Leveraging Importance Sampling for Effective Risk assessment of Supply Chain Networks

Jiajun Chen[1,a], Yutao Hu[2,b], Yuzhou Hu[2,c], Jiajie Su[2,d], Siqi Yu[*,3,e]

[a]13699276290@163.com, [b] hu20090123@gmail.com, [c] 25yuzhouh@gmail.com;
[d] Jsu120860@gmail.com ,[e] * nicoleyu993@gmail.com,

[1]Jilin Jianzhu University, Changchun, Jilin, China, 130118
[2] Stamford American International School of Singapore, Singapore, 35768
[3]Stamford American International School of Singapore, Singapore, 35768,

Abstract. Risk assessment has become a crucial aspect of managing supply chain networks due to their inherent complexity and vulnerability to various disruptive events. This research paper focuses on the application of importance sampling as a powerful technique to accurately estimate the reliability of supply chain networks and identify critical areas of risk. The proposed methodology involves two key steps. Firstly, a comprehensive mathematical model is developed to represent the intricate relationships and dynamics within the supply chain network. This model encompasses factors such as demand uncertainty, transportation disruptions, supplier reliability, and inventory management policies. Secondly, importance sampling is employed to efficiently simulate risk scenarios and estimate the likelihood of rare events, such as severe disruptions or critical failures within the network. Importance sampling offers significant advantages over traditional Monte Carlo simulation methods by concentrating computational resources on the events of interest. By selectively sampling rare and extreme scenarios, the approach enables a more accurate estimation of the network's reliability and vulnerability to potential disruptions. This, in turn, facilitates proactive risk mitigation strategies and enhances decision-making in supply chain management. Additionally, the methodology allows for sensitivity analysis, enabling the evaluation of the impact of various risk factors on the overall network performance. Decision-makers can prioritize their risk management efforts and allocate resources effectively to minimize potential losses and enhance the resilience of supply chain networks.

Keywords: Supply Chain Networks, Risk Assessment, Importance Sampling, Reliability, Monte Carlo Simulation, Rare Events, Computational Efficiency.

1. Introduction

Supply chain networks are of utmost importance due to their integral role in the functioning of modern-day economies [1]. These networks encompass the intricate and interconnected flow of goods, services, and information from suppliers to manufacturers, distributors, retailers, and end customers [2]. Their significance can be attributed to several many factors. However, reliance solely on traditional Monte Carlo-based supply chain network risk assessment methods presents certain limitations [3]. Firstly, these methods can be computationally intensive, particularly as network complexity and interdependencies increase. The repeated sampling from probability distributions becomes time-consuming and requires significant computational resources. Additionally, the emphasis on probability distributions may result in an underestimation of rare events with low probabilities that possess significant impact on network reliability. Traditional Monte Carlo simulation may not accurately capture extreme disruptions or failures,

ICPDI 2023, September 01-03, Chongqing, People's Republic of China

DOI 10.4108/eai.1-9-2023.2338823

compromising the overall risk assessment accuracy. Lastly, Monte Carlo methods usually operate under the assumption of static inputs, which do not account for real-time adaptability in dynamic supply chain environments. As a result, these methods may not effectively capture the changing risk profiles and evolving nature of supply chain risks, limiting their applicability in practice.

Concerning this limitation, this research paper proposes an innovative framework that incorporates importance sampling techniques to address the challenges of risk assessment in supply chain networks. The development of a comprehensive mathematical model represents a significant advancement, as it captures not only the structural aspects but also the dynamic nature of the network. By considering various risk factors such as demand uncertainty, transportation disruptions, supplier reliability, and inventory management policies, this model provides a more realistic representation of the complexities within the supply chain. The integration of importance sampling allows for the accurate estimation of rare event probabilities, which are essential for understanding the potential severe disruptions or critical failures that can impact the network's performance. By selectively sampling these rare events, decision-makers can gain insights into the vulnerabilities existing within the supply chain network. This information empowers them to allocate resources strategically and implement targeted risk mitigation strategies in the identified high-risk areas. Moreover, the conducted sensitivity analysis enhances the decision-making process by quantifying the sensitivity of the network's reliability to different risk factors. This analysis allows decision-makers to prioritize and allocate resources based on the relative importance and impact of each risk factor. As a result, the proposed methodology provides a practical and effective means to enhance the overall performance and resilience of supply chain networks.

