An Optimal Bilateral Contract Transaction Model for Natural Gas Suppliers and Large Consumers Based on Mixed Game Theory

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Abstract: In a gradual liberalization environment of natural gas market, the competitions in bilateral contract transacting include not only the competitions between the contracting parties, and generation companies and large consumers, but also the competitions in contracted natural gas prices between multiple suppliers to obtain more gas supply. In this context, a mixed game model is constructed for bilateral contract transactions between natural gas multiple suppliers and large consumers. The objective of the game is to determine the optimal contract price by playing a non-cooperative Bertrand game and Stackelberg game with other suppliers in order to maximize the profitability of the gas sold through bilateral contracts, while the large consumers choose their gas purchase strategy based on the price offered by each supplier, the forecast of the spot LNG price, and the gas demand at each time period in order to minimize their gas purchase cost. Moreover, the existence proof and solution of the Nash equilibrium for the game model are presented. Through scenario design and simulation, the results show that the mixed game bilateral optimal trading model proposed in this study effectively coordinates the bilateral trading between gas suppliers and large users in the natural gas sales market, and both suppliers and large users can benefit from participating in the bilateral contract.

Keywords: natural gas market; multi-agent modeling; bilateral contract; mixed game; Nash equilibrium

1 Introduction

China has become a big energy consumer. In order to achieve the carbon peaking and carbon neutrality goals to reduce carbon emissions, green and clean energy is the best choice. However, it takes a process to move from the consumption of fossil energy to the completely use of green and clean energy. At this time, the role of natural gas becomes prominent, since the natural gas has become a bridge for the transition from fossil energy to green and clean energy in worldwide[1]. With the deepening of China's upstream natural gas market reform [2], the monopoly of the domestic natural gas industry is gradually being broken [3]. As a crucial component of China's gas market reform, the natural gas sector is transitioning from monopolistic control to regulated deregulation and ultimately towards marketization[4], thereby fully leveraging the benefits of competitive markets.

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With the development of natural gas reform, bilateral contract trading has gradually become a hot topic in natural gas market research. The previous approach to natural gas price prediction relied on time series models [5]. However, due to the immature domestic natural gas market and limited historical data, the predictive efficacy of time series models for China's LNG market regional average transaction price is unsatisfactory. However, Li F (2021) [6] has introduce a new time series analysis method that has been used to study the development of natural gas in China. Chai J (2021) [7] used the NARDL model to analyze the impact of price spreads on market imbalance in the natural gas sector, while the TVP-SV-VAR model was utilized to examine the intensity of this impact on the natural gas pipeline market under different event contexts. The findings demonstrate that the asymmetric adjustment effect of the dual-track pricing system on the natural gas market imbalance serves as a crucial reference for future reforms in China's natural gas pricing mechanism. The study conducted by Rui X [9], utilizing the trading market cooperative game model and bibliometric analysis, elucidated that for China's proposed market-oriented natural gas trading platform to be successful, it is imperative to ensure competitive gas sources, enable independent operation of the official website to facilitate "third-party access," fully liberalize control over natural gas prices, and establish a pricing mechanism based on market supply and demand. On the basis of Granger causality test, Nie (2014) [11] explores the transmission effect of natural gas price and the influence degree of price fluctuation on various economic variables through the construction of VAR model, and provides reference for the formulation and implementation of natural gas price reform. Wang [12] and Jiang T [14] both studied the electricity market and the natural gas market, and concluded that the cooperative joint venture mode has a better effect in alleviating the price fluctuation of the natural gas market, improving the market competitiveness of power suppliers and the total social welfare of the electricity and gas markets. And the possibility that vertically integrated firms with assets in both gas and electricity markets manipulate gas and electricity prices by withholding gas pipeline capacity. By establishing a linear programming model for natural gas sales of gas suppliers in line with the reality of China's natural gas market, Chen (2020) [15] suggests that gas suppliers determine constraint conditions and establish a natural gas sales optimization model suitable for themselves, so as to provide decision-making basis for natural gas sales and maximize the benefits of natural gas sales. Gong (2022) [16] uses Stackelberg game to consider the pricing mechanism and income situation of stakeholders in the natural gas market under consumers' low-carbon preference and enterprises' carbon emission reduction level, and studies the decision-making mode of different subjects under their own leading and centralized decision-making. The results show that in order to maximize the overall benefits of the supply chain, the natural gas market should adopt the way of centralized decision-making to further promote the marketization reform of natural gas in China. Liu S (2023) and Gong C (2020) both studied the natural gas supply market, and proposed the multi-objective optimization model and the elasticity evaluation method of natural gas supply system based on ecological network analysis respectively, putting forward reasonable suggestions for improving the resilience of natural gas supply in China. The research subjects of the aforementioned literature primarily consist of either a sole gas supplier and a solitary major user, or a single major user in conjunction with multiple gas suppliers. However, in the open natural gas market environment, there should also be a competitive relationship between gas supply enterprises: the signing of bilateral contracts should not be limited to one-to-one interactions, but rather involve multiple parties in the process. Gas supply enterprises have the flexibility to establish contracts with multiple large users, while large users also have the option to enter into agreements with multiple gas supply enterprises.

The primary challenges faced by gas supply enterprises and large users during contract transactions involve the selection of bilateral contract quotations by gas supply enterprises to maximize profits, as well as the determination of natural gas contract quantities by large users to minimize their own procurement costs.

