A Rasch Model Analysis of Mathematical Understanding of Prospective Mathematics Teachers in The Conic Concept

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Abstract. This study aims to obtain a general description of the analysis of the mathematical understanding of prospective mathematics teachers in the conic concept with Rasch models. This study was pre-experimental with a type of one-shot case study involving students of prospective mathematics teachers in the second semester. This study uses a test instrument in the form of an essay question consisting of three questions designed by a research team that is a lecturer in analytic geometry courses. The results of the analysis of the mathematical understanding of prospective mathematics teachers in analytic geometry lectures with the Rasch model show that only a small proportion of students have high mathematical understanding. The results show that the items used are not reliable and cannot be accepted to measure the mathematical understanding of prospective mathematics teachers in the conic concept.

Keywords: Mathematical Understanding, Prospective Mathematics Teachers, Analytic Geometry, Rasch Model

1 Introduction

Mathematical understanding is important for prospective mathematics teachers because it becomes their provision to teach at school. At least for the past 60 years, a collection of research evidence has been collected which shows the teaching benefits for understanding in mathematics [1]. Geometry is one of the important concepts in school [2]. So that prospective students of mathematics must have a good mathematical understanding of geometry. One important concept in geometry is conic because it has been studied extensively among them by Ayoub [3], Buczkowske [4], dan Nhi, Tinh, and Phuong [5].

Alternative measurements are needed to assess the quality of reliable and valid instruments including the Rasch Model, which is one of Item-Response Theory (IRT). Research on instrument testing is needed which plays an important role in it is data collection. Conclusions from appropriate research can be drawn from the research sample [6]. And the main indicators of the quality of research instruments are validity and reliability [7]. The Rasch model can determine the reliability and validity of an instrument [8] [9]. Analysis using the Rasch model can produce more precise and preferred instruments [10]. The advantages of the Rasch model are being able to provide linear scales at the same interval, predict missing data, provide more precise estimates, detect model inaccuracies and produce measurements that can be replicated [10]. Therefore, this study aims to obtain an overview of instrument analysis and mathematical

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understanding of prospective mathematics teachers in analytic geometry lectures with Rasch models.

2 Method

This study follows the research conducted by Sudihartinih and Wahyudin [11]. This study was pre-experimental with a type of one-shot case study involving prospective mathematics teachers in the second semester consisting of 10 men and 29 women at one university in Indonesia. The research step is the syllabus analysis by selecting concepts for research; selection of research participants, design and validation of teaching materials; instrument design and validation; implementation of learning for 3 x 250 minutes; test; data analysis; report and dissemination.

This study uses a description test instrument in the conic concept as many as three questions designed by several of the members of the research team who are lecturers of analytic geometry courses. Students fill out questions using paper and pencil within 90 minutes during the midterm. The following are the test.

First, determine the parabola equation with the focus (10,0) and the directrix x = 2. Also, determine the vertex and the length of the latus rectum. Then now the focus F (0, ae) and the directrix equation (1) Prove that the simple equation of the ellipse is equation (2), and then determine the position of the points that have the nature that the difference in the distance of the point to the point (0, -7) and (0.7) is 8.

$$y = \frac{a}{e}. \tag{1}$$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \tag{2}$$

General description of instrument analysis and mathematical understanding of prospective mathematics teachers in analytic geometry lectures was obtained by analyzing student answers in questionnaires using Item-Response Theory (IRT) with Rasch models assisted by Mini step software.

3 Result and Discussion

3.1 Student Mathematical Understanding

Based on the results of the study, the following scores data on students' mathematical understanding in each question.

Table 1 shows the minimum score of students' mathematical understanding is 3.00 and the maximum score is 27.00, while the average score is 36.41 with a standard deviation of 25.03 meaning that the score is very diverse. Based on a statistical test with a difference test of two on average using the t test (using SPSS 23) on the scores of the three questions. That is question

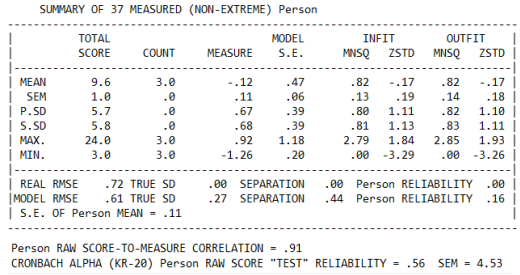
number 1 with question number 2, question number 1 with question number 3, and question number 2 with question number 3 it is known that there is no significant difference at the 95% significance level.

Table 1. Scores of student mathematical understanding in each question.

