

# The Effect of Metacognitive Guidance Assisted by Rich Mathematical Tasks on Mathematical Representation Ability in Terms of Students' Initial Ability

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**Abstract.** The low mathematical representation ability (MRA) of students necessitated innovative solutions. This study aimed to: 1) compare MRA of students receiving metacognitive guidance (MG) assisted by rich mathematical tasks (RMT) with students receiving conventional instruction; and 2) examine the interaction between MG and students' prior knowledge on their MRA. A quasi-experimental design with a  $2 \times 2$  factorial arrangement was employed. Sample consisted of 56 fifth-grade students at SDK Ruteng VI that selected through simple random sampling technique. Data were collected using test and analyzed by using two-way ANOVA. The findings revealed that the MRA of students utilizing MG was significantly higher than those without MG ( $p = 0.00$ ). Furthermore, no significant interaction was found between the implementation of the MG model and students' prior knowledge ( $p = 0.305$ ). Thus, MG effectively enhanced students' MRA regardless of their prior knowledge, providing equitable benefits for both high- and low-ability students.

**Keywords:** mathematics representation, metacognitive guidance, rich mathematical task

## 1 Introduction

Mathematical representation abilities are increasingly recognized as critical in students' overall understanding of mathematics; however, the capabilities of Indonesian students in this area remain a concern in both national and international assessments. Reports from PISA and TIMSS have highlighted significant difficulties among students in constructing meaningful mathematical representations, whether symbolic, graphical, or verbal in nature[1]. Specifically in Manggarai Regency (including Langke Rembong District), East Nusa Tenggara Province, the 2025 Education Report indicates that elementary students' numeracy skills remain at a moderate level[2]. At SDI Ruteng VI in particular, more than 50% of students have not yet reached the minimum competency threshold for numeracy. This indicates that students' basic mathematical skills are generally limited to fundamental computational abilities and the ability to solve simple, routine mathematical problems. They have not yet demonstrated the capacity to apply mathematical knowledge in more varied contexts, nor to reason in solving complex and non-routine problems based on the mathematical concepts they possess.

These limitations suggest that current mathematics instruction strategies may be insufficient in promoting reflective thinking and the flexible transformation of mathematical ideas. Numerous studies point to the need for instructional practices that foster critical engagement and flexibility in students' reasoning and conceptual understanding[3], [4]. These persistent difficulties indicate structural weaknesses in prevailing instructional practices, which remain largely teacher-centered and focused on procedural accuracy[5]. Research has illustrated that when

mathematical representations are presented in isolation, it hinders the establishment of conceptual connections among symbolic, graphical, and verbal forms of mathematics[6]. Therefore, enhancing students' metacognitive skills through targeted strategies, such as metacognitive guidance, offers a promising approach to support deeper understanding of mathematics, especially through richer tasks that encourage various representations.

The concept of mathematical representation is crucial in the context of elementary education, as it allows students to develop a diverse toolbox for understanding and solving mathematical problems. According to Sorimin & Ratuanik[7], mathematical representation involves translating real-world scenarios into mathematical models, generating graphical illustrations, and articulating mathematical concepts through various modes of communication, such as verbal and written expressions. These representations play an integral role in engaging students with mathematical content, as they foster a deeper understanding and facilitate the application of mathematics in everyday situations. For elementary students, effective mathematical representation is linked to the ability to identify relevant mathematical concepts, comprehend the connections between different representations, and apply these skills in problem-solving contexts. Indicators of mathematical representation ability among elementary students can include their proficiency in creating visual models and translating calculations into graphical formats.

Mathematical representation is a central construct in elementary mathematics learning because it mediates how students perceive, process, and apply mathematical ideas. However, from a cognitive perspective, constructing and transforming representations imposes substantial cognitive load on elementary students, particularly in the domain of plane geometry. Learners are required to simultaneously process visual-spatial information, symbolic notation, and verbal explanations, which can strain working memory when instructional support is limited[8]. In plane geometry, these demands are intensified by the need to coordinate abstract properties of shapes, spatial relationships, and visual transformations. Many students struggle not because of a lack of procedural knowledge, but due to difficulties in managing intrinsic and extraneous cognitive load when linking diagrams, symbols, and verbal reasoning[9]. This challenge underscores the importance of instructional approaches that deliberately support representational processing, as unstructured tasks may overwhelm students' cognitive resources and hinder meaningful geometric understanding.