Under the above background, based on game theory, this study examines the natural gas market characterized by bilateral transactions between multiple gas suppliers and large users. Gas suppliers engage in competitive bidding to secure a greater share of contracts. Large users devise purchasing strategies within the contract period based on quotations from each supplier and predicted spot market prices for natural gas during each contractual period. Specifically, these strategies encompass determining the volume of contracted natural gas from each supplier as well as the volume of natural gas purchased from the spot market. Consequently, a master-slave game model is formulated, encompassing the non-cooperative game among the upper gas suppliers and the optimization of gas procurement costs for lower large users. Subsequently, a solution algorithm for this game is presented, followed by its verification and analysis through an illustrative example.

2 Game-theoretical models in natural gas sales market

In order to describe and simulate the multi-agent operation of the natural gas market, a non-cooperative static game under complete information between gas suppliers (Bertrand model) and a non-cooperative dynamic game under complete information between gas suppliers and large users (Steinberg model) are constructed. The players are not only familiar with their own strategy space and profit function, but also possess knowledge of the strategy space and profit function of other players. Due to the influence of various complex environments in the multi-agent modeling of the natural gas market, the complexity of the model will be significantly heightened. In this game model, we consider the physical environment (such as pipeline transportation capacity and gas storage scale), social environment (including macroeconomic development, foreign trade, and industry cycles), and random factors (such as epidemic impacts, extreme weather events, natural gas leakage, and explosion accidents) among agents as static variables. The focus is on analyzing game behaviors and strategies (e.g., pricing, supply and demand) among agents. Additionally, an equilibrium solution method for mixed games is also developed.

Without loss of generality, a bilateral transaction scenario is established among multiple gas suppliers and multiple large users (where urban gas users can be simplified as a specific type of large user). Assuming that the bilateral contract transaction spans a certain time period, with I number of gas suppliers and J number of large users participating in the transaction. In the initial stage of contract signing, gas supplier i's contract quotation to large user j is $(p_{i,j}, b_i)$ Where $p_{i,j}$ is the initial gas price of the contract, b_i is the growth coefficient of gas price with respect to contract gas volume also treated as the slope of the offer curve. Therefore, at the $(t = 1,2, \dots, T)$ time, when the contract gas volume signed by large user j at supplier i, its contract gas price $p_{i,j}$ shall be $p_{i,j}^t = p_{i,j} + b_i q_{i,j}^t$. After getting the quotation of gas suppliers, large users decide the contract gas volume signed with each gas supplier according to the quotation of each gas supplier, the forecast real-time spot gas price, and the gas demand of each period. Large users can reduce their own gas purchase cost and gas consumption risk by optimizing gas purchase from spot market and multiple gas suppliers. The gas market is characterized by intense competition among suppliers. Figure 1 illustrates the decision relationship in the master-slave game between multiple gas suppliers and large industrial customers.

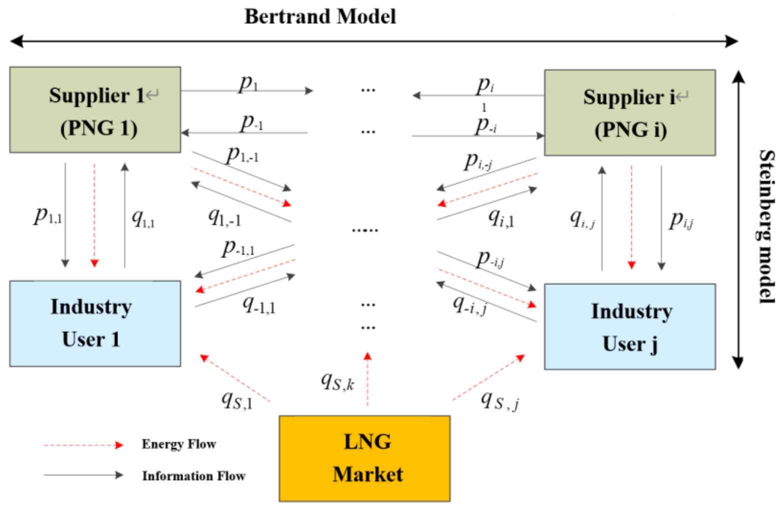

Figure1: Natural gas market participant relationship diagram.

In Figure 1, $p_{-i} = [p_1, p_2, \dots, p_{i-1}, p_{i+1}, \dots]$ is the combination of contract prices and spot gas prices of all other gas suppliers except p_i, where $p_i = [p_{i,1}, p_{i,2}, \dots, p_{i,J}]$represents gas supplier i 's contract quotation group for all large users, $P_S = [P_S^1, P_S^2, \dots, P_S^T]$ is the collection of spot gas prices at each point in the contract period. $p_{-i,j} = [p_{1,j}, p_{2,j}, \dots, p_{i-1,j}, p_{i+1,j}, \dots]$represents the combination of contract quotation and spot gas price of large user j by other gas suppliers except gas supplier i, $p_{i,-j} = [p_{i,1}, p_{i,2}, \dots, p_{i,j-1}, p_{i,j+1}, \dots, p_{i,J}]$ represents the collection of contract offers made by supplier i to other large users other than large user j, $q_{i,j} = [q_{i,j}^1, q_{i,j}^2, \dots, q_{i,j}^T]^T$ represents the collection of the optimal contract gas volume of the large user j at the supplier i at all times during the contract duration period T. And the $q_{S,j} = [q_{S,j}^1, q_{S,j}^2, \dots, q_{S,j}^T]^T$, where the $q_{-i,j}$ and $q_{i,-j}$ are defined similarly to $p_{-i,j}$ and $p_{i,-j}$, respectively.

