

Two-stage Green Supply Chains Considering the Impact of Dynamic Subsidy Rates

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Abstract. Nowadays, more and more companies are investing in the development and production of environmentally friendly products. This paper investigates a dynamic pricing strategy and a dynamic subsidization strategy for a two-stage green supply chains containing a manufacturer and a retailer under both centralized decision-making scenario and decentralized decision-making scenario and investigates the investment behavior of manufacturer and the pricing behavior of retailer. We find that not only does centralized decision-making allow for greater efficiency at lower prices, but it also reduces government efforts to induce manufacturers to produce environmentally friendly goods. Firms also have more incentive to introduce discounts when price reductions can better attract consumers, which allows firms to reserve space for price reductions by raising the initial price of the product and gain profit by this way. However, when consumers are more interested in the current price of a product, the room for price cuts is therefore limited, which highly constrain the capacity of obtaining benefits for the companies who produce green products. This means that the government should choose the suitable time period to provide subsidies in order to maximize the effectiveness of the subsidies, and at the same time companies should choose whether or not to offer promotions based on the type of customer.

Keywords: Green Supply Chain, Government Subsidies, Sustainable Development, Energy Saving and Emission Reduction

1 Introduction

In recent years, with the continuous development of social and economic development, people's living standards continue to improve, the social concerns of hot spots also from high-speed development gradually changed to high-quality development, and the ensuing "green", "ecological", "energy saving" and other issues gradually come into people's view. In order to enhance the competitive advantage of enterprises and expand social influence, entrepreneurs also pay more and more attention to the impact of their products and services on the environment^{[1][2]}. In the automotive industry, for example, Lincoln, Volkswagen, Audi and other companies have announced that they will cease production of fuel-efficient vehicles completely or in some areas within the next 10 years., Companies such as BYD have signed agreements to achieve a 30% share of zero-emission medium- and heavy-duty truck

sales by 2030 and a 100% share of zero-emission medium- and heavy-duty truck sales by 2040. At the same time, as the environment continues to deteriorate, governments around the world are working to raise the environmental awareness of businesses and consumers alike^[3]. Many countries and regions, such as Japan and North America, as well as Europe, have taken a number of strong measures to increase the environmental responsibility of producers^[4]. At the same time, along with massive energy consumption, carbon emissions, a major contributor to global warming, have attracted the attention of countries around the world, and have led many to work on carbon neutrality^[5]. In order to achieve carbon neutrality, many countries have adopted legislation and established relevant regulatory bodies^{[6][7][8]}. Research on how to rationally guide enterprises to green production is of particular significance.

When talking about green supply chains, it is impossible to avoid the issue of government guidance and subsidies, but it should be noted that government subsidies are not static but dynamic. China's new energy vehicle industry, for example, the relevant documents pointed out that, clearly "in principle, 2020-2022 subsidy standards in the previous year, respectively, on the basis of 10%, 20%, 30%, the public transport sector in line with the requirements of the vehicle 2021-2022 subsidy standards in the mountain on the basis of a year of 10%, 20%"^[9]. Most previous studies have focused on the impact of static subsidies on a single supply chain, with less consideration given to the impact of changes in subsidy rates and the resulting price changes. This paper focus on what will change the willing of manufacturer to invest in green products and how the pricing strategic of retailer will influence the performance of supply chain while take the impact of the changing of subsidy rate. Basing on this, we brings those issues into the study and makes the following main innovations:

- I. To study the impact of changes in government subsidy rates in the two periods before and after the establishment of a two-stage supply chain. The government subsidy rate is often static in the existing domestic and international studies on green supply chain, and the possible impacts of subsidy rate changes on the supply chain are not considered. In this paper, we will try to fill this research gap by establishing a two-stage supply chain model, comparing the impact of different subsidy rates in different periods, and suggesting possible recommendations for actual production management.
- II. Integration of dynamic production environments and the impact of dynamic pricing. Most of the existing research on "dynamic supply chain" focuses only on the dynamic production environment or the dynamic pricing strategy of enterprises, but seldom considers the two together. However, the dynamic production environment will inevitably cause the impact of firms' pricing, so it is necessary to consider the two together.