Among many innovative strategies, metacognitive guidance, is one that offers a promising approach to support deeper understanding of mathematics, especially through richer tasks that encourage various representations. As McKeivett et al.[10] point out, students' difficulties in utilizing mathematical representation can be mitigated through targeted instructional frameworks that embed metacognitive prompts. These prompts foster self-regulation and reflective practices, enabling students to become more aware of their cognitive processes during problem-solving tasks. For instance, present studies demonstrate that students who are guided to reflect on their mathematical reasoning exhibit improved performance in representation tasks at the elementary level. Additionally, students' engagement with multiple forms of representation—including symbolic, graphical, and verbal—has been shown to enhance their overall mathematical proficiency. This finding aligns with the work of Mainali[11], who highlighted the critical role of various forms of representation in teaching and learning mathematics, asserting that utilizing multiple modes significantly benefits students' learning processes. Furthermore, Jamaludin[12] provided evidence that contextual teaching materials

developed using a systematic approach improve students' mathematical representation abilities, indicating the positive impact of structured educational environments. Therefore, fostering an educational environment that emphasizes mathematical representation along with the necessary metacognitive skills can significantly improve the mathematical outcomes of elementary students, providing them with the foundational tools needed for future academic success.

Given these empirical insights on the benefits of metacognitive support for strengthening students' use of mathematical representations, a more structured instructional approach—such as metacognitive guidance—becomes essential to systematically cultivate the cognitive awareness needed for effective mathematical learning. Metacognitive guidance is a vital pedagogical approach that emphasizes the enhancement of students' self-awareness and regulation regarding their cognitive processes. It involves guiding students through systematic steps aimed at improving their planning, monitoring, and evaluation strategies when approaching learning tasks, particularly in mathematics [13], [14].

In the planning phase, students identify relevant information and select representations that align with problem demands. Monitoring involves checking coherence among diagrams, symbols, and verbal explanations and revising them when inconsistencies appear. Evaluation requires judging whether representations meaningfully support problem solving rather than merely yielding correct results. These processes are especially important in elementary geometry, where frequent shifts between visual, symbolic, and verbal forms occur. Without explicit prompts, students may engage mechanically. Research indicates that embedded metacognitive prompts can enhance representational coherence and support adaptive reasoning across different initial ability levels [15].

Furthermore, this approach entails several critical components: first, explicit instruction on metacognitive strategies, such as self-questioning and reflection, should be integrated into the curriculum. Second, educators must model these strategies in real-world problem-solving contexts, enabling students to observe and understand how to apply them effectively. Third, promoting active student engagement through group discussions and collaborative tasks facilitates peer learning and the sharing of metacognitive strategies. Lastly, comprehensive feedback and self-reflection opportunities allow students to assess their cognitive approaches and adjust their strategies accordingly [16]. This structured use of metacognitive guidance not only boosts students' mathematical problem-solving capabilities but also fosters a culture of autonomous learning, where students take ownership of their educational journeys and develop critical thinking skills crucial for their academic success.

The effectiveness of metacognitive guidance in enhancing students' mathematical representation abilities can be significantly amplified when coupled with rich mathematical tasks. Rich mathematical tasks are characterized by their complexity, requiring students to engage in deeper levels of thinking, thereby promoting exploration and application of various mathematical concepts. These tasks push students beyond rote memorization and encourage them to deploy metacognitive strategies such as planning, monitoring, and evaluating their approaches in real-time. A study by Cahyani et al. [17] found that when students are faced with tasks that necessitate strategic problem-solving and critical thinking, their engagement with metacognitive strategies improves, resulting in better mathematical reflective thinking. Moreover, integrating rich tasks into the educational framework compels students to articulate their reasoning and processes, fostering a deeper understanding of mathematical representations and enhancing their ability to

connect theory with practical applications. This merging of metacognitive guidance with rich tasks creates an environment conducive to developing critical problem-solving skills necessary for success in mathematics.