In this mixed game model, given the hierarchical positioning of the gas supplier at the upper level and the big user at the lower level, it is imperative to employ backward induction in accordance with the game relationship. This entails initially constructing the big user model, followed by establishing the supplier model (which encompasses non-cooperative games among gas suppliers), and ultimately resolving for a mixed game Nash equilibrium.

2.1 Industrial customer gas purchase model

The large industrial users of natural gas are in the lower level of the master-slave game and can only passively accept the contract quotation given by various gas suppliers, but the gas purchasing strategy of large users will affect the final income of gas suppliers. The optimization model of large user j is as follows:

$$Min\sum_{t=1}^{T}(p_j^t+b^TQ_j^t)^TQ_j^t$$

$$s.t\begin{cases}0\le q_{i.j}^t\le \bar{q}_{i,j}^t\\ \sum_{i=1}^{I}q_{i.j}^t=D_j^t\\ q_{S,j}^t\ge 0\end{cases} \tag{1}$$

Where $p_j^t=\left[p_{1,j},p_{2,j},\dots,p_j^I,P_S^t\right]^T$; P_S^t is the LNG spot gas price in the predicted time period t; $b=diag\left[b_1,b_2,\dots,b_I,0\right]$ is a diagonal matrix; $Q_j^t=\left[q_1^t,q_2^t,\dots,q_{I,j}^t,q_{S,j}^t\right]^T$; $q_{i,j}^t$ is the contract gas volume of time t in the bilateral contract signed by large user j with gas supplier i; $q_{S,j}^t$ is the amount of gas purchased by large user j in the LNG spot market during the t period; $\bar{q}_{i,j}^t$ is the upper limit of contract gas volume that large user j can obtain from gas supplier i during period t; D_j^t is the gas demand of large user j during period t.

It should be noted here that the gas purchase strategy of large users does not have any impact on the LNG spot market price, and its data can be accurately predicted based on historical LNG data. In addition, this paper assumes that the gas supplier has an independent quotation curve for each major user (user type difference), that is, the gas supplier's final contract gas price for each major user is only affected by the gas purchase strategy of the major user. Since the gas purchase arrangement of large users is independent from each other, it is only affected by the spot gas price and contract gas price of LNG during the period, so the gas purchase optimization result of large users can be regarded as the set of individual optimization results of each period. The optimization objective of each period is a strictly convex quadratic programming problem with a non-empty feasible solution set. For this optimization problem, when b_i is a constant positive value and p_j^t is a parameter matrix, the problem (1) has a solution for any p_j^t, its optimal solution ${q_{S,j}^t}^*={q_{S,j}^t}^*(p_j^t)$ is unique, and ${q_{S,j}^t}^*(p_j^t)$ is a piecewise linear function of p_j^t.

2.2 Gas supplier model

Gas suppliers are at the upper level of the master-slave game. For a certain gas supplier, if the bidding strategies of other gas suppliers remain unchanged, it can choose its own quotation by predicting the gas purchasing strategies of large users to maximize its gas selling profits. The purpose of this paper is to maximize the revenue of the gas supplier in the bilateral contract market, so the profit of the gas supplier in the LNG spot market is ignored in the modeling (it is assumed that the gas supplier's decision does not affect the LNG spot market price). When it comes time to sell in the LNG spot market, suppliers can optimize their decisions in the LNG spot market based on their existing bilateral contracts.

The gas sales revenue of supplier i is the sum of the gas purchase costs of all large users at supplier i:

$$\sum_{j=1}^{J}(p_{i,j}E_{1J}q_{i,j}+b_iq_{i,j}^Tq_{i,j}) \tag{2}$$

Where E_{1J} is the identity matrix of order $1\times$ J; The gas supply cost of supplier i can be fitted with a quadratic function:

$$\left(P_iE_{1J}+B_i(\sum_{j=1}^{J}q_{i,j})^T\right)\sum_{j=1}^{J}q_{i,j} \tag{3}$$

Where: P_i and B_i are the cost coefficients of the gas supplier. Therefore, the gas profit function of gas supplier i can be expressed as follows:

$$\sum_{j=1}^{J}(p_{i,j}E_{1J}q_{i,j} + b_i q_{i,j}^T q_{i,j}) - \left(P_i E_{1J} + B_i(\sum_{j=1}^{J} q_{i,j})^T\right)\sum_{j=1}^{J} q_{i,j} \tag{4}$$

Since b_i is fixed for each supplier, the supplier's strategy only includes its initial contract offer portfolio p_i for large users. For any strategy $(p_{i,j}, p_{-i,j})$, large users will have the corresponding optimal gas purchase strategy $q^*(p_{i,j}, p_{-i,j})$. Therefore, the profit function of gas supplier i can be rewritten as follows:

$$f_i(p_i, p_{-i}) = \sum_{j=1}^{J}(p_{i,j}E_{1J}q_{i,j} - P_i E_{1J})q_{i,j}^* + \sum_{j=1}^{J} b_i\, q_{i,j}^{*T} q_{i,j}^* - B_i(\sum_{j=1}^{J} q_{i,j}^*)^T \sum_{j=1}^{J} q_{i,j}^* \tag{5}$$

Similarly, the profit function is strictly convex quadratic programming problem with a non-empty feasible solution set, so $q_{i,j}^{t*}(p_{i,j}, p_{-i,j})$ is piecewise linearly smooth, $q_{i,j}^*(p_{i,j}, p_{-i,j})$ is also piecewise linearly smooth, and thus the profit function is also piecewise smooth in the feasible domain. Therefore, the problem of maximizing profit of gas suppliers can finally be expressed as follows:

$$\begin{cases} \max f(p_{i,j}, p_{-i,j}) \\ s.t\ p_{i,j} \in [P_{min}, P_{max}] \end{cases} \tag{6}$$

Where P_{min}, P_{max} are respectively the lower limit and upper limit of the gas supplier contract quotation.

