Optimization Analysis of Portfolio Method Based on High Order Moment

Kai Shi

3538598297@qq.com

School of Statistics, Shandong Technology and Buisiness University, Laishan District, Yantai City, Shandong Province, China

Abstract. In this paper, a dynamic high order moment parametric portfolio investment decision model (B-S-K) is proposed to solve the deficiency of risk measurement model in existing high order moment portfolio investment models. This model uses MIDAS-QR model and parametric portfolio investment strategy to improve the timeliness, accuracy and robustness of risk measurement, reduce the number of parameters to be estimated and improve the solving efficiency of the model. In the empirical study, the model is applied to the individual stock and industry index of the Chinese stock market, and the results show that the model has advantages in risk measurement and portfolio investment decision. Specifically, the dynamic high-order moment risk measure based on MIDAS-QR model takes into account the time-varying characteristics of financial risk and has little influence on outliers. In addition, P/E ratio, book value ratio and dynamic skewness risk are positively correlated with portfolio weight, while conditional volatility and dynamic kurtosis risk are negatively correlated with portfolio weight, which provides explanations for portfolio investment decisions. Compared with other models, B-S-K model shows advantages in terms of return, risk and risk-adjusted return. In short, B-S-K model improves the risk measurement and portfolio investment decision through MIDAS-QR model and parametric portfolio investment strategy, and has better performance and effect.

Keywords: High moment, Portfolio investment, Parametric strategy, B-S-K modle, Bayesian optimization

1 Introduction

Financial theory and practice show that the return on financial assets often does not follow the normal distribution, but has asymmetric and peak and fat tail characteristics. On the one hand, the left tail of the income series distribution is usually longer than the right tail, and there is a negative skew, which means that the possibility of income decline is higher than the possibility of income increase. On the other hand, the return series distribution is steeper than the normal distribution, with excess kurtosis, which greatly increases the possibility of black swan events. Therefore, it is difficult to fully and accurately characterize the behavior characteristics of financial assets' return only from the first second moment of the return sequence, and it is necessary to study the behavior of its higher moments (skewness and kurtosis).

In the measurement of skewness and peak, such as the traditional moment measurement method (corresponding to the third standard moment and the fourth standard moment), its value is amplified to the third power and the fourth power, which is easy to be affected by outliers.

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Therefore, the robustness measurement of high order moment risk has been paid more attention by scholars. For example, GROENEVELD[1] and HOGG[2] proposed a quantile-based skewness measure. WHITE [3] and Tao[4] designed a kurtosis measurement method based on quantile; David [5] and Luis[6] proved the robustness and effectiveness of quantile-based high-order moment risk measurement method through Monte Carlo experiment and empirical research. To further investigate the dynamic high-order moment risk, Wen[7] and Yin[8] construct a mixed-frequency data quantile regression (MIDASQR) model and use it to estimate conditional quantiles, and give a skewness risk measurement method with time-varying characteristics. However, the robust measurement method of time-varying kurtosis risk needs further research.

Based on the above content, the innovative work of this research is as follows. Firstly, we propose a robustness measurement method of dynamic kurtosis risk based on MIDAS-QR model. By considering the time-varying characteristics of kurtosis risk in financial market and using high-frequency data information, the accuracy and robustness of kurtosis risk measurement are improved. Secondly, we use parametric portfolio strategy to introduce dynamic skewness risk and dynamic kurtosis risk into portfolio weight function and combine with constant absolute risk aversion (CARA) utility function to construct dynamic high order moment parametric portfolio decision model (BSK model). By greatly reducing the number of parameters to be estimated, the model effectively avoids the difficulty of solving the optimization with high order moment as the objective function. Third, we designed a three-step solution scheme, which can effectively distinguish the impact of asset characteristic variables, dynamic skewness risk and dynamic kurtosis risk on asset allocation weight and portfolio investment performance, and provide a basis for investors to dynamically modify portfolio investment selection and effectively prevent high-order moment risks. Finally, we conducted an empirical study on individual stocks and industry sector indices in China's stock market. The results show that the dynamic kurtosis risk measurement method based on MIDAS-QR model is effective and robust, and the P/E ratio, book value ratio and dynamic skewness risk have a significant positive impact on portfolio weight. Conditional volatility and dynamic kurtosis risk have significant negative effects on portfolio weight. These research results provide a better financial theory explanation for portfolio investment decision-making, and prove that the BSK model considering dynamic high-order moment risk is an effective tool, which can help investors grasp and avoid investment risks more comprehensively, and provide higher risk-adjusted returns.

