x + 4 \Leftrightarrow 2x - 2x + 10 = 5x - 2x + 4 \Leftrightarrow 10 = 3x + 4 \Leftrightarrow 10 - 4 = 3x + 4 - 4 \Leftrightarrow 6 = 3x \Leftrightarrow \frac{6}{3} = \frac{3x}{3} \Leftrightarrow 2 = x$.

4 Conclusion

Based on previous exposure, it can be concluded the following points. The learning sequence on linear equation in one variable with the balance-scales model begins by introducing the model as the linear equation in one variable—using realistic contextual problems. Then, the composition of learning materials is completed by providing equations problem which is more abstract. This problem is expected to be solved by students using the principle of the balanced-scale model. From this description of learning materials, we are of the view that the balanced-scales model helps students understand the abstract problem to be more concrete so that it will be more meaningful in their thinking.

References

[1]Kemdikbud.: Kurikulum 2013. Kompetensi Dasar: SekolahMenengahPertama/Madrasah Tsanawiyah. Jakarta: Kementerian Pendidikan dan Kebudayaan (2013)

[2]Katz, V. K.: Algebra: Gateway to a Technological Future. (The Mathematical Association of America, Washington DC).

[3]Jupri, A., Drijvers, P., and Van den Heuvel-Panhuizen, M.: Mathematics Education Research Journal 26 683-710 (2014).

[4]Jupri, A., Drijvers, P. and Van den Heuvel-Panhuizen, M.: International Electronic Journal on Mathematics Education 9 39-55 (2014).

[5]Jupri, A., Drijvers, P.: Eurasia Journal of Mathematics, Science and Technology Education, 12 2481-2502 (2016).

[6]Nuharini, D. and Wahyuni, T.: Matematika 1: Konsep dan Aplikasinya: Untuk Kelas VI SMP/MTs Jakarta: Pusat Perbukuan, Departemen Pendidikan Nasional, (2008).

[7]Adinawan, M.C.: Matematika Untuk SMP/MTs Kelas VII Semester 1. Jakarta: Erlangga (2016)

[8] Bakker, A.: Design research in statistics education: On symbolizing and computer tools. (Utrecht, the Netherlands: Freudenthal Institute (2004).