3 Mixed game equilibrium solution

3.1 Game equilibrium analysis

The relationship between gas suppliers and large users can be characterized as a master-slave game. In order to establish the existence of a game equilibrium solution, it is necessary to demonstrate the quasi-concavity of the profit function within the strategy space for both types of players. Furthermore, proving the quasi-concavity of the profit function for gas suppliers also serves as a basis for establishing the existence of a Nash equilibrium solution.

In general, for a game with players numbers of I, if the payment function of player i is $f_i(x_i, x_{-i})$, and its strategy set is X_i, when the whole game process has the following properties:
① The policy space $X_i \in R_i^n$ is a non-empty set of real numbers.
② For any $x_{-i} \in X_{-i} = \prod X_j$, the set $X_i(x_{-i}) := \{x_i \in X_i : f_i(x_i, x_{-i}) \geq 0\}$ is a non-empty convex set;
③ For any $x_{-i} \in X_{-i}$, the function $f_i(\cdot, x_{-i})$ is quasi-concave in the set $X_i(x_{-i})$;
④ $f_i(\cdot)$ is continuous in space $\prod X_i$. So, there is at least one Nash equilibrium in this game which makes the profit for all players non-negative;
To prove that the non-cooperative game between gas suppliers satisfies the above four properties, the following propositions need to be proved first.

For a given $p_{-i,j}$, define $\partial_{i,j} q_{i,j}^{*}(p_{i,j}, p_{-i,j})$ as the generalized gradient of $q_{i,j}^{*}(p_{i,j}, p_{-i,j})$. According to the model, for any gas supplier $i = 1,2, \ldots, I$, the contract gas quantity $q_{i,j}^{t*}(p_{i,j}, p_{-i,j})$ signed with large user j during the period t is a non-increasing function for $p_{i,j}$ when the quotation strategy p_{-i} of others and its own quotation strategy $p_{i,-j}$ for other large users are unchanged. And the directional derivative of $q_{i,j}^{t*}(p_{i,j}, p_{-i,j})$ in any direction satisfies:

$$\left| q_{i,j}^{\dot{t}*}(p_{i,j}, p_{-i,j}) \right| \leq \frac{|u|}{2b_i} \tag{7}$$

That is, $\partial_{i,j} q_{i,j}^{*}\left(p_{i,j}, p_{-i,j}\right) \subseteq \left[-\frac{1}{2b_i}, 0\right]$, indicating that the contract gas volume signed by the gas supplier and the large user decreases monotonically with the increase of the initial contract quotation. Consider the mapping $p_{i,j} \mapsto q_{i,j}^{t*}\left(p_{i,j}, p_{-i,j}\right)$ at time t, where $p_{-i,j}$ is known to be invariant, $q_{i,j}^{t*}\left(p_{i,j}, p_{-i,j}\right)$ is piecewise linearly smooth with respect to $p_{i,j}$, and the derivative of $q_{i,j}^{t*}\left(p_{i,j}, p_{-i,j}\right)$ with respect to $p_{i,j}$ has a breakpoint. If the number of breakpoints is W (the value of W depends on the number of KKT conditions), then except for W breakpoints $0 \leq \omega_1 = \omega_1\left(p_{-i,j}\right) \leq \cdots \leq \omega_W = \omega_W\left(p_{-i,j}\right)$, $q_{i,j}^{t*}\left(p_{i,j}, p_{-i,j}\right)$ is differentiable with respect to $p_{i,j}$ in the feasible domain, and the derivative is $q_{i,j}^{\dot{t}*}(p_{i,j}, p_{-i,j})$. At each break point ω_k, there is at least one gas supplier h, whose contract gas volume is zero a certain period of time t for the large user j, that is, $q_{h,j}^{t*}\left(p_{i,j}, p_{-i,j}, p_{-i}\right) = 0$. At the break point and near the break point, $q_{i,j}^{\dot{t}*}(p_{i,j}, p_{-i,j})$ has the following properties:

$$\lim_{v \uparrow \omega_{i,j}} q_{i,j}^{\dot{t}*}\left(v, a_{-i,j}\right) \geq \lim_{v \downarrow \omega_{i,j}} q_{i,j}^{\dot{t}*}\left(v, a_{-i,j}\right) \tag{8}$$

Where $v \uparrow \omega_{i,j}$ represents $v \to \omega_{i,j}$ and $v < \omega_{i,j}$; The definition of $v \downarrow \omega_{i,j}$ is similar. Under this property, suppose that for the contract gas volume of t period signed by the gas supplier i and the large user j, if any $p_{-i,j}$ and p_{-i} meet the breakpoint $\omega_{i,j}$ of $f_i\left(\omega_{i,j}, p_{i,j}, p_{-i}\right) > 0$, if equation (8) is satisfied, then there is at least one Nash equilibrium in the game problem between gas suppliers formed by equation (6). In order to prove the validity of equation (8), it is necessary to prove that the game between gas suppliers and large users satisfies the following four properties:

(a)The policy space $X_I \subseteq R_i^n$ is a non-empty set of real numbers, and this condition is obviously satisfied;

(b) For any $x_{-i} \in X_{-i} = \prod X_j$, the set $X_i(x_{-i}) \coloneqq \{x_i \in X_i : f_i(x_i, x_{-i}) \geq 0\}$ is a non-empty convex set. The profit function of the gas supplier i is equation (5). In this paper, it is assumed that the profit function of the gas supplier is always positive under a limited quotation interval, $p_{i,j} > P_i \gg b_i > B_i$, and the gas supply does not exceed the upper limit, Therefore, the set $X_i(x_{-i}) \coloneqq \{x_i \in X_i : f_i(x_i, x_{-i}) \geq 0\}$ is a non-empty convex set.