2 Dynamic high order moment risk robustness measure

2.1 Skewness

Given the different holding periods, TAYLOR stressed the need and importance of studying multi-period returns. In this paper, we mainly discuss the risk characteristics of monthly (n=22) returns to weigh the timeliness and stability of data information and research results.

Considering the sensitivity of traditional skewness measurement methods to outliers, the robustness measure of skewness risk has been discussed in existing literature. Among them, GHYSELS et al. (4) proposed a quantile based skewness measurement method, which combined with Cornish-Fisher (CF) expansion, and fully considered the time variability of financial market risks. They propose a robust hybrid measure of dynamic skewness risk (1):

$$\mathrm{S}K_{INT,t-1}(r_{t,n}) = 6 \times RA_{INT,t-1}(r_{t,n}) \frac{\int_{0.5}^{1} q_{\alpha,t-1}(r_{t,n})d\alpha}{\int_{0.5}^{1} q_{\alpha,t-1}^{2}(r_{t,n})d\alpha} \quad (1)$$

Where $q_{\alpha,t-1}(\gamma_{t,n})$is the conditional quantile of the low frequency period (month) estimated using the high frequency (day) return information based on the MIDAS-QR model:

$$q_{a,t-1}(r_{t,n};\theta_{\alpha,n}) = \beta_{\alpha,n}^{(0)} + \beta_{\alpha,n}^{(1)} \sum_{d=0}^{D} \vartheta_d(K_{\alpha,n}) r_{t-1-d} \quad (2)$$

In addition, MIDAS-QR model is used to directly model the original high-frequency data, which avoids the problem of information loss caused by traditional cofrequency aggregation method, and improves the timeliness and accuracy of conditional quantile estimation.

2.2 Kurtosis

Similar to skewness, traditional momency-based demeanor measurement methods are easily affected by outliers. Therefore, HOGG proposed a quantile based kurtosis measurement method:

$$\mathrm{R}K_{INT}(\tau_{t,n}) = \frac{\beta}{\alpha} \times \frac{\int_{1-\alpha}^{1} q_\tau(r_{t,n})d\tau - \int_{0}^{\alpha} q_\tau(r_{t,n})d\tau}{\int_{1-\beta}^{1} q_\tau(r_{t,n})d\tau - \int_{0}^{\beta} q_\tau(r_{t,n})d\tau} \quad (3)$$

In this paper, Ping[9] the quantile-based kurtosis measurement method in equation (3) is combined with the fourth-order CF expansion, while considering the time-varying kurtosis risk in financial markets, and a hybrid method for robust measurement of dynamic kurtosis risk under conditional information set Ω_{t-1} is given:

$$\mathrm{K}R_{INT,t-1}(r_{t,n}) = \frac{24\times\int_{1-\beta}^{1} F(\tau)d\tau \times \frac{\alpha}{\beta}A - 24\times\int_{1-\alpha}^{1} F(\tau)d\tau}{\int_{1-\alpha}^{1} G(\tau)d\tau - \frac{\alpha}{\beta}A\int_{1-\beta}^{1} G(\tau)d\tau} + \frac{\frac{2}{3}\times SK_{INT,t-1}^{2}(r_{t,n})\left\{\int_{1-\alpha}^{1} H(\tau)d\tau - \frac{\alpha}{\beta}A\int_{1-\beta}^{1} H(\tau)d\tau\right\}}{\int_{1-\alpha}^{1} G(\tau)d\tau - \frac{\alpha}{\beta}A\int_{1-\beta}^{1} G(\tau)d\tau} \quad (4)$$

$q_{\tau,t-1}(r_{t,n})$Estimated by MIDAS-QR model in equation (2), $\mathrm{R}K_{INT,t-1}(r_{t,n})$, a hybrid method for dynamic kurtosis risk measurement based on MIDAS-QR model, can effectively extract and timely supplement useful information from high-frequency data. The kurtosis risk measurement of $\mathrm{R}K_{INT,t-1}$ is not only time-varying, but also more timely, robust and accurate. Furthermore, the robust measure $\mathrm{S}K_{INT,t-1}$ of dynamic skewness risk and the robust measure $K_{INT,t-1}$ of kurtosis risk are calculated by combining equations (1) and (4).

3 Experiments

3.1 Simulated data experiment

Table1. Parameter estimation results of portfolio investment model.