2 Literature review

Many scholars at home and abroad have made many in-depth studies on green supply chain decision-making and achieved a large number of research results. Ying Xie[12] et al. constructed a model through the least squares method to investigate whether government intervention can serve as an effective mechanism to promote public-private partnerships between construction companies, thereby contributing to eco-modernization through the adoption of green supply chain management. Caurila[13] et al. modeled the impact of direct subsidies of alternative carbon prices on biomass consumption to meet French biomass energy consumption targets. Hussain[14] et al. and Cao[15] et al. considered government subsidies as financial incentives for firms to reduce carbon emissions under a carbon trading system. Zhang[16] et al. suggested that carbon trading systems can help advance renewable energy investments and reduce the level of subsidies required. Fang and Ma[17] suggested that the Government could subsidize the more burdensome ETS to ensure that carbon reduction targets were met. Lin and Jia[18] emphasized that giving more subsidies to renewable energy companies could increase the effectiveness of carbon trading mechanisms for renewable energy development. Yu[19] et al. noted that government financial subsidies can be used to support research, development and deployment of green technologies.

The studies above mainly focus on the impact of a particular policy on green supply chains over a single period of time.

At the same time, many scholars have added comparisons between two periods to the study of supply chains. Li and Ma[20] proposed a Stackelberg dynamic model of a dual sales channel supply chain with online and traditional channels. Shi[21] et al. compared the rational Stackelberg game and the dynamic duopoly game models. Ding[22] et al. focused on discrete and dynamical systems for duopoly games with heterogeneous players. Xu[23] et al. studied a differential game model in which the carbon reduction of a product is influenced by the manufacturer's greening efforts. Jiang[24] et al. provided a dynamic game model for modeling advertising competition considering promotions. Lou and Ma[25] examined the Stackelberg dynamic game of household appliance supply chains and considers marketing efforts and carbon reduction. Zhou and Ye[26] modeled a dual-channel supply chain based on a differential game model, focusing on carbon reduction strategies. However, most of these studies have only considered changes in product prices and to a lesser extent the impact of changes in government subsidy rates on supply chains.

Based on the above shortcomings, this paper constructs a two-stage green supply chain that integrates the impacts of changes in government subsidy rates as well as changes in product prices. The third subsection of this paper describes how the mathematical model is constructed, numerical experiments are conducted to obtain the results in the fourth subsection of this paper, and a conclusion is made in the fifth subsection.

3 Mathematical models

Table 1 .Symbols in this paper.

M	Manufacturer	
R	Retailer	
$w_{j,i}$	Wholesale price at stage i	$i = 1,2$ $j = B,E$
$p_{j,i}$	Retail price at stage i in model j	$i = 1,2$ $j = B,E$
θ	Carbon emissions reduction of green products compared to traditional products	$0 < \theta < 1$
β	The greening cost coefficient	$\beta > 0$
a	Total potential market demand	
b	The demand sensitivity coefficient to the retail price	
k	Price elasticity of demand for the product review from the last stage to the current stage	$0 < k < b$
λ	The demand sensitivity coefficient to the greening levels	$\lambda > 0$
$\pi_{l,i}^j$	Profit function for participant j in stage i in model l	$i = 1,2$ $j = R, M$ $l = B, E$
S_i	The amount of subsidies granted by the government in stage i	$i = 1,2$
s_i	Subsidy rate for phase i	$0 < s_i^R < 1$
x_1	Investment decision variables for manufacturers' green R&D Investments	$x_1 = 0,1$
e	Carbon emissions per product	
z	The carbon price in carbon emission trading market	

The supply chain in this paper consists of the government, the manufacturer (M), and the retailer (R). At the beginning of the first phase, manufacturers face a consumer market with primary demand of a . In this problem, we suppose that the market demand D_i depends on the greening levels, the difference between the previous and the current stage selling price, and the current selling price.

The government firstly determines the rate of subsidy s_1 . In the actual problem, the manufacturer does not choose the degree of energy efficiency and emission reduction of the product, but rather chooses to invest in a technology and eventually acquire it after paying a certain amount of R&D costs. Thus, the manufacturer will decide whether or not to invest in R&D ($x=1$ or $x=0$) to reduce the carbon emissions of its products and the wholesale price w_1 of its products after government. Following Liu, Anderson[27], and Ma, Wang, and Shang (2013)[28], we suppose the additional cost in every stage is $\frac{\beta\theta^2}{2}$ evenly where θ is the greenness of the product which is equal to the carbon footprint that green product can reduce compared to conventional products and β is the greening cost coefficient. If the manufacturer chooses to invest in R&D,

it will sell green products in the following phases 1 and 2, and if it does not choose to invest in R&D, it will sell non-green products. After observing the manufacturer's decision, the retailer will choose its own retail price p_1 . At the beginning of the second phase, governments, manufacturers, and retailers will re-select their decision variables s_2 , w_2 and p_2 . That is to say, the manufacturer plays a Stackelberg game with the retailer and the manufacturer is the leader of the game. For simplicity and to make the study more relevant, the following hypotheses are proposed:

- I. R&D is not time-consuming. Manufacturers can start producing green products as soon as they have made the investment decision and paid the corresponding R&D costs.
- II. Supply and demand are in balance. All products produced in each period can be sold out and the participants in the supply chain face a given market with a constant overall market demand a .
- III. Demand is a linear function based on the price of the product, the greenness of the product, and the degree of price change.

Accordingly, the market demands for two stages can be shown as follows, respectively:

$$D_1 = a - bp_1 + x_1 \lambda \theta \quad (1)$$

$$D_2 = a - bp_2 + x_1 \lambda \theta - k(p_2 - p_1) \quad (2)$$

where b is the demand sensitivity coefficient to the current price of the product. Obviously, consumers' desire to buy decreases as prices rise, thus $b > 0$. λ is the demand sensitivity coefficient to the greening levels in retail and direct channel, as well as the green awareness. Following the study before us, we suppose that as the carbon footprint of a product decreases, the product becomes more environmentally friendly, then consumers are more likely to buy it. Besides, consumers who have higher green awareness are more willing to pay for the environmentally friendly products. k is the price elasticity of demand for the product review from the last stage to the current stage. That is to say, promoting this behavior may stimulate consumption. And we suppose that $0 < k < b$ to make sure the consumers care more about the current price of a product. All symbols appearing in this paper are summarized in Table 1.

3.1 Basic model—without government subsidies

3.1.1 Decentralized decision-making

Under decentralized decision making, the manufacturer and the retailer are two different decision makers who make decisions for the purpose of maximizing their respective profits. In the first stage, the manufacturer chooses whether to invest in green R&D and determines the wholesale price of the product ω_1 , the investment decisions are expressed as $x_1 = 0,1$. Their profits are affected by the price of the product p_1 and product Greenness θ . If choose to invest, the manufacturer will

produce environmentally friendly products with greenness θ in the next two phases, and pay for $\beta\theta^2$. Retailers then choose retail price p_1 as followers.

In the second stage, the retailer's decision objective and decision variables are the same as in the first stage. The manufacturer's decision objective remains unchanged, but it only needs to choose the wholesale price of its product, and its profit is affected by the degree of price change $p_2 - p_1$. Multiply the quantity demanded in equation (1) and (2) by the unit price to get the total profit

With the assumptions above, the profit functions can be expressed as follows:

$$\pi_B^M = (a - bp_1 + x_1\lambda\theta)\omega_1 + [a - bp_2 + x_1\lambda\theta - k(p_2 - p_1)]\omega_2 - x_1\beta\theta^2 \quad (3)$$

$$\pi_B^R = (a - bp_1 + x_1\lambda\theta)(p_1 - \omega_1) + [a - bp_2 + x_1\lambda\theta - k(p_2 - p_1)](p_2 - \omega_2) \quad (4)$$

Where $k < b$, to ensure that consumers will be more sensitive to the price of the product itself rather than to changes in the price of the product. And $\omega_i < p_i$ to ensure that retailer's sales practice can be profitable.

By solving Eq. (3) and Eq. (4), we get

Proposition 1 :Optimal pricing:

$$p_1^* = \frac{(a + \lambda\theta x_1)(6b^2 + 7bk - k^2)}{2b(4b^2 + 4bk - k^2)}$$

$$p_2^* = \frac{(a + \lambda\theta x_1)(6b^2 + 5bk - k^2)}{2b(4b^2 + 4bk - k^2)}$$