The significance of this research lies in its contribution to both industry and academia. The practical applications of the proposed framework enable decision-makers to proactively manage risks, minimize disruptions, and improve the efficiency of supply chain operations. Additionally, the advancements in the academic knowledge of supply chain risk management are achieved through the innovative integration of importance sampling and the development of a comprehensive mathematical model. The findings of this research contribute to enhancing the understanding and management of risks in supply chain networks, thereby facilitating the achievement of supply chain resilience and the ability to withstand and recover from disruptions effectively. This research paper serves as a valuable resource for researchers, practitioners, and decision-makers interested in developing advanced risk assessment methodologies and enhancing the performance of supply chain networks.

2. Importance sampling-based network work reliability analysis

2.1. Network analysis of supply chain

A supply chain network encompasses the intricate connections, operations, and entities involved in the seamless flow of products, services, information, and finances within a supply chain. This interconnected structure links suppliers, manufacturers, distributors, retailers, and customers, facilitating the efficient exchange of goods and services. The optimization of a supply chain network focuses on enhancing coordination, collaboration, and integration across the entire supply chain, as depicted in Fig. 1. Key factors in designing and managing this network include determining optimal facility locations and capacities, establishing effective inventory policies, selecting suitable transportation modes, and implementing efficient information systems. Assessing the maximum flow capability of the supply chain is a common practice within this network. Furthermore, if demand exceeds the supply chain's capacity, it is considered a failure. Therefore, reliability plays a critical role in ensuring the effectiveness of supply chain networks.

Reliability, within the context of supply chain networks, refers to the consistent and dependable delivery of products, services, and information, even in the face of disruptions, uncertainties, and challenges. The significance of reliability lies in its direct impact on overall performance, customer satisfaction, and profitability. A reliable supply chain network ensures products are available when and where needed, minimizing stockouts, delays, and disruptions. Consequently, this enhances customer service levels, fosters customer loyalty, and strengthens brand reputation. To address this crucial aspect, this research proposes an efficient method for analyzing network reliability within the framework of importance sampling. By integrating importance sampling into the reliability analysis, this novel approach allows for accurate estimation of rare event probabilities associated with severe disruptions and critical failures in the supply chain network. The use of importance sampling enables a focused analysis on rare events, ensuring their impact is properly considered. This method improves the accuracy of probability estimates and enhances decision-making regarding risk mitigation strategies. Additionally, importance sampling reduces computational burden, enhancing the efficiency of the analysis compared to traditional methods such as subset simulation.

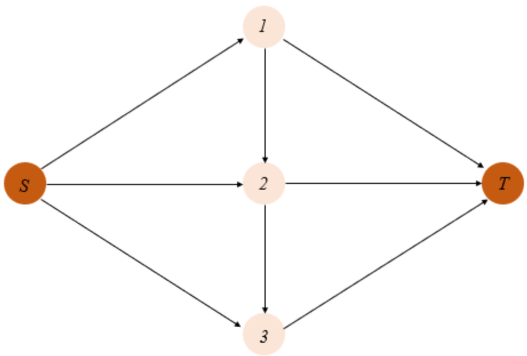

Fig. 1. Illustrative interpretation of supply chain network.

2.2. Reliability analysis with importance sampling

Reliability analysis primarily revolves around assessing the vulnerability of components or systems to factors such as degradation, normal operating loads, or extreme disturbances. The failure condition is identified through the limit state function, and the joint probability density function characterizes the distribution of the associated random variables. The quantification of the failure probability involves integrating the joint probability density function over the region where the limit state function is less than or equal to zero. Precise estimation of the failure probability encompasses various methodologies, including sampling techniques, optimization approaches, and advanced surrogate modeling methods. Within this framework, importance sampling is leveraged as a powerful technique for accurately evaluating the failure probability, denoted as P_f in this context.:

$$P_f = \Pr(g(\boldsymbol{x}) \leq 0) = \int_{x \in \Omega_f} f(\boldsymbol{x}) d\boldsymbol{x} \tag{1}$$