(c)For any $x_{-i} \in X_{-i}$, the function $f_i(\cdot, x_{-i})$ is quasi-concave in the set $X_i(x_{-i})$. Consider the gas supplier's return $f_i^t\left(p_{i,j}, p_{i,-j}, p_{-i}\right)$ at time t, and let $\omega_1, \omega_2, \ldots, \omega_W$ represent the breakpoints of the derivative of $q_{i,j}^{t*}\left(p_{i,j}^t, p_{-i,j}^t\right)$, where the derivative $f_i^{t'}\left(p_{i,j}, p_{i,-j}, p_{-i}\right)$ of the profit function $f_i^t\left(p_{i,j}, p_{i,-j}, p_{-i}\right)$ does not exist. In the interval (ω_j, ω_{j+1}), the second derivative of the profit function $f_i^t\left(p_{i,j}, p_{i,-j}, p_{-i}\right)$ of the gas supplier is:

$$f_i^{t''}(a_{i,j}) = 2\dot{q}_{i,j}^{t*} + (2b_i\dot{q}_{i,j}^{t*} - 2B_i\dot{q}_{i,j}^{t*}) = 2[1 + (b_i - B_i)\dot{q}_{i,j}^{t*}]\dot{q}_{i,j}^{t*} \tag{9}$$

Since $\dot{q}_{i,j}^{t*} \subseteq [-1/2b_i, 0]$, we know that $f_i^{t''}(a_{i,j}) \leq 0$ is always true. So $f_i^{t'}(p_{i,j}, p_{i,-j}, p_{-i})$ does not increase at any two breakpoints. Next consider the concavity of $f_i^t(p_{i,j}, p_{i,-j}, p_{-i})$ near the break point. According to inequality (8), we have the following inequality:

$$\begin{aligned}\lim_{v\uparrow\omega_j} f_i^{t'}(v, p_{-i,j}, p_{-i}) &= \lim_{v\uparrow\omega_j}\begin{bmatrix} q_{i,j}^{t*}(v, p_{-i,j}) + 4(v - P_i)\dot{q}_{i,j}^{t*}(v, p_{-i,j}) \\ +2\left[b_i q_{i,j}^{t*T} - B_i \sum_{j=1}^{S} q_{i,j}^{t*}(v, p_{-i,j})\right]\dot{q}_{i,j}^{t*}(v, p_{-i,j}) \end{bmatrix} \\ &\geq \lim_{v\downarrow\omega_j}\begin{bmatrix} q_{i,j}^{t*}(v, p_{-i,j}) + (v - P_i)\dot{q}_{i,j}^{t*}(v, p_{-i,j}) \\ +2\left[b_i q_{i,j}^{t*T} - B_i \sum_{j=1}^{S} q_{i,j}^{t*}(v, p_{-i,j})\right]\dot{q}_{i,j}^{t*}(v, p_{-i,j}) \end{bmatrix} \\ &= \lim_{v\downarrow\omega_j} f_i^{t'}(v, p_{-i,j}, p_{-i})\end{aligned} \tag{10}$$

In formula (10), it can be obtained by $v > P_i \geq b_i \geq B_i$, and $v - P_i + 2b_i q_{i,j}^{t*T} - 2B\sum q_{i,j}^{t*}(v, p_{-i,j}) > 0$ is always valid in the feasible domain. Combining equations (9) and (10), we can see that $f_i^t(p_{i,j}, p_{i.-j}, p_{-i})$ is a concave function of $p_{i,j}$ in the feasible domain, and thus we can further see that $f_i(p_{i,j}, p_{i,-j}, p_{-i})$ is a concave function of $p_{i,j}$ in the set $X_i(x_{-i})$.

(d) $f_i(\cdot)$ is continuous in space $\prod X_i$; Since $q_{i,j}^{t*}(\cdot, p_{-i,j})$ is piecewise linear and continuous with respect to $p_{i,j}$. Therefore, $f_i(\cdot)$ is also continuous in space $\prod X_i$.

3.2 The steps to achieve game equilibrium solution

Since the four properties of the game equilibrium are satisfied and the inequality (8) is satisfied, the game between gas suppliers has a Nash equilibrium solution. The solution of the game Nash equilibrium is as follows: Firstly, the gas purchasing strategy of the large user is determined by the quotation of each gas supplier, and then the gas supplier takes the gas purchasing strategy of the large user as the input to play the game with other gas suppliers to obtain the game equilibrium solution.