The concept of a mathematical rich task involves engaging students in high-level cognitive processes, encouraging them to explore concepts through inquiry and problem-solving rather than mere procedural application. As indicated by Sasson & Tifferet[18], such tasks should be designed to stimulate critical thinking, allowing students to analyze, synthesize, and evaluate mathematical information in ways that are both meaningful and relevant. Key characteristics of rich mathematical tasks include their open-ended nature, allowing for multiple solution strategies, and their connection to real-world contexts that enhance students' motivation and interest. Furthermore, these tasks are often designed to be collaborative, fostering discussion and reflection among peers, which inherently supports the metacognitive process by allowing learners to articulate their thought processes while learning from each other. The integration of rich mathematical tasks within a metacognitive framework not only empowers students to take ownership of their learning but also equips them with essential skills for critical thinking, ultimately resulting in improved mathematical representation and problem-solving capabilities.

Moreover, students' initial ability plays a critical role in shaping the effectiveness of metacognitive guidance within rich mathematical tasks and warrants careful theoretical examination. From the perspective of differentiated instruction, students' initial ability is theorized to moderate the effectiveness of instructional approaches, as learners with different readiness levels are expected to respond differently to the same learning design[19], [20]. Differentiation theory assumes that instructional supports, task complexity, and scaffolding should be adjusted to students' prior knowledge to optimize learning outcomes[21], [22]. Consequently, when metacognitive guidance is implemented uniformly, differentiated instruction would predict a significant interaction effect, particularly in representation-intensive mathematical tasks where cognitive demands vary substantially across learners [14].

However, the absence of a significant interaction between instructional model and students' initial ability in this study stands in contrast to the assumptions of differentiated instruction, which theoretically predict differential learning effects based on learners' readiness levels[23]. The relatively uniform effects observed in this study suggest an alternative explanatory mechanism that metacognitive guidance embedded within rich mathematical tasks may function as an inclusive design approach rather than an ability-based adaptation. The open-ended and multi-representational nature of rich tasks provides flexible entry points that allow students with diverse initial abilities to engage in meaningful sense-making, while metacognitive prompts scaffold regulation processes without requiring explicit differentiation [24], [25]. This finding positions the present study as a theoretical counterpoint to strict differentiation assumptions and highlights a need for further empirical investigation into whether adaptive task design combined with metacognitive support can systematically mitigate ability-based learning disparities in elementary mathematical representation[26], [27].

Against this theoretical background, the primary focus of this research is to examine how metacognitive guidance embedded in rich mathematical tasks influences students' mathematical representation ability when viewed in relation to their initial ability levels. Although prior studies have documented the separate contributions of metacognitive support and task richness to mathematical learning, empirical investigations analyzing their interaction with students'

initial ability, particularly within elementary geometry, remain limited. Existing research in this area tends to emphasize general achievement outcomes or focus on higher-grade mathematics, while neglecting how students with differing entry-level competencies construct and transform geometric representations[28]. Consequently, there is insufficient evidence explaining whether metacognitive guidance benefits students uniformly or differentially across ability levels. Addressing this gap is essential, as elementary geometry relies heavily on representational reasoning, and understanding this interaction can inform more equitable and responsive instructional designs in mathematics education contexts such as Indonesia[29]. This research contributes to the existing body of literature by offering an innovative perspective on the intersection of metacognition and mathematics education, particularly in contexts characterized by diverse student demographics. The novelty of this investigation lies in its emphasis on integrating metacognitive guidance with rich mathematical tasks, which is currently underexplored within existing research. By shedding light on this interdisciplinary approach, the study aims to establish a new pedagogical framework that can serve as a model for improving mathematics instruction and learning outcomes across various educational settings[7].

The expected outcomes of this research include improved mathematical representation abilities among students and heightened awareness and application of metacognitive strategies in their problem-solving processes. It is anticipated that through targeted interventions, students will be better equipped to approach complex mathematical tasks with confidence and competence, potentially leading to an overall enhancement in their academic and cognitive development. Additionally, findings from Rajagukguk et al.[30] demonstrate how metacognitive strategies may play a crucial role in mathematics learning, suggesting that students who engage in these strategies are likely to achieve better academic outcomes. Moreover, this research aims to stimulate further discussion and investigation into the critical role of metacognition in mathematics education, especially in light of increasingly demanding educational standards worldwide, as evidenced by Cahyani et al.[17]. The exploration of the relationship between metacognitive guidance, rich mathematical tasks, and mathematical representation abilities presents a vital avenue for enhancing educational practices. This research seeks to fill existing gaps in the literature by providing empirical evidence and theoretical insights that can inform innovative approaches to mathematics teaching and learning[31]. By situating this investigation within the broader context of global educational trends and challenges, it underscores the significance of adaptive pedagogical strategies that cater to the diverse needs of learners in mathematics, ultimately advocating for a future where all students can achieve their full potential in this critical area of study.