Model	Baseline	M-V	B-MODEL	B-S	B-S-K
	(1)	(2)	(3)	(4)	(5)
*IFPLEDGE*t	-0.02	-0.03	-0.02	-0.05***	-0.03*
	(0.31)	(0.29)	(0.51)	(**0.00**)	(**0.07**)
*DTA*t	0.21***	0.27***	0.18**	0.09**	0.12***
	(0.00)	(0.00)	(0.01)	(0.03)	(0.00)
*TAT*t	-0.04	-0.02	-0.05	-0.03*	-0.03
	(0.12)	(0.59)	(0.12)	(0.10)	(0.17)
*BTM*t	-0.14***	-0.16***	-0.11***	-0.09***	-0.09***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
*ROA*t	0.60***	0.30	0.89***	0.27*	0.59***
	(0.00)	(0.28)	(0.00)	(0.08)	(0.00)
*SIZE*t	0.02**	0.03*	0.01	0.03***	0.02**
	(0.03)	(0.07)	(0.32)	(0.00)	(0.01)
*TOPHLD*t	0.06	-0.02	0.11	0.07	0.06
	(0.29)	(0.87)	(0.49)	(0.12)	(0.17)
*SOE*t	0.02	0.02	0.01	0.02	0.00
	(0.31)	(0.48)	(0.53)	(0.14)	(0.82)
__cons	-0.72**	-0.73	-0.90**	-0.65*	-0.07
	(0.03)	(0.17)	(0.03)	(0.10)	(0.76)
Year	YES	YES	YES	YES	YES
Indu	NO	NO	YES	YES	YES
N	7,268	3,344	3,924	3,591	3,677
R^2	0.0337	0.0461	0.0293	**0.6694**	**0.7043**

*, **, *** represent significant at the 10%, 5%, and 1% levels.

As can be seen from Table 1 the parameters of the dynamic high-order moment parametric portfolio investment model are estimated by using the three-step method to maximize the utility of investors. Under different risk aversion levels, we investigate the effects of asset characteristic variables, dynamic skewness risk and dynamic kurtosis risk on portfolio weight. By observing the parameter estimation results in Table 1, we can draw the following conclusions: First, conditional volatility has a significant negative impact on portfolio weight, while P/E ratio and book value ratio have a significant positive impact on portfolio weight. This indicates that investors are more inclined to hold stocks with lower conditional volatility, higher P/E ratios and higher book value ratios. Secondly, the coefficient of dynamic skewness risk is positive at the significant level of 1%, indicating that investors are more inclined to hold stocks with greater skewness risk. Third, the coefficient of dynamic kurtosis risk is significantly negative, indicating that dynamic kurtosis risk has a significant negative impact on portfolio weight. In other words, in an optimal asset allocation, the greater the dynamic kurtosis risk of a stock, the less weight it will be assigned. This is consistent with the investor's investment philosophy of minimizing kurtosis risk. Finally, for different risk aversion levels, asset characteristic variables, dynamic skewness risk and dynamic kurtosis risk have the same influence direction on portfolio weight. This shows that the parametric portfolio investment decision model constructed in this paper is robust and can provide a better mechanical explanation for portfolio investment choice. At the same time, Bayesian optimization can be introduced to realize the adaptive adjustment of parameters

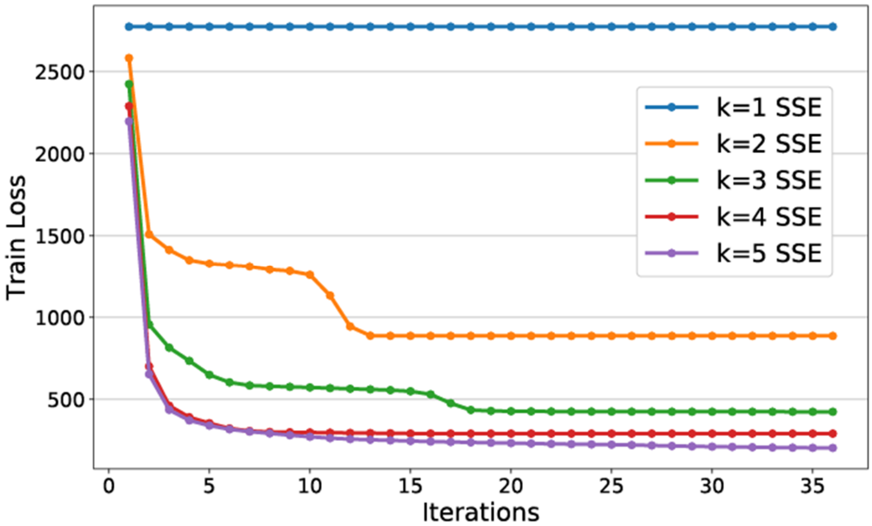

Fig. 1. Iterations & Train Loss.