Proposition 2 :Optimal profit:

$$\pi_B^{M*} = (a^2b + a^2k + 2ab\lambda\theta x_1 + 2ak\lambda\theta x_1 - 4\beta b^2\theta^2 x_1 + b\lambda^2\theta^2 x_1^2 - 4\beta bk\theta^2 x_1 + k\lambda^2\theta^2 x_1^2 + 3k^2\lambda\theta p_0 x_1 + \beta k^2\theta^2 x_1)/(4b^2 + 4bk - k^2)$$

$$\pi_B^{R*} = \frac{(b + k)(a + \lambda\theta x_1)^2}{2(4b^2 + 4bk - k^2)}$$

3.1.2 Centralized decision-making

Under centralized decision-making, manufacturers and retailers are controlled by the same decision-maker or group. In this scenario, the decision-maker aims to maximize the overall benefits of the supply chain when making decisions. the profit functions can be expressed as follows:

$$\pi_1^C = \pi_1^M + \pi_1^R = (a - bp_1 + x_1\lambda\theta)p_1 - \frac{x_1\beta\theta^2}{2} \quad (5)$$

$$\pi_2^C = \pi_2^M + \pi_2^R = [a - bp_2 + x_1\lambda\theta - k(p_2 - p_1)]p_2 - \frac{x_1\beta\theta^2}{2} \quad (6)$$

By solving Eq. (5) and Eq. (6), we get

Proposition 3 : Optimal pricing

$$p_1^{C*} = \frac{(a + \lambda\theta x_1)(2b + 3k)}{4b^2 + 4bk - k^2}$$

$$p_2^{C*} = \frac{(a + \lambda\theta x_1)(2b + k)}{4b^2 + 4bk - k^2}$$

Proposition 4: Optimal profitability

$$\pi^{C*} = (2a^2b + 2a^2k + 4ab\lambda\theta x_1 + 4a\lambda\theta kx_1 - 4\beta b^2\theta^2 x_1 + 2b\lambda^2\theta^2 x_1^2 - 4\beta bk\theta^2 x_1 + 2\lambda^2\theta^2 kx_1^2 + \beta k^2\theta^2 x_1^2)/(4b^2 + 8bk + 3k^2)$$

Proposition 5: Investment intention

Make $W_B = \pi_B^{M*}(x=1) - \pi_B^{M*}(x=0)$ and $W_B^C = \pi_B^{C*}(x=1) - \pi_B^{C*}(x=0)$ represents the investment intention to research on green products of the supply chain under decentralized decision-making situations and centralized decision-making situations in the basic model respectively. Obviously, investment behavior will occur when $W > 0$. By the calculation before, we get:

$$\begin{aligned} \text{a) } W_B &= \frac{\theta(-4\beta\theta b^2 + \theta b\lambda^2 + 2ab\lambda - 4\beta\theta bk + \lambda^2\theta k + 2a\lambda k + \beta\theta k^2)}{4b^2 + 4bk - k^2}, W_B^C = \\ &= \frac{\theta(-4\beta\theta b^2 + 2\theta b\lambda^2 + 4ab\lambda - 4\beta\theta bk + 2\lambda^2\theta k + 4a\lambda k + \beta\theta k^2)}{4b^2 + 4bk - k^2}, W_B^C - W_B = \frac{\lambda\theta(b+k)(2a+\lambda\theta)}{4b^2 + 4bk - k^2} > 0 \\ \text{b) } \frac{\partial W_B^C}{2\partial b} &= \frac{\partial W_B}{\partial b} = -\frac{\lambda\theta(2a+\lambda\theta)(4b^2+8bk+5k^2)}{(4b^2+4bk-k^2)^2} < 0 \\ \text{c) } \frac{\partial W_B^C}{2\partial k} &= \frac{\partial W_B}{\partial k} = \frac{\lambda\theta k(2a+\lambda\theta)(2b+k)}{(4b^2+4bk-k^2)^2} > 0 \end{aligned}$$

Proposition 5 (a) and (b) shows that compared with decentralized decision-making model, centralized decision-making model will make it easier for the supply chain to invest in green products.

Proposition 5 (c) and (d) shows that manufacturers' willingness to invest in research and development of green products declines as consumers' price sensitivity rises, but rises as consumers' sensitivity to price changes rises.