## **2 Method**

This study is a quantitative research employing a quasi-experimental design with a 2x2 factorial structure in the form of a pretest-posttest control group design. The research population comprised fifth-grade students at SDK Ruteng VI, from which two classes were purposively selected to minimize selection bias. In addition to equivalence in mathematics report card scores, the selection considered similarities in curriculum implementation, instructional time, teacher qualifications, and classroom learning conditions. Preliminary discussions with teachers and a review of prior assessment records indicated comparable learning experiences across the two classes. These criteria were applied to ensure that differences in outcomes could be more

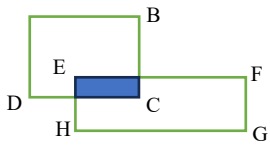
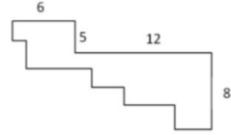
confidently attributed to the instructional intervention rather than pre-existing contextual or instructional disparities.

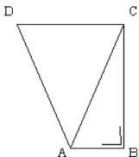
The sample comprised 56 fifth-grade students of SDK Ruteng VI, with class VA (29 students) serving as the experimental group and class VB (27 students) as the control group. The experimental group was taught using metacognitive guidance supported by rich mathematical tasks, whereas the control group received conventional instruction aligned with the teacher's regular practices. To ensure instructional fidelity, the control group lessons were monitored through classroom observations and lesson plan reviews, confirming the absence of explicit metacognitive prompts or rich task structures. This monitoring helped ensure that observed differences were attributable to the intervention rather than unintended instructional overlap. The instructional intervention was carried out over ten sessions, during which the experimental group was provided explicit metacognitive guidance across three phases (planning, monitoring, and reflection) concurrently with exploration of open-ended mathematical tasks.

Prior to the intervention, students' initial mathematical ability was determined from their first semester mathematics grades in the 2024/2025 academic year, which were used to categorize students into high and low initial ability groups based on the median score. The independent variable in this study was the instructional model (metacognitive guidance with rich mathematical tasks vs. conventional instruction), the moderating variable was initial ability (high vs. low), and the dependent variable was students' mathematical representation ability. Instruction in the experimental class began with activation of prior knowledge and strategic planning, followed by exploration of mathematical tasks and concluded with reflective discussions to evaluate the representation processes used. In contrast, the control class engaged in expository learning consisting of concept explanations, worked examples, and direct practice problems.

Data collection was conducted through a written test comprising five essay questions developed by the research team to assess students' mathematical representation abilities (MRA) in the topic of plane geometry, covering verbal-visual, visual-symbolic, and visual-verbal-symbolic aspects.

Table 1. Test Items of MRA

| Item No | Aspect of RMA     | Test Items   |
|---------|-------------------|--|
| 1       | Verbal - visual   | A father intends to construct a rectangular frame by bending a wire with a total length of 18 decimeters. Determine the dimensions of the length and width of the frame so that the enclosed area is maximized, with no wire remaining unused.   |
| 2       | Visual - simbolik | Shown alongside are a square ABCD and a rectangle EFGH. If the area of the unshaded region is $68 \text{ cm}^2$ , determine the area of the shaded region. <div style="display: flex; align-items: center; justify-content: center;">  </div> |
| 3       | Visual - simbolik | Observe the following plane figure. The lengths of its sides are given in centimeters. Determine the perimeter of the figure. <div style="display: flex; align-items: center; justify-content: center;">  </div>                              |

|   |                            |   |   |
|---|----------------------------|---|---|
| 4 | Verbal - visual - simbolik | <p>In the following figure, triangle ABC is a right triangle with the right angle at B. Triangle ACD is an isosceles triangle (<math>AC = AD</math>) with an area of 20 square units. Determine the area of trapezoid ABCD in square units.</p> |  |
| 5 |                            | <p>A rectangular garden has a length of 15 meters and a width of 12 meters. A path with a width of 2 meters is constructed around the garden and will be paved with tiles. What is the area of the tiled path?</p>                              |   |

After the intervention, all students completed a posttest to measure learning outcomes. Data were analyzed using descriptive and inferential statistics (two-way ANOVA) with SPSS 22 to evaluate the main effects and interactions between instructional model and initial ability on mathematical representation ability. All research procedures adhered to ethical principles, including obtaining approval from the school and ensuring the confidentiality of participants' identities.