As can be seen from Fig.1 through these results, investors can better understand the impact of different factors on portfolio investment weights and make effective asset allocation according to their own level of risk aversion. B-S-K model considering dynamic kurtosis risk also shows significant advantages in portfolio performance. First, B-S-K model performs best in terms of portfolio return, risk and risk-adjusted return. Specifically, the B-S-K model has the highest expected return, adjusted Sharpe ratio, and Sortino ratio, while the standard deviation and downside risk are the lowest. Therefore, B-S-K model can bring investors a higher level of returns and effectively diversify investment risks. Of course, we will also strengthen the real-time monitoring of market dynamics. According to market changes, the weight of the combination is adjusted in time to improve the real-time performance and investment effect of the model

3.2 Datasets

This research selects individual stocks in China's stock market as the research object, and randomly selects six stocks for representative analysis. The research data includes the daily closing price, price-earnings ratio and book-value ratio of these stocks. The data comes from Juling Finance and Economics, and the sample range is from January 4, 2012 to September 31, 2021. In addition, conditional volatility is calculated by mixing frequency data sampling method.

By statistical description of the daily returns of six stocks, it is found that the return series of six stocks do not follow normal distribution, and have obvious characteristics of peak and fat tail. Therefore, in portfolio investment decision-making, it is insufficient to pay attention to the first second moment of the return sequence, and it is necessary to consider the higher moment characteristics of the return sequence to get a better portfolio investment.

3.3 Model

The solution of B-K-S model is a nonlinear optimization problem, and the common techniques include genetic algorithm, particle swarm optimization algorithm and gradient descent algorithm. However, these methods usually optimize the objective function as a whole at one time, and there are difficulties in calculation pressure and initial value selection. In this paper, the three-step method based on BFGS algorithm is used to solve the parametric portfolio investment model under the dynamic high-order moment risk step by step. Specifically, the first step is to estimate the coefficients λB of the characteristic variables in the B model$\lambda_B = (\lambda_{VOL}, \lambda_{PE}, \lambda_{BTM})$; The second step is to fix the result of the first step and estimate the coefficient of dynamic skewness risk λ_{SK}; The third step is to estimate the dynamic kurtosis risk coefficient λ_{KR} based on the expected utility maximization of investors on the basis of the fixed parameter estimates obtained in the first two steps.

Compared with the solution method of overall optimization, the three-step method has obvious advantages: First, the number of parameters to be optimized in each step is relatively small, thus reducing the calculation pressure, shortening the optimization time, and improving the solving efficiency of the model; Secondly, the optimization of the latter step can take the results of the previous step as input, which effectively reduces the sensitivity to the initial value and increases the stability of the parameter estimation results. Thirdly, the three-step method can show the intermediate results of B-K-S model in stages, which is convenient for comparing and analyzing the specific impact of asset characteristic variables, dynamic skewness risk and dynamic kurtosis risk on portfolio investment weight and performance.

Through the three-step solution scheme, we can solve the B-K-S model more effectively, get accurate parameter estimation results, and better understand and analyze the influence of different factors on the investment portfolio. This provides investors with a more reliable and actionable basis for decision making.

In order to investigate the effectiveness of B-S-K model, this paper considers three evaluation indicators of return, risk and risk-adjusted return, including expected return, standard deviation (σ), downside risk (δ), adjusted Sharp ratio (AShR) and Sortino ratio (SoR). Among them, the AShR proposed by YUE[10] and WANG[11] makes up for the deficiency in the definition of Sharp ratio under the normal distribution assumption, which is defined as follows:

$$AshR = ShR \times \left[1 + \left(\frac{S}{6}\right) \times ShR - \left(\frac{K}{24}\right) \times ShR^2\right] \tag{5}$$

Where S is the third-order standard moment of portfolio investment return; K is the fourth standard moment of portfolio return. Further combined with formula (5), we can successively obtain: reference (B) model, B-S model, B-S-K model.

3.4 Robustness test

Table 2. Independent Samples Test.

	t-test for Equality of Means		
	t	Sig. (1-tailed)	Mean Difference
DTA	-.253	.400	-0.001
TAT	.553	.285	0.004
BTM	-2.414	.008***	-0.040
ROA	-.617	.265	-0.001
SIZE	-3.305	.000***	-0.086
TOPHLD	-.245	.402	-0.001
SOE	-.730	.223	-0.008

As can be seen from Table 2 in rder to verify the robustness and validity of the dynamic high-order moment parametric portfolio investment decision model (B-S-K) constructed in this paper, we further analyze the portfolio investment results of 18 industry sector indexes, including the results of model parameter estimation and performance (detailed results are omitted).