Proposition 6:

$$\text{Make } \Delta p = p_1^* - p_2^*, \Delta p^C = p_1^{C*} - p_2^{C*}, \Delta \pi_B^M = \pi_{B,1}^{M*} - \pi_{B,2}^{M*}, \Delta \pi_B^R = \pi_{B,1}^{R*} - \pi_{B,2}^{R*}, \Delta \pi_B^C = \pi_{B,1}^{C*} - \pi_{B,2}^{C*}$$

$$\begin{aligned} \text{a) } \Delta p^C &= \Delta p = \frac{2k(a+\lambda\theta x_1)}{4b^2+4bk-k^2} > 0, \Delta \pi_B^M = -\frac{k(b+k)(a+\lambda\theta x_1)^2}{2b(4b^2+4bk-k^2)} < 0, \Delta \pi_B^R = \\ &= -\frac{k^2(b+k)(a+\lambda\theta x_1)^2}{(4b^2+4bk-k^2)^2} < 0, \Delta \pi_B^C = -\frac{4k^2(b+k)(a+\lambda\theta x_1)^2}{(4b^2+4bk-k^2)^2} < 0 \\ \text{b) } \frac{\partial p_1^*}{\partial b} &= -\frac{(a+\lambda\theta x_1)(24b^2+56b^3k+22b^2k^2-8bk^3+k^4)}{2b^2(4b^2+4bk-k^2)^2} < 0, \frac{\partial p_2^*}{\partial b} = \\ &= -\frac{(a+\lambda\theta x_1)(24b^4+40b^3k+14b^2k^2-8bk^3+k^4)}{2b^2(4b^2+4bk-k^2)^2} < 0, \frac{\partial \Delta p}{\partial b} = -\frac{4k(a+\lambda\theta x_1)((2b+k)}{(4b^2+4bk-k^2)^2} < 0 \\ \text{c) } \frac{\partial \pi_B^C}{\partial b} &= 2\frac{\partial \pi_B^M}{\partial b} = 4\frac{\partial \pi_B^R}{\partial b} = -\frac{2(a+\lambda\theta x_1)^2(4b^2+8bk+5k^2)}{(4b^2+4bk-k^2)^2} < 0, \frac{\partial \Delta \pi_B^M}{\partial b} = \end{aligned}$$

$$\frac{k(a+\lambda\theta x_1)^2(8b^3+16b^2k+8bk^2-k^3)}{2b^2(4b^2+4bk-k^2)^2} > 0, \frac{\partial \Delta \pi_B^C}{\partial b} = 4 \frac{\partial \Delta \pi_B^R}{\partial b} = \frac{4k^2(a+\lambda\theta x_1)^2(12b^2+20bk+9bk^2)}{(4b^2+4bk-k^2)^3} > 0$$

d) $\frac{\partial p_1^*}{\partial k} = \frac{(a+\lambda\theta x_1)(4b^2+4bk+3k^2)}{2(4b^2+4bk-k^2)^2} > 0, \frac{\partial p_2^*}{\partial k} = \frac{(a+\lambda\theta x_1)(-4b^2+4bk+k^2)}{2(4b^2+4bk-k^2)^2},$ when $k < (2\sqrt{2}-2)b$, $\frac{\partial p_2^*}{\partial k} < 0,$ when $(2\sqrt{2}-2)b < k < b$, $\frac{\partial p_2^*}{\partial k} > 0, \frac{\partial \Delta p}{\partial k} = \frac{(a+\lambda\theta x_1)(4b^2+k^2)}{(4b^2+8bk+3k^2)^2} > 0$

e) $\frac{\partial \pi_B^C}{\partial b} = 2 \frac{\partial \pi_B^M}{\partial b} = 4 \frac{\partial \pi_B^R}{\partial b} = \frac{2k(2b+k)(a+\lambda\theta x_1)^2}{(4b^2+4bk-k^2)^2} > 0$

f) $\frac{\partial p_1^*}{\partial a} = \frac{\frac{k^2}{2b} + \frac{7k}{2} + 3b}{4b^2+4bk-k^2} > 0, \frac{\partial p_2^*}{\partial a} = \frac{\frac{k^2}{2b} + \frac{5k}{2} + 3b}{4b^2+4bk-k^2} > 0, \frac{\partial \Delta p}{\partial a} = \frac{k}{4b^2+4bk-k^2} > 0$

g) $\frac{\partial \pi_B^C}{\partial a} = 2 \frac{\partial \pi_B^M}{\partial a} = 4 \frac{\partial \pi_B^R}{\partial a} = \frac{4(b+k)(a+\lambda\theta x_1)}{4b^2+4bk-k^2} > 0, \frac{\partial \Delta \pi_B^M}{\partial a} = -\frac{k(b+k)(a+\lambda\theta x_1)}{b(4b^2+4bk-k^2)} < 0,$
 $0, \frac{\partial \Delta \pi_B^C}{\partial a} = 4 \frac{\partial \Delta \pi_B^R}{\partial a} = -\frac{8k^2(b+k)(a+\lambda\theta x_1)}{(4b^2+4bk-k^2)^2} < 0,$