### 3 Results and Discussion

Prior to conducting the study, the researchers established students' initial mathematical ability (IMA) as a fundamental basis in designing the experiment to ensure internal validity and the equitable distribution of treatments. IMA is defined as the students' level of mastery over mathematical material before the intervention, which in the context of this research was determined through scores from the even semester's daily mathematics assessments in the 2024/2025 academic year. These scores were selected because they authentically and contextually reflect students' academic achievement and originate from regular classroom learning activities.

Based on these scores, students were classified into two ability categories (high and low) in order to ensure that both the experimental and control groups had a balanced distribution of initial ability. This grouping aimed to control variables that might affect the treatment outcomes, as well as to facilitate the researchers in interpreting the effectiveness of the intervention implemented.

Table 2. Classification of Initial Mathematical Ability (IMA)

| Groups     | Mean  | Range | Category                  |                       |
|------------|-------|-------|---------------------------|-----------------------|
|            |       |       | High ( $\geq$ mean score) | Low ( $<$ mean score) |
| Experiment | 74,12 | 28    | 12                        | 17                    |
| Control    | 75,29 | 25    | 15                        | 12                    |

The results of descriptive statistical analysis in this study provide an overview of the distribution of students' mathematical representation abilities in both the experimental and control groups. The analyzed statistics include the mean, standard deviation, maximum, and minimum values. The higher mean score achieved by the experimental group indicates that the instructional intervention contributed positively to students' mathematical representation ability. More importantly, the smaller standard deviation reflects a narrower spread of achievement, suggesting that the treatment supported more balanced learning outcomes among students. This

pattern implies that lower-achieving students benefited substantially from the structured metacognitive guidance, reducing performance disparities without diminishing gains among higher-achieving peers. Qualitatively, the intervention appears to have promoted more equitable engagement with representational tasks, leading to a more uniform distribution of learning gains across the group. These data serve as the preliminary basis for subsequent inferential analysis.

Table 3. Results of Descriptive Statistical Analysis

| Groups     | Mean     |         | Total   | Std. Deviation |         | Total   |
|------------|----------|---------|---------|----------------|---------|---------|
|            | High IMA | Low IMA |         | High IMA       | Low IMA |         |
| Experiment | 76.9792  | 70.4412 | 73.1466 | 7.14020        | 7.14020 | 8.07070 |
| Control    | 58.9500  | 48.6292 | 54.3630 | 7.16340        | 3.45513 | 7.74559 |

The results of the normality and homogeneity tests in this study indicate that the analyzed data meet the basic assumptions required for parametric statistical testing. The normality test, conducted using the Kolmogorov-Smirnov test, showed that the data in both the experimental and control groups were normally distributed, as evidenced by significance values ( $p > 0.05$ ). Furthermore, the homogeneity of variance test using Levene's Test produced significance values greater than 0.05, indicating that the variances of the two groups were homogeneous. These findings suggest that both groups had comparable data distributions and consistent variance, thereby allowing for valid subsequent analyses such as t-tests or ANOVA to examine differences between the groups.

Table 4. Results of Normality and Homogeneity Tests

| Tests of Normality            | Levene's Test of Equality of Error Variances <sup>a</sup> |    |      |       |     |     |      |
|-------------------------------|---|----|------|-------|-----|-----|------|
|                               | Statistic   | df | Sig. | F     | df1 | df2 | Sig. |
| Standardized Residual for RMA | .107  | 56 | .167 | 1.969 | 3   | 52  | .130 |

Design: Intercept + Kelas + IMA + Kelas \* IMA

The results of hypothesis testing using two-way ANOVA in this study revealed a significant partial effect of the treatment on students' mathematical representation abilities. The two-way ANOVA analysis yielded a significance value (p-value) of less than 0.05 for the treatment factor, indicating a significant difference between the experimental and control groups. In addition, the significance value for the initial mathematical ability factor was also below 0.05, demonstrating that differences in initial ability levels influenced students' learning outcomes. However, the interaction test between the instructional model and students' initial ability resulted in a significance value greater than 0.05, indicating that there was no significant interaction between the two variables with respect to students' mathematical representation abilities. Thus, the effectiveness of the instructional model applied was not affected by differences in students' initial abilities. This suggests that the model had a relatively uniform impact across all categories of initial ability—high, moderate, and low—making it applicable to all students without the need for differentiated treatment based on initial ability.