Respondent	Question 1	Question 2	Question 3	Average Student Score
01L	1	6	7	4.67
02P	5	9	1	5.00
03P	5	2	6	4.33
04P	8	2	1	3.67
05L	1	2	6	3.00
06P	1	2	0	1.00
07L	1	1	1	1.00
08P	7	1	9	5.67
09L	9	1	0	3.33
10P	5	2	1	2.67
11P	6	2	0	2.67
12P	1	6	2	3.00
13L	1	0	2	1.00
14P	1	2	1	1.33
15P	2	2	1	1.67
16P	8	6	8	7.33
17L	9	6	1	5.33
18L	4	1	1	2.00
19P	2	1	1	1.33
20P	1	1	1	1.00
21P	1	1	9	3.67
22P	8	2	9	6.33
23P	2	2	1	1.67
24P	1	1	9	3.67
25L	1	1	1	1.00
26L	1	1	2	1.33
27L	9	9	9	9.00
28P	1	9	1	3.67
29P	9	9	6	8.00
30P	9	2	1	4.00
31P	1	1	1	1.00
32P	4	2	1	2.33
33P	1	1	1	1.00
34P	5	6	4	5.00
35P	1	4	6	3.67
36P	5	6	1	4.00
37P	9	9	9	9.00
38P	1	1	1	1.00
39P	7	9	1	5.67
Average score	3.95	3.36	3.15	36.41
SD	3.20	2.99	3.26	25.03

3.2 Reliability Item

According to **Figure 1**, the mean measure is -0.12 means that the average score of students' mathematical understanding is negative. The average value which is less than logit 0.0 indicates the tendency of students who cannot answer the questions [10]. Cronbach Alpha value 0.56 means poor reliability [12]. The value of Person reliability is 0.00 which means weak [10]. Thus, the consistency of students is weak.



SUMMARY OF 37 MEASURED (NON-EXTREME) Person

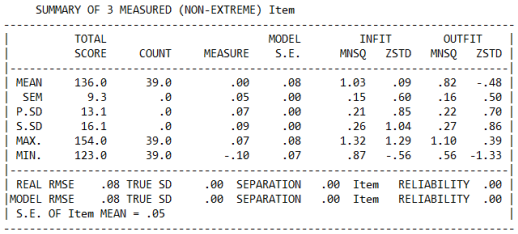
	TOTAL SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	9.6	3.0	-.12	.47	.82	-.17	.82	-.17
SEM	1.0	.0	.11	.06	.13	.19	.14	.18
P.SD	5.7	.0	.67	.39	.80	1.11	.82	1.10
S.SD	5.8	.0	.68	.39	.81	1.13	.83	1.11
MAX.	24.0	3.0	.92	1.18	2.79	1.84	2.85	1.93
MIN.	3.0	3.0	-1.26	.20	.00	-3.29	.00	-3.26

REAL RMSE	.72 TRUE SD	.00	SEPARATION	.00	Person RELIABILITY .00
MODEL RMSE	.61 TRUE SD	.27	SEPARATION	.44	Person RELIABILITY .16
S.E. OF Person MEAN = .11					

Person RAW SCORE-TO-MEASURE CORRELATION = .91
CRONBACH ALPHA (KR-20) Person RAW SCORE "TEST" RELIABILITY = .56 SEM = 4.53

Fig. 1. Measured Person.

According to **Figure 2**, the mean measure +0.00 means that the average item is 0.00 so the average question is medium. Item reliability value is 0.00 which means weak [10]. Thus, the quality of the question item is weak.



SUMMARY OF 3 MEASURED (NON-EXTREME) Item

	TOTAL SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	136.0	39.0	.00	.08	1.03	.09	.82	-.48
SEM	9.3	.0	.05	.00	.15	.60	.16	.50
P.SD	13.1	.0	.07	.00	.21	.85	.22	.70
S.SD	16.1	.0	.09	.00	.26	1.04	.27	.86
MAX.	154.0	39.0	.07	.08	1.32	1.29	1.10	.39
MIN.	123.0	39.0	-.10	.07	.87	-.56	.56	-1.33

REAL RMSE	.08 TRUE SD	.00	SEPARATION	.00	Item RELIABILITY .00
MODEL RMSE	.08 TRUE SD	.00	SEPARATION	.00	Item RELIABILITY .00
S.E. OF Item MEAN = .05					

Fig. 2. Measured Item.

In **Figure 3**, the Guttman scalogram pattern shows the answer of each student in a question item. The items are sorted according to the easiest questions on the left to the most difficult questions located on the right side of the Guttman scalogram. Students are sorted from students who have the highest score at the top of the Guttman scalogram. If seen further, the students who have the highest score are 27L and 37P, while the students with the lowest scores are 5 people, namely 20P, 25L, 31P, 33P, 38P. There are students who look inconsistent in their answers, for example, 08P students because of irregular scores from high to low scores. This means that he can answer more difficult questions, but easy questions cannot.