Suppose that the combination of the gas supplier's quotation to the large user j and the LNG spot gas price at the time period t is $p_j^t = [p_{1,j}, p_{2,j}, \ldots, p_{I,j}, P_S^t]^T$, then the function of the optimized gas purchase cost of the large user at this time is as follows:

$$Min\ (p_j^t + b^T Q_j^t) \tag{11}$$

The KKT condition for optimization problem (4) is:

$$\begin{cases} p_{i,j} + 2b_i q_{i,j}^t - \lambda_i - u = 0 & i = 1,2,\ldots,I \\ \lambda_i \geq 0, q_{i,j}^t \geq 0, \lambda_i q_{i,j}^t = 0 & i = 1,2,\ldots,I \\ P_S^t - \lambda - u = 0 \\ \lambda \geq 0, q_{S,j}^t \geq 0, \lambda q_{S,j}^t = 0 \\ q_{1,j}^t + q_{2,j}^t + \cdots + q_{I,j}^t + q_{S,j}^t = D_j^t \end{cases} \tag{12}$$

By solving the above KKT conditions, the optimal gas purchase strategy Q_j^{t*} for large users corresponding to p_j^t can be obtained. Then, the optimal gas purchasing strategy of large user j

in the whole contract period T can be obtained. Similarly, the optimal gas purchasing strategy of other large users can also be obtained. After taking the optimal gas purchase strategy of large users as the input of the bidding strategy of gas suppliers, the whole master-slave game problem can be reduced to a non-cooperative Bertrand competitive game problem among I gas suppliers, that is, problem (6), which is solved by the following steps:

Step 1: Initialize parameters.

Step 2: Randomly select the bidding strategy of gas supplier $i(i = 1,2,\dots,I)$ in the feasible region p_i.

Step 3: For supplier $i(i = 1,2,\dots,I)$, calculate $f_i\,(p_i, p_{-i})$ at this time, and maximize the profit $f_i^*\,(p_i, p_{-i}) = f_i\,(p_i, p_{-i})$ for supplier i.

Step 4: For gas supplier $i(i = 1,2,\dots,I)$, find the solution p_i^* of problem (2) and its corresponding $f_i\,(p_i^*, p_{-i})$ by traversing the feasible domain; If $f_i\,(p_i^*, p_{-i}) > f_i^*\,(p_i, p_{-i})$, let $p_i = p_i^*$ and $f_i^*\,(p_i, p_{-i}) = f_i\,(p_i^*, p_{-i})$.

Step 5: Repeat Step 4 until all suppliers do not modify their contract offers and Nash equilibrium is reached. At this time the quotation of each gas supplier is the quotation of each gas supplier under the Nash equilibrium solution.

4 Simulation and application

4.1 Scenario

With the deepening of the market-oriented reform of natural gas, local gate station prices have been gradually abolished, and the market-oriented pricing of bilateral contracts between gas suppliers and large users has become a trend. According to the mixed game model of the natural gas sales market established in this paper and the requirements of bilateral contracts in the natural gas sales market, the following game scenarios are constructed for simulation analysis: The contract is designed in an annual cycle and executed monthly, starting from April of the current year and ending in March of the next year. The contract period T =12; Contract prices are divided into two types: annual contract price (unified price), off-season contract price (off-season price, peak season price). The off-season refers to April to October of the year, a total of 7 months, and the peak season refers to November of the year to March of the next year, a total of 5 months. This paper assumes that the participating bilateral contracts in a regional natural gas sales market involve three major gas suppliers (CNPC, Sinopec, CNOOC) and three industrial large users. Among them, the three gas suppliers hold dominant positions in the game and determine their contract offers to the large users through non-cooperative competition based on the Bertrand Model. The three industrial consumers, operating at a lower level in the game, will employ the Stackelberg game to determine the contracted gas volumes with each gas supplier based on their prices and the spot market price of LNG, ensuring their own gas demand. In accordance with the game model, Table 1 presents relevant parameters for the three gas suppliers.

Table 1 Cost coefficient and parameters of gas supplier.

Parameter	Symbol	Gas supplier 1	Gas supplier 2	Gas supplier 3
Cost Coefficient	B	0.01	0.006	0.004
Adjustment Coefficient	b	0.01	0.008	0.005
Supply Capacity	$\bar{q}$	130	180	260
Upper Price limit	P_{max}	4.10	4.10	4.10
Lower Price limit	P_{min}	2.84	2.85	2.85

Table 1 reflects the difference between the supply cost and supply of each gas supplier, as well as the supply capacity constraints of each gas supplier. It should be noted here that although the bilateral contract between the gas supplier and the large user adopts market-oriented pricing, the contract price cannot exceed the price ceiling set by the regional government, nor will it be lower than the price expected lower limit of each gas supplier. Due to the differences in monthly gas source costs of each gas supplier, the supply cost of each gas supplier, the demand of each user, and the spot LNG market price are given based on historical data, as shown in Table 2.

In Table 2, it can be seen that there are differences in the demand of natural gas industrial users in the off-peak and peak seasons, especially in the heating season (peak demand season), the demand is significantly increased, and the gas source cost of gas suppliers will increase. In addition, because the LNG market has been fully market-oriented, its price fluctuations are relatively obvious, but in the period of 2021-2022, its cost is significantly higher than the lower cost of the three gas suppliers, but in some off-season demand, the comprehensive gas supply cost of the three gas suppliers will be higher than the LNG price, so there is a possibility for users to purchase LNG spot resources.

Table 2 Customer demand and spot LNG market reference price.

