Acturial Analysis with Risk Updating: Case Study of Insurance Premium Calculation for Mechanical Vehicles

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Abstract. This paper introduces an innovative approach to premium calculation in actuarial science by incorporating the concept of risk updating. Traditional methods rely on static data and assumptions, but with advancements in data collection and statistical techniques, there is an opportunity to redefine premiums based on continuously gathered client information. The objective of this research is to develop a dynamic framework for premium calculation that adapts to changes in individual risk profiles. By leveraging client-specific data and statistical tools, insurers can adjust premiums to reflect evolving risks. Through a comprehensive review of existing literature and incorporation of advanced statistical methodologies, this research proposes a method to accurately model various risk factors, including demographics, claim history, and behavior patterns. Key considerations include addressing concerns regarding data privacy, accuracy of predictions, and monitoring changes in risk over time. The benefits of this approach include improved risk management, enhanced customer satisfaction, and fairer pricing based on individual risk profiles. By introducing a dynamic and adaptive method for premium calculation, this research revolutionizes actuarial science. It highlights the importance of real-time data and statistical analysis in determining accurate pricing models and optimizing risk portfolios.

Keywords. Risk updating, Actuarial science, Premium calculation, Dynamic approach, Statistical tools, Individual risk profiles.

1. Introduction

Actuarial science is tailored to utilize mathematical and statistical methods to assess and manage risks in the insurance and financial sectors. It involves analyzing and quantifying uncertain events, such as accidents, illnesses, and natural disasters, to determine appropriate premiums and establish financial reserves. Actuaries employ sophisticated models and techniques to evaluate and predict future outcomes, ensuring the financial stability and profitability of insurance companies. Risk updating is an essential concept within actuarial science [1]. It refers to the continuous monitoring and adjusting of risk assessments and calculations based on evolving information and changing circumstances. Traditional actuarial practices rely heavily

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on historical data and assumptions, often resulting in static and potentially outdated risk evaluations. However, with the advent of advancing technologies and the availability of real-time data, the concept of risk updating has emerged as a critical factor in enabling actuaries to accurately reflect current and emerging risks.

The importance of risk updating in actuarial science cannot be overstated. First and foremost, it allows for a more precise and dynamic analysis of risks. By incorporating up-to-date information and trends, such as changes in demographics, economic conditions, and industry developments, actuaries can more accurately estimate the probability and potential impact of various risks. This enables insurance companies to make informed decisions regarding premium calculations, reserves, and risk management strategies. Furthermore, risk updating plays a vital role in maintaining fairness within the insurance industry [2]. As risk profiles evolve over time, it becomes imperative to ensure that premiums align with the actual risks faced by policyholders. By continually updating risk assessments, insurers can offer more tailored and individualized pricing, reflecting the specific characteristics and circumstances of each policyholder. This promotes fairness, as policyholders are charged premiums that accurately reflect their actual risk levels rather than being subjected to broad assumptions or outdated data. Additionally, risk updating enhances the overall resilience and stability of insurance companies [3]. By closely monitoring and quickly responding to shifts in risk profiles, insurers can proactively adapt their business strategies, risk appetites, and product offerings [4]. This allows insurers to effectively manage and mitigate emerging risks, minimize financial losses, and maintain a strong financial position in the face of evolving market conditions and unforeseen events.

The primary objective of this article is to incorporate risk updating within the framework of actuarial science, aiming to achieve insurance premium calculation based on dynamic risk assessments. By incorporating risk updating methodologies, the article proposes a novel approach for calculating premiums. Specifically, the study utilizes reliability updating techniques to estimate the posterior probability of failure, thereby assessing the likelihood of damage or loss for individual insurance claims. The method employs Monte Carlo Simulation to compute the posterior probability of failure, subsequently enabling the calculation of premiums based on the principles of the central limit theorem. The motivation behind this research stems from the recognition that traditional actuarial practices often rely on static assumptions and historical data, which may not accurately reflect current and evolving risk profiles. By introducing risk updating into the field, the article seeks to address this limitation and provide a more dynamic and accurate premium calculation method.

This article initially presents an advanced actuarial approach for determining premiums based on the Probability of bankruptcy. Subsequently, the concept of risk updating is incorporated into the premium calculation, followed by a comprehensive demonstration of the proposed framework through the computation and updating of motor vehicle premiums.