Consider Eq. (1), where $\boldsymbol{x}$ signifies the vector of random variables, Ω_f denotes the failure domain, $g(\boldsymbol{x})$ denotes the performance function or limit state function, and $f(\boldsymbol{x})$ represents the joint probability density function (PDF) of $\boldsymbol{x}$. A plethora of methods are available to estimate the probability of failure, P_f, as defined in Eq. (1). These methods encompass various approaches, including but not limited to sampling techniques such as Crude Monte-Carlo Simulation [4], [5], Importance Sampling (IS) [6], Subset Simulation (SS) [7], as well as optimization approaches such as First or Second Order Reliability Method (FORM & SORM) [8], [9], among others [10]–[12]. In this study, we employ the method of importance sampling to accurately estimate the probability of failure. Therefore, one can recalculate the failure probability, P_f^{im}, as,

$$P_f^{im} = \int_{x \in \Omega_f} I_g(\boldsymbol{x}) \frac{f(x)}{h(x)} h(\boldsymbol{x}) d\boldsymbol{x} \tag{2}$$

where Ω_f denotes the failure domain, I_g is the indicator for identify the failure labels of $\boldsymbol{x}$, $h(\boldsymbol{x})$ is the proposed probability density of sampling distribution. Importance sampling exhibits several compelling advantages over traditional Monte Carlo simulation. Firstly, it excels in computational efficiency by judiciously selecting importance distributions that concentrate on the most influential regions of the target space. This smart sampling strategy dramatically reduces the computational effort required, particularly for estimating rare events or calculating tail probabilities. Secondly, importance sampling effectively mitigates variance issues by leveraging the proximity between importance distributions and the target distribution, resulting in superior precision and reduced fluctuations in the estimates. Additionally, its flexibility allows for the adaptation of importance distributions to tackle intricate and high-dimensional problems, rendering it resistant to the challenges posed by the curse of dimensionality. Moreover, importance sampling accommodates tailored sampling, enabling researchers to strategically focus sampling efforts on specific areas of interest, such as extreme events or critical portions of the distribution.

3. Case study

Fig. 2 showcases the intricate structure of a multi-state two-terminal supply chain network, consisting of 20 nodes and 32 edges. The edges' capacities, representing the maximum flow that can traverse each edge, are distributed in a stochastic manner across four distinct states: 0, 3, 6, and 10. The corresponding probabilities associated with these states are 10-3, 0.05, 0.1, and 0.672, respectively. Notably, it is imperative to recognize that the states of the edges are completely independent, meaning that the behavior of one edge has no influence on the others. The primary objective revolves around estimating the probability that the maximum flow from the source node, denoted as 's', to the sink node, denoted as 't', adheres to or falls below a predefined demand for a maximum flow of 12. This estimation of probability assumes great significance in evaluating the network's performance and assessing the effectiveness of the proposed approach. Accurately quantifying this probability is a challenging task.

By implementing an importance sampling approach with a substantial sample size of 2×10^7, it becomes feasible to approximate the true value of this probability. The extensive importance sampling analysis reveals that the estimated probability is approximately 1.325×10^{-5}, presenting a robust benchmark for evaluating the precision and efficiency of the proposed methodology. This case study demonstrates the utilization of the Edmonds-Karp algorithm to solve the challenge of determining the maximum flow in a supply chain network. Moreover, the computational outcomes, as documented in Table 1, demonstrate the remarkable enhancement in computational efficiency achieved through the adoption of the importance sampling method for network reliability calculation. Though there is a slight reduction in computational accuracy, the importance sampling method only requires 1277 points to accomplish the same outcome as 2×10^7 samples in Monte Carlo simulations, in terms of computational cost. Moreover, the approach through Subset simulation is also as large as 5255, which demonstrates the computational efficiency of importance sampling for network reliability analysis. This marginal trade-off in accuracy does not significantly impact its overall computational capability.

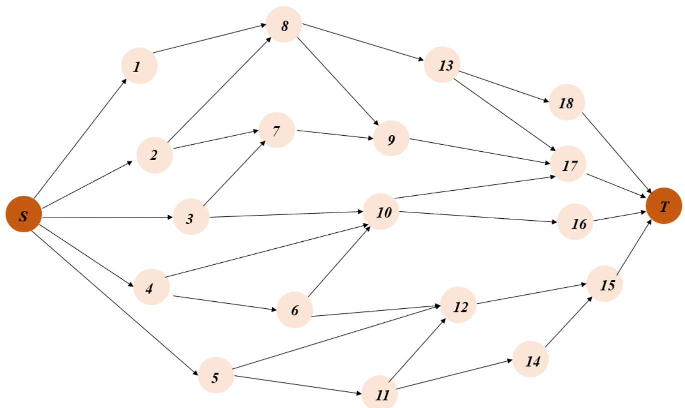

Fig. 2. Supply chain network of the numerical example

In the realm of network flow analysis, the Edmonds-Karp algorithm stands out as an effective approach for determining the maximum flow in a network. This algorithm operates by identifying augmenting paths, utilizing the Breadth-First Search (BFS) technique to discover the shortest path in each iteration. By iteratively updating the flow along these augmenting paths, the Edmonds-Karp algorithm converges to the maximum flow value within the network. On a different note, importance sampling emerges as a prominent method for estimating uncertain quantities. This technique involves generating random samples from a given distribution and utilizing these samples to approximate the desired quantity. In the specific context of supply chain network flow, importance sampling demonstrates its utility by accurately representing the uncertainties associated with various factors, including demand, supply, and transportation time.