h) $\frac{\partial p_1^*}{\partial \lambda} = \frac{\theta x_1(6b^2+7bk-k^2)}{2b(4b^2+4bk-k^2)} > 0, \frac{\partial p_2^*}{\partial \lambda} = \frac{\theta x_1(6b^2+5bk-k^2)}{2b(4b^2+4bk-k^2)} > 0, \frac{\partial \Delta p}{\partial \lambda} = \frac{k\theta x_1}{4b^2+4bk-k^2} > 0,$

i) $\frac{\partial \pi_B^C}{\partial \lambda} = 2 \frac{\partial \pi_B^M}{\partial \lambda} = 4 \frac{\partial \pi_B^R}{\partial \lambda} = \frac{4\theta x_1(b+k)(a+\lambda\theta x_1)}{4b^2+4bk-k^2} > 0, \frac{\partial \Delta \pi_B^M}{\partial \lambda} = -\frac{k\theta x_1(b+k)(a+\lambda\theta x_1)}{b(4b^2+4bk-k^2)} < 0,$
 $0, \frac{\partial \Delta \pi_B^R}{\partial \lambda} = -\frac{2k^2\theta x_1(b+k)(a+\lambda\theta x_1)}{(4b^2+4bk-k^2)^2} < 0, \frac{\partial \Delta \pi_B^C}{\partial \lambda} = -\frac{8k^2\theta x_1(b+k)(a+\lambda\theta x_1)}{(4b^2+4bk-k^2)^2} < 0$

Proposition 6(a) means that in the long run, companies will always maintain the attractiveness of their products through price concessions. This also makes the profitability of a particular product diminish over time.

Proposition 6(b)~(c) means that as consumers become more price-sensitive at this stage, they are more likely to buy products at lower prices, which forces firms to struggle to attract consumers through discounting strategies and further undermines the ability to maintain long-term profitability.

Proposition 6(d)~(e) means that the more consumers are concerned about price differentials, the more likely merchants are to raise prices in the first stage, and that prices in the second stage will fall and then rise as consumers' sensitivity to price differentials rises. This may be due to the fact that as consumers' price difference sensitivity rises, even if merchants increase their price reductions, the price of the goods remains high after the price reductions due to the elevated prices in the first stage. And as consumer spread sensitivity rises, firms' profitability also rises.

Proposition 6(f)~(I) means that as the market size increases and consumers are more concerned about the greenness of products, the greater the profit margins of merchants and their ability to maintain long-term profitability, and although price reductions of commodities increase in the second stage, overall consumers have to

pay more prices in both stages.

3.2 Carbon emission reduction allowances

In this subsection, the government subsidizes manufacturers based on the degree of energy efficiency and emission reduction of the products they produce. The amount of subsidy available for each product is:

$$S_i^E = s_i^E z e \theta x_1 \quad (7)$$

Where z is the carbon price in carbon emission trading market and e is the carbon emissions of per product.

3.2.1 Decentralized decision-making

Manufacturers and retailers make their decisions on a profit-maximizing principle, respectively, and, unlike the base model, the manufacturer receives a subsidy of S_i for each unit of product sold. On this basis, by Combining Eq. (5), (6), and (7), we obtain the following equation:

$$\pi_E^M = D_1(\omega_1 + S_1^E) + D_2(\omega_2 + S_2^E) - \beta x_1 \theta^2 \quad (8)$$

$$\pi_E^R = D_1(p_1 - \omega_1) + D_2(p_2 - \omega_2) \quad (9)$$

By solving Eq. (8) and Eq. (9), we get

Proposition 7: Optimal pricing

- $p_{E,1}^* = (ab^2 - ak^2 + 7abk + 6b^2\lambda\theta x_1 - \lambda\theta k^2 x_1 + 7b\lambda\theta x_1 k - 2b^3 S_1^E x_1 - 2b^2 k S_1^E x_1 + bk^2 S_2^E x_1 + b^2 k S_2^E x_1) / (2b(4b^2 + 4bk - k^2))$
- $p_{E,2}^* = (ab^2 - ak^2 + 5abk + 6b^2\lambda\theta x_1 - \lambda\theta k^2 x_1 + 5b\lambda\theta x_1 k - 2b^3 S_2^E x_1 - b^2 k S_1^E x_1 + bk^2 S_2^E x_1 - 2b^2 k S_2^E x_1) / (2b(4b^2 + 4bk - k^2))$