Here, we also consider contrast models such as equal weight scheme, M-V model, B model and B-S model, and also consider different risk aversion levels of low, medium and high, corresponding to γ values of 3, 5 and 7, respectively.

Based on the above analysis results, we find that the results obtained by using the index data of 18 industry sectors are consistent with the results obtained by using the data of six individual stocks. Therefore, we can conclude that the parametric B-S-K model constructed in this paper is not only applicable to the portfolio investment decision-making problem between single stocks requiring flexible allocation, but also can be applied to the research of industry sector indexes that reflect the overall development trend of the market and have medium and long-term investment value, and has good robustness and effectiveness.

4 Conclusion

In this paper, a dynamic high-order moment parameterized portfolio investment decision model (B-S-K) is proposed. There are two key aspects to the construction of this model. Firstly, based on MIDAS-QR model, we propose a robustness measurement method for dynamic kurtosis risk, which improves the timeness, robustness and accuracy of dynamic kurtosis risk measurement by fully mining the original high-frequency data information. Secondly, we introduce a parametric portfolio investment framework, considering dynamic skewness risk and dynamic kurtosis risk, and build a parametric portfolio investment model including dynamic high-order moment risk, and use BFGS algorithm to give a three-step solution. This model not only greatly reduces the number of parameters to be estimated and avoids the difficulty of solving the portfolio investment model with high order moment as the objective function, but also quantifies the specific contributions of asset characteristic variables, dynamic skewness risk and dynamic kurtosis risk to portfolio investment weight and performance, and increases the interpretability of portfolio investment selection. It provides the decision basis for investors to dynamically modify asset allocation.

In order to verify the validity of the B-S-K model, we selected six individual stocks and 18 industry indices of the Chinese stock market for empirical research. The results show that: First, the robustness of dynamic high-order moment risk is an effective method to measure high-order moment risk in financial markets, and has little influence on outliers; Secondly, dynamic skewness risk and dynamic kurtosis risk have a significant impact on portfolio investment decisions. Investors with different risk aversion levels prefer stocks with greater skewness risk and smaller kurtosis risk when choosing portfolio investments. Third, conditional volatility, P/E ratio, book value ratio, dynamic skewness risk and dynamic kurtosis risk are significantly correlated with portfolio investment weight, which provides a good financial mechanistic explanation for portfolio investment decision. Finally, compared with equal-weight scheme, M-V model, benchmark (B) model and B-S model, the B-S-K model constructed by us shows significant and stable advantages in terms of return, risk and risk-adjusted return.

References

[1] GROENEVELD, MEEDEN G. Measuring skewness and kurtosis [J]. The Statistician,31(3):129–137, 2013.

[2] HOGG R V. Adaptive robust procedures: A partial review and some suggestions for future applications and theory[J]. The Review of Financial Studies,25(2):623-675, 2019.

[3] Elias S Sotirchos, Kathryn C Fitzgerald, and Ciprian M Crainiceanu. Report- ing of r2 statistics for mixed-effects regression models. JAMA neurology, 76(4):507–507, 2020.

[4] Tao Jing, Fangjin Hao, and Zhiqian Luan. Equity pledge, interest rate level and stock price crash wind. Review of Investment Studies, 3(65-80), 2020. ISSN 1003-7624.

[5] David L Hepner and Mariana C Castells. Anaphylaxis during the periope rative period. Anesthesia & Analgesia, 97(5):1381–1395, 2021.

[6] Jos´e Luis Montiel Olea and James Nesbit. (machine) learning parameter regions. Journal of Econometrics, 222(5):725–758, 2021.

[7] Wen Zhongling, Hou Jietai, and Zhang Lei. Comparison and application of moderating and mediating effects(in chinese). Acta Psychologica Sinica, 37(2):268–274, 2019. ISSN 0439-755X.

[8] Yugang Yin and Rongfu Tian. Investor sentiment, financial report qual- ity and stock price crash risk: Role of short-sales constraints. Emerging Markets Finance and Trade, 53(3):493–510, 2021.

[9] Ping Yu and Peter CB Phillips. Threshold regression with endogeneity. Journal of Econometrics, 203(1):50–68, 2020.

[10] Miao Wang, Liu Chunchen, and Gen Zhi. Statistical approaches for causal inference (in chinese). Sci Sin Math, 48(12):1753–1778, 2019.

[11] Wuyi Wang, Peter CB Phillips, and Liangjun Su. Homogeneity pursuit in panel data models: Theory and application. Journal of Applied Econometrics, 33(6):797–815, 2020.