2. Premium calculation based on probability of bankruptcy

Within the realm of actuarial science, the concept of insolvency probability delves into the potentiality of an insurance company or financial institution failing to fulfill its financial obligations or meet policyholder claims. It arises from a delicate balance between insufficient reserves and overwhelming liabilities. Essentially, it serves as a gauge of an insurer's financial fragility. The insolvency probability stands as a pivotal metric for assessing the financial soundness and solvency of an insurance company. Its purpose is to quantify the peril that an insurer's financial resources may prove inadequate to cover its liabilities, thus entailing bankruptcy or an inability to honor contractual commitments. Let us denote C(t) as the capital of an insurance company at time t. As such, C(t) can be characterized by the equation:

$$C(t) = C_i + \mathrm{I}(t) - X(t) \quad (1)$$

Here, C_i represents the initial capital of the insurance company, I(t) denotes the continuously evolving total premium income with respect to time t, while $X(t)$ corresponds to the cumulative claim amount up until time t, also recognized as the risk process. Additionally, we introduce a stochastic process denoted as $S(t)$, representing the total number of claims up to time t. Hence, $S(t)$ can be further derived by the equation:

$$X(t) = \sum_{i=1}^{S(t)} X_i \quad (2)$$

Accordingly, the probability of bankruptcy can be defined as follows:

$$P_{rup} = \Pr(C(t) \leq 0, \text{where t} \in [0, \mathrm{T}]) \quad (3)$$

Thus, the probability of bankruptcy efficiently captures the evolving risk of bank insolvency. Drawing upon the Central Limit Theorem (CLT), the cumulative claim amounts up to time t, *X(t)*, can be assumed to follow a normal distribution with mean value, μ_X, and variance, σ_X^2:

$$X(t) \sim N(\mu_X, \sigma_X^2\ ) \quad (4)$$

It can be inferred that $X(t)$ follows a normal distribution. By combining above equations, one can express the optimal premium as follows:

$$P^* = \underset{P \in \xi, P_{rup} \geq P_{min}}{\arg\max}\ C(t) \quad (5)$$

Here, P* signifies the optimal premium, ξ denotes all the feasible premium price, maximizing capital while ensuring that the probability of insolvency does not surpass the prescribed value of P_{min}.

Risk updating is essential in calculating premiums based on bankruptcy probability, especially in car insurance. It involves continuously assessing and adjusting risks to ensure premiums accurately reflect policyholders' evolving risk profiles. Dynamic factors such as driver behavior, accident history, and vehicle characteristics constantly change. Risk updating allows insurers to consider these changes and accurately evaluate the likelihood of accidents or claims. Premiums are adjusted accordingly to align with each policyholder's actual risk exposure [5]. For instance, if a policyholder with a clean record has recent accidents, risk updating helps recognize the increased probability of claims and adjust premiums accordingly. Similarly, improved driving behavior or advanced vehicle safety features can lead to lower premiums. Continuous premium updates maintain a fair pricing structure that benefits policyholders and insurers. Policyholders pay premiums based on their individual risk levels, promoting fairness and affordability, while insurers can account for future claims' financial impact, ensuring the portfolio's financial viability. Risk updating enhances precision and fairness in premium determination, particularly in car insurance, where risks frequently change. It empowers insurers to make informed pricing decisions that reflect real-time risk exposure, benefiting both insurers and policyholders.

3. Premium updating with risk updating

By denoting failure events as $X'(t)$ and observational data as D, the updated risk, represented as the conditional probability of risk, $\Pr(X'(t)|D)$, can be computed as follows [1]:

$$\Pr(X'(t)|D) = \frac{\Pr\big(D\big|X'(t)\big) \cdot \Pr\big(X'(t)\big)}{\Pr(D)} \tag{6}$$

The conditional probability of risk given information D, denoted as $\Pr(X'(t) \mid \mathrm{D})$ or the posterior probability of risk, and the probability of the joint event X∩D, denoted as $\Pr(X'(t) \cap \mathrm{D})$. Therefore, the optimal premium should be updated according to the updated risk of each claims. One can estimate the posterior optimal premium according to the following equations,

$$P^* = \underset{P \in \xi, P_{rup} \geq P_{min}}{\arg\max} \; C\big(t|X'(t)\big) \tag{7}$$

where $C\big(t|X'(t)\big)$ denotes the capital after the collection of the data information. Upon obtaining updated risk indicators based on data, the use of actuarial methods to update insurance premiums involves a meticulous and comprehensive process. Actuaries initiate the risk assessment phase by analyzing the newly obtained risk indicators, considering factors such as the frequencies of emergency incidents, instances of speeding, and tire wear and tear. Mathematical modeling is then employed, incorporating historical claims data, actuarial tables, and statistical techniques to quantify the relationship between the updated risk indicators and expected losses. Actuaries carefully calibrate these models to ensure accurate pricing [6]. Based on this analysis, premiums are calculated using established actuarial techniques, incorporating factors like coverage limits, deductibles, and claims processing expenses. External factors such as inflation, interest rates, and regulatory requirements are also considered. Actuaries collaborate with underwriters and business executives to develop a pricing strategy aligned with market dynamics, determining appropriate risk margins and profit targets. The impact of updated premiums on overall profitability and policyholder behavior is assessed. Ongoing monitoring and review are conducted to assess the effectiveness and accuracy of the updated premiums. Actuaries analyze claims experience, market trends, and changes in risk factors, refining the premium calculation process as necessary. This comprehensive approach enables insurers to offer premiums that accurately reflect risks, enhance profitability, and maintain a balanced insurance portfolio. Ultimately, this meticulous use of actuarial methods ensures fair and sustainable coverage for policyholders.