Table 5. Results of Hypothesis Testing

| Source | Type III Sum of | df | Mean Square | F | Sig. |
|--------|-----------------|----|-------------|---|------|
|--------|-----------------|----|-------------|---|------|

|              | Squares    |   |            |          |      |
|--------------|------------|---|------------|----------|------|
| Intercept    | 222567.936 | 1 | 222567.936 | 4877.501 | .000 |
| Groups       | 5433.106   | 1 | 5433.106   | 119.065  | .000 |
| IMA          | 972.833    | 1 | 972.833    | 21.319   | .000 |
| Groups * IMA | 48.980     | 1 | 48.980     | 1.073    | .305 |

The findings from this study offer essential insights into the effects of the implemented instructional model on enhancing students' mathematical representation capabilities. The two-way ANOVA analysis revealed that the treatment factor had a statistically significant effect on students' mathematical representation skills, with a p-value less than 0.05. This indicates that the metacognitive guidance adopted in the experimental group outperformed traditional teaching methods found in the control group. Moreover, the significant p-value associated with initial mathematical ability underscores the critical role of a student's starting level of competence in their learning process. This is consistent with existing literature indicating that prior knowledge significantly influences current learning outcomes, particularly in mathematics[1].

However, an intriguing aspect of the results is the non-significant interaction effect between the treatment and students' initial ability levels with a p-value greater than 0.05. The absence of a significant interaction between the instructional treatment and students' initial ability levels suggests that metacognitive guidance assisted by rich mathematical tasks functioned consistently across diverse learner profiles. One plausible explanation is the open-ended structure of rich mathematical tasks, which allows for multiple solution pathways and varying depths of engagement. Such tasks may provide natural entry points for students with lower initial ability while simultaneously challenging higher-ability students to extend their reasoning[32]. As a result, differences in prior competence may have been moderated by the flexibility inherent in the task design.

Furthermore, the metacognitive guidance embedded within the instruction likely supported all students in regulating their learning processes, reducing reliance on prior knowledge. Rather than tailoring content explicitly by ability level, the guidance emphasized planning, monitoring, and evaluation strategies that are broadly applicable. This approach may have minimized performance gaps by helping students engage meaningfully with representations at their own level[28]. Consequently, the non-significant interaction does not negate differentiation theory but suggests that task openness and cognitive regulation can implicitly address learner diversity within a unified instructional framework.

In relation to existing literature, the results align well with studies advocating for enriched instructional methods over traditional practices. Research has demonstrated that strategies such as problem-based learning, inquiry-based learning, and integrative approaches yield positive outcomes in students' mathematical understanding and skills[33], [34]. Students exposed to richer contextual tasks showed greater engagement and comprehension in mathematics than their peers receiving conventional instruction[35]. This finding is also supported by empirical studies indicating that instructional models integrating reflective prompts and open-ended tasks can enhance representational understanding compared to conventional, procedure-oriented instruction[36]. Contemporary research in elementary mathematics has shown that students exposed to metacognitive scaffolding demonstrate improved abilities to construct, interpret, and connect multiple representations, suggesting consistency with current instructional trends[37].

Further reinforcement arises when comparing this study with investigations contrasting enriched learning environments with traditional teaching approaches. Studies in similar elementary settings have indicated that rich tasks promote active engagement and deeper cognitive processing, leading to improved performance in representation-based assessments[38]. For example, Arianto[39] highlights the positive effects of metacognitive strategies on problem-solving abilities in elementary students, reinforcing how enriched instructional methods bolster cognitive engagement. Furthermore, research by Jumanto & Mustofa[38] emphasizes that metacognitive-based teaching materials significantly affect learning outcomes, corroborating the notion that rich, contextual tasks enhance student performance in mathematics. The alignment of these outcomes implies that the effectiveness noted in this study is not merely context-specific but reflects a broader trend identified in recent literature, thereby strengthening the external validity of the findings.

Nonetheless, discrepancies in research advocating personalized and differentiated strategies based on students' initial aptitude present a contrasting perspective. These differences may arise from variations in sample demographics, instructional contexts, or research methodologies. Several contemporary studies emphasize that personalized and differentiated instruction, particularly for students with lower initial ability, leads to stronger learning gains when task difficulty and scaffolding are explicitly adjusted[40]. In contrast, the present study did not implement formal differentiation based on initial aptitude; instead, the use of rich mathematical tasks combined with metacognitive guidance provided flexible entry points and multiple solution pathways, potentially reducing the need for explicit personalization. This distinction suggests that discrepancies across studies may be explained by differing approaches to addressing learner variability, either through structured differentiation or adaptable task design[41].