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GUTTMAN SCALOGRAM
Person |Item
       |123
       |---
    27 +999  27L
    37 +999  37P
    29 +996  29P
    16 +868  16P
    22 +829  22P
     8 +719  08P
    39 +791  39P
    17 +961  17L
     2 +591  02P
    34 +564  34P
     1 +167  01L
     3 +526  03P
    30 +921  30P
    36 +561  36P
     4 +821  04P
    21 +119  21P
    24 +119  24P
    28 +191  28P
    35 +146  35P
     9 +910  09L
     5 +126  05L
    12 +162  12P
    10 +521  10P
    11 +620  11P
    32 +421  32P
    18 +411  18L
    15 +221  15P
    23 +221  23P
    14 +121  14P
    19 +211  19P
    26 +112  26L
     6 +120  06P
     7 +111  07L
    13 +102  13L
    20 +111  20P
    25 +111  25L
    31 +111  31P
    33 +111  33P
    38 +111  38P
       |---
       |123
```

Fig. 3. Gutmann Scalogram.

3.3 Item Validity

Based on **Figure 4** about measure order items, MNSQ 1.01 INFIT mean and MNSQ 0.82 mean OUTFIT, if it is closer to 1.00 then it is better, meaning this question has good conditions for measurement [10]. The ZSTD INFIT mean is 0.1 and the mean ZSTD OUTFIT is -0.5. If the ZSTD value is closer to 0.0, the better means that the data has a logical estimate [10]. Item I0001 with +0.07 logit shows item number 3 is the most difficult to answer correctly by students, while item I0001 with -0.1 logit shows items that are most easily solved by students [10].

If the value is in PTMea Corr. positive (+) then the item measures the construct, and conversely. So if PTMea Corr. negative (-) then the item must be discarded or refined because it is too difficult/easy or does not lead to the question (out of focus) [8]. The findings of this study indicate that all items that have PTMea Corr. positive so that no items are discarded or revised.

Item STATISTICS: MEASURE ORDER

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-AL CORR.	PTMEASUR-AL EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	Item
3	123	39	.07	.08	1.32	1.29	1.10	.39	.61	.65	24.3	29.1	I0003
2	131	39	.03	.08	.88	-.47	.79	-.49	.68	.66	35.1	29.5	I0002
1	154	39	-.10	.07	.87	-.56	.56	-1.33	.71	.68	40.5	30.5	I0001
MEAN	136.0	39.0	.00	.08	1.03	.1	.82	-.5			33.3	29.7	
P.SD	13.1	.0	.07	.00	.21	.9	.22	.7			6.7	.6	

Fig. 4. Item statistic: measure order.

3.4 Analysis of Mathematical Understanding of Prospective Mathematics Teacher Students

The Person-Item Distribution Map (PIDM) describes the ability of students to be latent in responding to the difficulty of an item [13]. As shown in **Figure 5** and **Figure 6**, almost all students have difficulty answering questions, except students 27L and 37P. Although the third question is the most difficult, but the third position is close, so all the questions are difficult.

OUTFIT mean square (MNSQ) must be in the interval between 0.60 and 1.40 and the ZSTD INFIT and OUTFIT values must be in the interval between -2 to +2 [10]. Based on Figure Person statistics: Order measurement, more than half of the students outside the interval, for example, students 38P, 33P, 31P. This means that these students are not logical in answering questions. In the next table, diagnostic samples from students are given with illogical answers.

Based on the Analysis of Mathematical Understanding of Prospective Mathematics Teacher Students it is known that more than half the number of students is still low. Therefore, further research is still needed to improve the Mathematical Understanding of Prospective Mathematics Teacher Students.