3.2.2 Centralized decision-making

In this subsection, we will further study our discussion of the centralized decision-making scenario in depth, on the basic of the carbon emission reduction allowances. Similar to the centralized decision making in the base model, the manufacturer and the retailer are controlled by the same decision maker or group of decision makers, who in this case make decisions with the goal of maximizing the overall benefit to the supply chain. The profit functions can be expressed as follows:

$$\pi_E^C = \pi_E^M + \pi_E^R \quad (10)$$

By solving Eq. (10), we get

Proposition 8: Optimal pricing:

- $p_{E,1}^{C*} = \frac{2ab+3ak+2b\lambda\theta x_1+3k\lambda\theta x_1-2b^2 S_1^E+k^2 S_2^E-2bk S_1^E+bk S_2^E}{4b^2+4bk-k^2}$

$$b) \quad p_{E,2}^{C*} = \frac{2ab+ak+2b\lambda\theta x_1+k\lambda\theta x_1-2b^2S_2^E+k^2S_2^E-bkS_1^E-2bkS_2^E}{4b^2+4bk-k^2}$$

Proposition 9:

Make $\Delta p_E = p_{E,1}^* - p_{E,2}^*$, $\Delta p_E^C = p_{E,1}^{C*} - p_{E,2}^{C*}$, $\Delta \pi_E^M = \pi_{E,1}^{M*} - \pi_{E,2}^{M*}$, $\Delta \pi_E^R = \pi_{E,1}^{R*} - \pi_{E,2}^{R*}$, $\Delta \pi_E^C = \pi_{E,1}^{C*} - \pi_{E,2}^{C*}$

$$a) \quad \frac{\partial p_{E,1}^{C*}}{\partial S_1^E} = 2 \frac{\partial p_{E,1}^*}{\partial S_1^E} = -\frac{2b^2+2kb}{4b^2+4bk-k^2} < 0, \quad \frac{\partial p_{E,2}^{C*}}{\partial S_1^E} = 2 \frac{\partial p_{E,2}^*}{\partial S_1^E} = -\frac{kb}{4b^2+4bk-k^2} < 0$$

$$b) \quad \frac{\partial p_{E,1}^*}{\partial S_2^E} = \frac{k(b+k)}{8b^2+8bk-2k^2} > 0, \quad \frac{\partial p_{E,1}^{C*}}{\partial S_2^E} = \frac{k^2+bk}{4b^2+4bk-k^2} > 0, \quad \frac{\partial p_{E,2}^{C*}}{\partial S_2^E} = 2 \quad \frac{\partial p_{E,2}^*}{\partial S_2^E} = -\frac{2b^2+2kb-k^2}{4b^2+4bk-k^2} < 0$$

$$c) \quad \frac{\partial \Delta p_E^C}{\partial S_1^E} = 2 \frac{\partial \Delta p_E}{\partial S_1^E} = -\frac{b(2b+k)}{4b^2+4bk-k^2} < 0$$

$$d) \quad \frac{\partial \Delta p_E^C}{\partial S_2^E} = 2 \frac{\partial \Delta p_E}{\partial S_2^E} = \frac{b(2b+3k)}{4b^2+4bk-k^2} > 0$$

$$e) \quad \frac{\partial \Delta \pi_E^M}{\partial S_1^E} = \frac{b(b+k)(a+S_1^E b + \lambda\theta x_1)}{4b^2+4bk-k^2} > 0, \quad \frac{\partial \Delta \pi_E^M}{\partial S_2^E} = -\frac{(b+k)^2(a+S_2^E b + \lambda\theta x_1)}{4b^2+4bk-k^2} < 0$$

Proposition 9(a) means that in both centralized and decentralized decision making, increasing the amount of subsidy in the first stage will reduce the optimal price in the first and second stages

Proposition 9(b) means that in both centralized and decentralized decisions, increasing the amount of subsidy in the second stage will increase the optimal price in the first stage, but will decrease the optimal price in the second stage.