4. Case study

In this section, the utilization of the proposed method on risk-based premium calculation in studying the issue of premium updates for vehicles will be demonstrated. Specifically, the frequencies of emergency forklift incidents, instances of speeding, as well as the degree of tire wear and tear will be employed to update the probabilities of vehicle repairs and damages occurring [7]. Subsequently, based on the derived probabilities, they will be incorporated into the overall model for premium calculation to determine the resulting premium amount. Through this analysis, a comprehensive framework for assessing and adjusting premiums using risk-based approaches will be provided, integrating key factors related to vehicle safety and maintenance [8]. By leveraging these insights, insurers can enhance their pricing strategies and offer more accurate premiums that reflect the underlying risks associated with each insured vehicle. Fairness in premium allocation and the promotion of safer driving practices will be encouraged, resulting in reduced accident rates and improved overall risk management for the insurance industry.

As shown in the framework represented in Fig. 1, The annual premiums for vehicles undergo continuous fluctuations, and it would be unfair to apply the same premium pricing to different policyholders. Individuals with good driving habits should be rewarded with lower premiums, as they pose lower risks. On the other hand, individuals with poor driving habits should have higher premiums, as they are more likely to cause traffic accidents and file claims for compensation. Tailoring premiums to reflect individual risk profiles helps promote fairness and align insurance costs with actual risk exposures. By introducing a dynamic pricing mechanism based on individual driving behavior and risk indicators, insurers can incentivize safe driving practices while appropriately managing risk and maintaining profitability. This personalized approach not only ensures fairness in premium allocation but also encourages policyholders to improve their driving habits, ultimately contributing to safer roads and reduced accident rates. Therefore, the risk of accident of vehicle, R', should be updated according to the three parameters:

$$R' = M(E, S, T) \tag{8}$$

where R' represents the posterior risk of the vehicle, E denotes the frequency of emergency forklift incidents, S denotes the frequency of speeding incidents and T denotes the degree of tire wear and tear. In this case, the prior premium P^* is set as 125$ for each client, $\mathrm{N}(t)$ follows a Poisson random process, the initial capital, C_i is defined as 800k$, and the client's premium is collected annually and independently. After one year, it is observed of a client that E, S and T are estimated as 12, 16 and 7% with coefficient of variable of 18%, 26% and 15%.

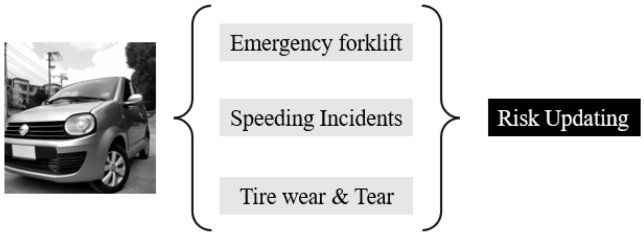

Fig. 1 Risk updating of the location of transmission based on frequency of emergency forklift incidents, frequency of speeding incidents and degree of tire wear and tear

By integrating the equations elucidated in section 2 and 3, a meticulous estimation of the optimal premium, symbolized as P^*, reveals a final calculation of 89.7$ per annum. However, a supplementary estimate surfaces, bearing testament that P^* amounts to 142.4 monetary units under the premise that E, S, and T are appraised at 21%, 22%, and 11% respectively, alongside a coefficient of variation of 15%, 22%, and 12% correspondingly. This contrasting valuation emerges as a consequence of the amplified risk engendered by perilous driving behaviors.

5. Conclusion

This paper introduces the revolutionary integration of risk updating into actuarial science, paving the way for a paradigm shift in premium calculation and risk management [9, 10]. By embracing a dynamic approach that harnesses real-time data and sophisticated statistical tools, insurers can transcend the constraints of traditional static methods and attain unprecedented levels of accuracy, fairness, and efficacy. Given the exponential growth in data availability and the advancements in data analytics techniques, the actuarial profession is compelled to adapt and evolve. The notion of risk updating presents a transformative solution by enabling insurers to continuously redefine premiums based on the ever-evolving risk profiles of individual policyholders. This approach not only allows for more precise and personalized pricing but also ensures that insurers remain agile and proactive in mitigating emergent risks. The potential benefits of incorporating risk updating into actuarial science are manifold. Enhanced risk management stands out as a primary advantage, equipping insurers with the ability to identify and monitor risk factors in real-time, adjust premiums accordingly, and fortify their defenses against potential losses.

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