Date	Gas Demand（$10^4 m^3$）			Cost of Supplier（¥/m^3）			LNG Price(¥/m^3)
	Client 1	Client 2	Client 3	Supplier 1	Supplier 2	Supplier 3	
202104	56.99	93.50	127.97	1.63	1.87	2.00	3.90
202105	54.10	79.12	119.45	1.85	1.97	2.11	3.85
202106	37.28	87.01	64.27	1.75	1.86	1.99	3.17
202107	51.00	63.53	90.78	1.72	1.88	2.00	2.91
202108	49.50	73.17	90.82	1.84	1.95	2.09	3.57
202109	56.53	66.74	136.85	1.76	1.92	2.05	4.00
202110	44.50	94.99	187.84	2.18	2.18	2.20	4.35
202111	110.00	112.07	212.10	2.53	2.70	2.73	4.28
202112	37.75	131.99	227.50	2.56	2.65	2.72	4.54
202201	37.75	234.05	273.77	2.69	2.70	2.73	4.51
202202	69.00	140.19	183.94	2.38	2.39	2.41	5.16
202203	37.75	106.44	199.40	2.13	2.23	2.38	3.87

4.2 Game equilibrium results

The model established in this paper is introduced based on data from three gas suppliers and three major users, as well as the expected LNG spot price. Subsequently, MATLAB software is used for simulation programming following the game equilibrium solution steps outlined in Section 3.2. Finally, Table 3 presents the equilibrium results of three gas suppliers under different contract types.

According to the game equilibrium results in Table 3, it can be found that, no matter under the annual contract or the off-peak season contract, the three gas suppliers have a very small difference in their equilibrium offers, and this result is fully consistent with the result of the non-cooperative Bertrand price competition game, that is, in the same market, homogeneous products are provided, and the price difference is ultimately small. In addition, it can be found that the contract quotation of gas suppliers is lower than the LNG spot market price in most of the months, indicating that gas suppliers try to adopt a low price strategy for fully market-oriented LNG gas sources to reduce the impact of the LNG market.

Table 3 Game equilibrium results of gas supplier contract quotation(¥/m3).

Contract type		Supplier 1	Supplier 2	Supplier 3
Annual	Equilibrium price	3.51	3.48	3.53
Seasonal	Off-season price	3.01	2.97	3.04
	Peak-season price	3.90	3.85	3.92

According to the results of gas suppliers' balanced quotation, the distribution characteristics of gas purchase of large users under different contract trading modes can be found. (1) Under the annual bilateral contract, due to the uniform price of each month, when the spot LNG price is lower than the unified equilibrium price (in the off-season of demand from June to August), users will buy a large number of spot LNG resources to reduce the cost of gas; when the spot LNG price is higher than the price of gas suppliers, they will basically not buy LNG resources, only during part of the peak season of demand. Due to the shortage of gas supply commercial gas resources, spot LNG resources will be purchased at a small price to meet the demand for gas. (2) Under the bilateral contract in off-peak season, due to the large price difference between off-season and peak season, the LNG resource competition faced by the user is obviously different in the off-peak season. In the off-season, due to the lower price of the gas supplier itself, the lower price of LNG may be hedged; in the peak season, although the LNG price is higher, the gas supplier's price is also higher in the peak season. Therefore, the purchase decision of the user will also change greatly. From the comparison between Figure 2 and Figure 3, it can be found that the gas purchase decision of the user 1 is significantly different under the two bilateral contracts.

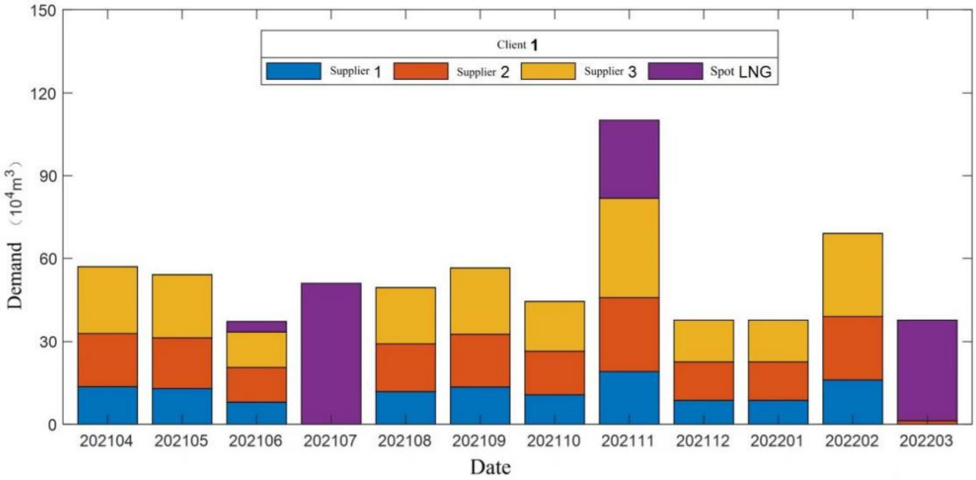

Figure 2: Gas demand in seasonal contract.

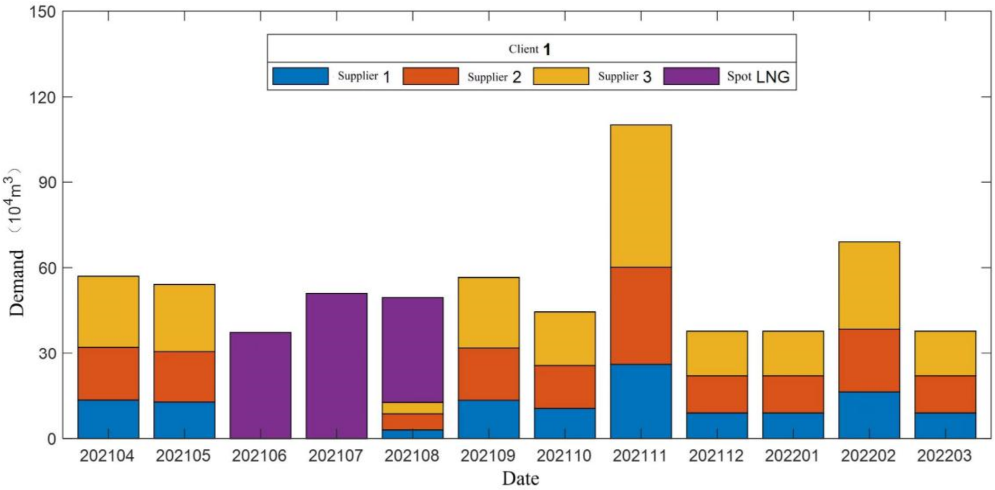

Figure 3: Gas demand in annual contract.