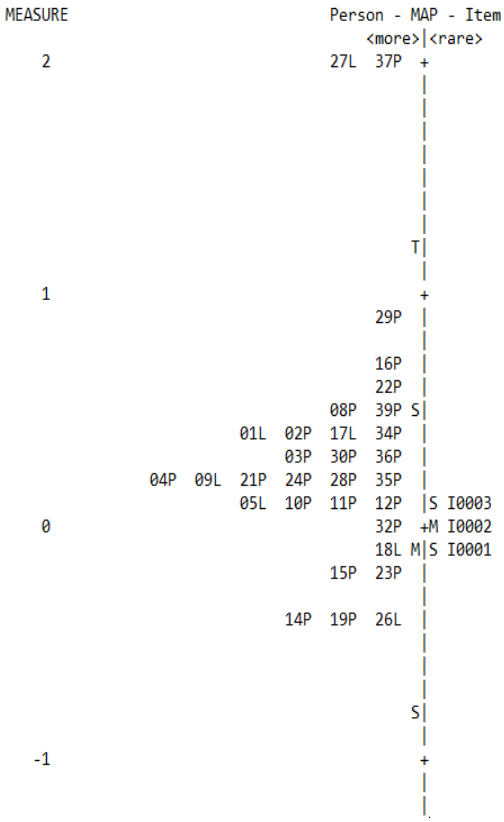

Fig. 5. Person-item measurement.

Person STATISTICS: MEASURE ORDER

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-AL CORR.	PTMEASUR-AL EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	Person
27	27	3	2.08	1.49	MAXIMUM MEASURE				.00	.00	100.0	100.0	27L
37	27	3	2.08	1.49	MAXIMUM MEASURE				.00	.00	100.0	100.0	37P
29	24	3	.92	.36	.63	.17	.57	.11	.71	.11	.0	13.1	29P
16	22	3	.74	.27	.19	-.90	.17	-.89	.25	.15	33.3	9.0	16P
22	19	3	.57	.22	1.34	.68	1.24	.55	.12	.19	.0	16.0	22P
8	17	3	.48	.20	1.47	.94	1.38	.79	.01	.20	.0	12.1	08P
39	17	3	.48	.20	1.19	.52	1.12	.39	.52	.20	.0	12.1	39P
17	16	3	.44	.20	.89	-.07	.89	-.05	.92	.21	.0	12.2	17L
2	15	3	.40	.20	1.16	.48	1.13	.42	.26	.21	.0	11.5	02P
34	15	3	.40	.20	.09	-3.29	.09	-3.26	.26	.21	33.3	11.5	34P
1	14	3	.36	.20	1.24	.63	1.25	.65	-.99	.21	.0	10.7	01L
3	13	3	.32	.20	.38	-1.50	.39	-1.46	.01	.21	33.3	6.6	03P
30	12	3	.28	.20	1.07	.30	1.03	.24	.99	.21	.0	4.7	30P
36	12	3	.28	.20	.44	-1.18	.47	-1.08	.57	.21	33.3	4.7	36P
4	11	3	.24	.21	.82	-.14	.78	-.20	.99	.21	.0	2.0	04P
21	11	3	.24	.21	2.30	1.84	2.45	1.93	-.71	.21	.0	2.0	21P
24	11	3	.24	.21	2.30	1.84	2.45	1.93	-.71	.21	.0	2.0	24P
28	11	3	.24	.21	2.06	1.60	2.05	1.55	-.25	.21	.0	2.0	28P
35	11	3	.24	.21	.91	.03	.89	.01	-.99	.21	.0	2.0	35P
9	10	3	.20	.22	1.72	1.12	1.61	.97	.99	.20	.0	2.0	09L
5	9	3	.15	.23	1.04	.31	1.12	.41	-.83	.19	.0	2.0	05L
12	9	3	.15	.23	.93	.16	.90	.14	-.43	.19	.0	2.0	12P
10	8	3	.09	.25	.30	-.81	.29	-.75	1.00	.18	33.3	16.3	10P
11	8	3	.09	.25	.79	.05	.78	.06	1.00	.18	33.3	16.3	11P
32	7	3	.02	.27	.19	-.84	.19	-.74	1.00	.16	33.3	16.5	32P
18	6	3	-.07	.32	.42	-.13	.37	-.13	.96	.14	.0	24.6	18L
15	5	3	-.19	.40	.07	-.77	.09	-.69	.71	.11	66.7	24.1	15P
23	5	3	-.19	.40	.07	-.77	.09	-.69	.71	.11	66.7	24.1	23P
14	4	3	-.44	.62	.28	-.26	.30	-.24	-.25	.07	66.7	70.3	14P
19	4	3	-.44	.62	.19	-.43	.17	-.47	.96	.07	66.7	70.3	19P
26	4	3	-.44	.62	.31	-.22	.35	-.15	-.71	.07	66.7	70.3	26L
6	3	3	-1.26	1.18	2.73	1.32	2.79	1.34	.26	.04	33.3	78.2	06P
7	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	07L
13	3	3	-1.26	1.18	2.79	1.34	2.85	1.36	-.26	.04	33.3	78.2	13L
20	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	20P
25	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	25L
31	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	31P
33	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	33P
38	3	3	-1.26	1.18	.00	-1.42	.00	-1.42	.00	.04	100.0	78.2	38P
MEAN	10.5	3.0	.00	.53	.82	-.2	.82	-.2			33.3	29.7	
P.SD	6.8	.0	.81	.44	.80	1.1	.82	1.1			37.2	30.8	

Fig. 6. Person statistics: Order measurement.

4 Conclusion

The results of the analysis of the mathematical understanding of prospective mathematics teachers in analytic geometry lectures with the Rasch model show that only a small proportion of students have high mathematical understanding. The results of the Rasch model measurement to examine the instruments show that the items used are not reliable and cannot be accepted to measure the mathematical understanding of prospective mathematics teachers in analytic geometry lectures. So that further research is needed on the instruments and mathematical understanding of prospective mathematics teachers in analytic geometry lectures with the Rasch model.

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