Moreover, recent comparative research indicates that instructional context plays a critical role in determining the effectiveness of differentiation. Studies conducted in classrooms with limited instructional time or large student-teacher ratios often report stronger effects for explicit differentiation. In contrast, settings that emphasize reflective learning and open-ended tasks show more uniform benefits across ability levels. The current findings reinforce this latter perspective, indicating that well-designed instructional models can implicitly accommodate learner diversity while maintaining consistent learning outcomes[42].

The theoretical implications of the study's outcomes can be understood through relevant educational theories such as constructivism and metacognitive learning theory. Constructivist theories suggest that learners construct knowledge through active engagement and reflection, with the integration of metacognitive strategies enhancing this process[43]. The combination of metacognitive guidance and rich mathematical tasks promotes students' reflection on their learning processes, facilitating deeper understanding and retention of mathematical concepts. Furthermore, the absence of a significant interaction with initial ability levels suggests that the model possesses universal applicability across diverse student populations, reinforcing the potential for broad implementation to provide equitable access to quality mathematics education[1].

From a practical standpoint, the results have significant implications for mathematics educators and institutions. Given the instructional model's effectiveness across differing ability levels, educators should adopt this approach broadly within their teaching strategies, recognizing that

all students can benefit from a rich learning environment[44]. This finding aligns with the trend towards inclusive teaching practices, moving away from tracking or differentiating instruction solely based on initial competencies. Instead, the focus should be on creating opportunities for all students to engage analytically with mathematical concepts, fostering a collaborative classroom atmosphere where diverse learners thrive[45].

Moreover, the successful implementation of metacognitive guidance underlines the importance of teacher training in adopting innovative pedagogical practices. Educators require support and ongoing professional development to effectively employ strategies that promote metacognitive engagement and task-rich learning environments. Collaborative initiatives between education authorities and institutions can enhance teaching quality, ensuring educators are equipped for modern classrooms filled with diverse learners, as evidenced by the ongoing necessity for adaptive teaching methods[46].

In terms of future research directions, several avenues emerge from this investigation. Future studies should replicate these findings in different educational settings with larger and more diverse samples to verify the stability of results across various contexts. Investigating additional factors, such as motivational variables and students' emotional responses in learning, can provide valuable insights into their interactions with instructional models and inform effective teaching practices[47]. Furthermore, exploring how technological advancements and digital learning tools can complement metacognitive strategies and rich tasks presents a significant opportunity for future educational research.

The research findings underscore the effectiveness of a metacognitive guidance and rich mathematical task model in enhancing students' mathematical representation skills. The inclusive nature of this instructional model, which accommodates students at varying competency levels, suggests significant implications for mathematics education, advocating for shifts towards inclusive practices and continuous professional growth among educators. The potential for further advancement in this research area encourages ongoing inquiry into the multifaceted nature of mathematics learning, ultimately aiming to foster a more equitable and effective educational landscape for all students.

## **4 Conclusion**

The findings indicate that metacognitive guidance integrated with rich mathematical tasks contributes meaningfully to the development of students' mathematical representation ability in heterogeneous elementary classrooms. For broader implementation, teacher professional development should move beyond general training and be organized into focused modules that address the design of multi-representational tasks, the practical use of metacognitive prompts during instruction, and the interpretation of formative assessment evidence to support students' regulation processes. Embedding these modules within lesson study and instructional coaching frameworks may support systematic reflection, peer-based feedback, and closer monitoring of instructional fidelity, thereby increasing the sustainability of classroom implementation.

This study also recognizes the relatively small sample size as a primary methodological limitation, as it may reduce statistical power and necessitate caution when interpreting the non-significant interaction between instructional model and students' initial ability. Moreover, the focus on a single school context restricts broader generalization. Future research should

therefore replicate this model with larger and more diverse samples. Longitudinal designs are particularly needed to examine whether improvements in representation ability persist over time and extend to other mathematical domains. Additional studies may also investigate how different configurations of task complexity and metacognitive scaffolding shape learning outcomes across varied instructional settings.

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