4.3 Result analyses

The gas supply of three gas suppliers and the LNG purchase of three users are analyzed here, and the results are shown in Table 4. It can be found that compared with a single annual contract, the off-peak season contract can generally increase the gas supply of the gas supplier and reduce the amount of LNG purchased by the user from the spot market, in which the total gas supply of the gas supplier increases by 0.5% and the LNG purchase decreases by 1.4%.

Table 4 Gas supply by gas suppliers and LNG purchases by users.

Contract Type	Supplies（$10^4 m^3$）			LNG Demand($10^4 m^3$)		
	Supplier 1	Supplier 2	Supplier 3	Client 1	Client 2	Client 3
Annual	517.00	1002.23	1327.84	125.15	280.57	586.85
Seasonal	522.75	935.57	1402.77	119.40	347.23	511.92

In order to further compare the results of different types of bilateral contracts, changes in revenue and profit of gas suppliers and changes in gas purchase cost expenditures of users are calculated here, as shown in Table 5 and Table 6 respectively.

Table 5 Gas supply revenue and net profit of gas suppliers.

Contract Type	Gas supplier income（10^4¥）			Gas supplier profit (10^4¥)		
	Supplier 1	Supplier 2	Supplier 3	Supplier 1	Supplier 2	Supplier 3
Annual	2563.60	3401.10	4916.10	502.30	731.80	1002.40
Seasonal	2495.40	3428.10	4717.90	578.20	804.40	1100.00

Table 6 Client gas purchase cost and LNG cost.

Contract Type	Client gas cost（10^4¥）			Client LNG cost (10^4¥)		
	Supplier 1	Supplier 2	Supplier 3	Supplier 1	Supplier 2	Supplier 3
Annual	2289.50	4774.80	7464.40	398.20	959.80	2189.80
Seasonal	2265.40	4802.90	7354.80	422.00	1321.70	2038.00

It can be seen from Table 5 and Table 6 that although the total revenue of the gas supplier decreased by 2% compared with the annual contract, the profit increased by 11%. This is because the off-peak season contract increased the total gas supply of the gas supplier and increased the economic benefit through the off-peak season price difference. For customers, the total cost of gas purchased by customers decreased by 4%, mainly because customers purchased less LNG.

The outcome of the game between the entire gas supplier and the major natural gas consumer reveals that, in the master-slave dynamic, although the latter occupies a subordinate position, it passively accepts contract offers from the former. However, compared to procuring all natural gas demand from the spot LNG market, engaging in bilateral contracts during off-peak seasons can partially mitigate price fluctuations risks inherent in this market and reduce costs for consumers. While gas suppliers face fierce competition from other players as leaders in this game, manipulating the market and attaining excessive monopoly profits is challenging. Nevertheless, they can leverage disparities between off-peak and peak season prices alongside their own gas source expenses to expand their supply capacity and capture a portion of LNG's spot market share since LNG resources are fully globalized and driven by market forces. Due to its global nature, significant changes in price fluctuations are unlikely due to local factors; thus local gas suppliers can capitalize on their own resources through bilateral contract transactions during off-peak seasons to enhance profitability. Given gradual deregulation of pricing controls within today's natural gas sales market context, conducting bilateral contract transactions among natural gas suppliers, sales markets, and major consumers becomes both advantageous and necessary.

5 Conclusion

With the gradual advancement of natural gas market reform, the implementation of bilateral contracts in the natural gas sales market can mitigate market risks arising from price fluctuations in the spot LNG market for participants involved in supply, sale, and utilization. This paper utilizes non-cooperative Bertrand competition game theory and Steinberg master/slave game theory to establish a game model for bilateral contract transactions between multiple gas suppliers and large users. It proves the existence of Nash equilibrium solution among multiple gas suppliers simultaneously and provides a solution method along with concrete steps for the game model. The findings demonstrate that during both peak and off-peak seasons of bilateral trading contracts, gas suppliers can maximize profits from contract sales while reducing operational risks within their enterprises. Large users also benefit by lowering their own gas purchase costs and securing a stable supply of natural gas.

The bilateral transaction model established in this study can serve as a valuable reference for formulating contract natural gas prices in future bilateral contract transactions between multiple suppliers and large users. In the game model of this study, the decision variable for gas suppliers is their initial contract offer to different large users, excluding the slope of the offer curve. The next step will involve further optimization of the game model by incorporating both the initial contract offer and the slope of the price curve as decision variables for gas suppliers, along with providing a solution method for the game model. Furthermore, as competition in the natural gas sales market gradually takes shape, there is potential to enhance and optimize the bilateral contract transaction model by extending off-peak season contracts to monthly bilateral contracts and fully exploring new methods of quoting contracts after loosening market sales prices. These efforts aim to improve market participants' enthusiasm, reduce market risks, and increase overall market benefits.

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