Table 1. Simulation results of case study

	MCS	Subset Simulation	Importance Sampling
P_f	1.325×10^{-5}	1.021×10^{-5}	1.297×10^{-5}
C.O.V	0.045	0.242	0.076
N_s	2×10^{7}	5255	1277

4. Conclusion

This research paper has demonstrated the importance and effectiveness of leveraging importance sampling for risk assessment of supply chain networks. By incorporating this advanced simulation technique, we have been able to estimate the reliability of supply chain networks more accurately and identify critical areas of risk and vulnerability. The utilization of importance sampling allows decision-makers to focus computational resources on rare and extreme events that can significantly impact the performance and resilience of supply chain networks. This approach enables the estimation of rare event probabilities more efficiently than traditional Monte Carlo simulation methods. The identification and quantification of these rare events provide valuable insights into the potential disruptions and failures within the network. The results of the case study support the effectiveness of the proposed methodology. By applying importance sampling-based risk assessment to a realistic supply chain network, we were able to obtain more precise estimations of the network's reliability and vulnerability. This enhanced understanding enables decision-makers to develop proactive risk mitigation strategies, effectively allocate resources, and prioritize risk management efforts within the supply chain network.

References

[1] E. J. Hearnshaw and M. M. Wilson, “A complex network approach to supply chain network theory,” *International Journal of Operations & Production Management*, vol. 33, no. 4, pp. 442–469, 2013.

[2] M. Eskandarpour, P. Dejax, J. Miemczyk, and O. Péton, “Sustainable supply chain network design: An optimization-oriented review,” *Omega*, vol. 54, pp. 11–32, 2015.

[3] N. L. Dehghani, S. Zamanian, and A. Shafieezadeh, “Adaptive network reliability analysis: Methodology and applications to power grid,” *Reliability Engineering & System Safety*, vol. 216, p. 107973, 2021.

[4] R. Y. Rubinstein and D. P. Kroese, *Simulation and the Monte Carlo method*. John Wiley & Sons, 2016. Accessed: May 12, 2017.

[5] G. Fishman, *Monte Carlo: concepts, algorithms, and applications*. Springer Science & Business Media, 2013. Accessed: May 12, 2017.

[6] B. Echard, N. Gayton, M. Lemaire, and N. Relun, “A combined importance sampling and kriging reliability method for small failure probabilities with time-demanding numerical models,” *Reliability Engineering & System Safety*, vol. 111, pp. 232–240, 2013.

[7] X. Huang, J. Chen, and H. Zhu, “Assessing small failure probabilities by AK–SS: an active learning method combining Kriging and subset simulation,” *Structural Safety*, vol. 59, pp. 86–95, 2016.

[8] O. Ditlevsen and H. O. Madsen, *Structural reliability methods*, vol. 178. Wiley New York, 1996. Accessed: May 12, 2017.

[9] M. Lemaire, *Structural reliability*. John Wiley & Sons, 2013. Accessed: May 12, 2017.

[10] Z. Wang and A. Shafieezadeh, “REAK: Reliability analysis through Error rate-based Adaptive Kriging,” *Reliability Engineering & System Safety*, vol. 182, pp. 33–45, Feb. 2019, doi: 10.1016/j.ress.2018.10.004.

[11] Z. Wang and A. Shafieezadeh, “ESC: an efficient error-based stopping criterion for kriging-based reliability analysis methods,” *Struct Multidisc Optim*, vol. 59, no. 5, Art. no. 5, May 2019, doi: 10.1007/s00158-018-2150-9.

[12] X. Xiao, Q. Li, and Z. Wang, “A novel adaptive importance sampling algorithm for Bayesian model updating,” *Structural Safety*, vol. 97, p. 102230, Jul. 2022, doi: 10.1016/j.strusafe.2022.102230.