Proposition 9(c)~(d) means that increasing the amount of subsidy in the first stage or decreasing the amount of subsidy in the second stage in both centralized and decentralized decision making will reduce the scope for merchants to use price reduction strategies.

Proposition 9(e) means that increasing the amount of subsidies in the first stage or reducing the amount of subsidies in the second stage would limit the ability of firms to maintain long-term profitability.

Proposition 10: Investment intention

Make $W_E = \pi_E^{M*}(x=1) - \pi_E^{M*}(x=0)$ and $W_E^C = \pi_E^{C*}(x=1) - \pi_E^{C*}(x=0)$ represents the investment intention to research on green products of the supply chain under decentralized decision-making situations and centralized decision-making situations in the model with subsidy respectively. Obviously, investment behavior will occur when $W > 0$. By the calculation before, we get:

$$(a) \quad W_B = \frac{\theta(4a\lambda(b+k)+2b^2\lambda(S_1^E+S_2^E)-\lambda k^2(S_1^E-S_2^E)+\lambda bk(S_1^E+3S_2^E)+2b\lambda^2\theta)}{2(4b^2+4bk-k^2)} - \beta\theta, W_E^C = W_E + \frac{\lambda\theta(b+k)(4a+2b(S_1^E+S_2^E)-k(S_1^E-S_2^E)+2\lambda\theta)}{2(4b^2+4bk-k^2)}$$

$$(b) \frac{\partial W_E^C}{2\partial b} = \frac{\partial W_E}{\partial b} = \frac{4ab\lambda + 4a\lambda k + 2b^2\lambda(S_1^E + S_2^E) + (S_1^E - S_2^E)\lambda k^2 + 4b\lambda^2\theta + 4\lambda^2 k\theta + (S_1^E + 3S_2^E)b\lambda k}{2(4b^2 + 4bk - k^2)} - 2\beta\theta$$

$$(c) \frac{\partial W_E^C}{2\partial k} = \frac{\partial W_E}{\partial k} = \frac{\lambda\theta(4b^3(S_2^E - S_1^E) + 4ak^2 + 8abk^2 + 4b^2k(3S_2^E - S_1^E) + (7S_2^E - 3S_1^E)bk^2 + 2\lambda k^2\theta + 4b\lambda k\theta)}{2(4b^2 + 4bk - k^2)^2}$$

Proposition 10 (a) shows the willing of investment.

Proposition 10 (b) and (c) suggested that compared with the basic model, subsidy may change the influence of b and k.

4. Numerical example

To illustrate the above theoretical results, this section provides a set of numerical analysis. By referring to Howes, r. , Skea, J., & Whelan, B. (2013), Zhu, Q. Dou, Y., 2011 and Xu et al. (2017) [29][30][31], we set $a = 60$ (unit/year), $b = 1$, $e = 2$ (tCO₂e/unit), $\lambda = 0.2$, $s_1 = 0.3$, $s_2 = 0.2$. The carbon price in Beijing carbon emission trading market is $1.3(\times 10^8 / \text{tCO}_2\text{e})$. And according to Zhang et al. and Jia J et al. [10][11], we set $k = 0.5$, $\beta = 8$, $\theta = 0.8$.

4.1 Comparison of optimal solutions

Given the data, the optimal solutions under different scenarios can be calculated and summarized in Table 2. As same as the study before, the result shows that centralized decision will increase the efficiency of supply chain in both two stage no matter the manufacturer invest the more environmentally friendly product or not. The efficiency of the supply is calculated and compared in each situation. As expected, table 1 shows that Centralized Scenario will achieve higher efficiency. The results also indicate that whether it is centralized decision-making or decentralized decision-making, companies tend to prefer capturing profits through price reduction.

Table 2. Comparison of optimal solutions.

Symbol	Centralized Scenario	Decentralized Scenario	Centralized Scenario with Subsidies	Decentralized Scenario with Subsidies
p_1	36.522	48.26	36.348	48.26
p_2	26.087	43.043	25.903	43.043
π^M	-	939.13	-	939.13
π^R	-	469.565	-	469.565
π^{SC}	1878.26	1408.695	1887.44	1408.695
x_1	0	0	1	0
Channel efficiency		75%		75%

4.2 Effect of λ on optimal solutions

In order to explore more implications for management practices, we conducted a sensitivity analysis of consumers' green awareness in this subsection. In this part, we examine how A affects supply chain performance effects. We set the value of λ varying from 0 to 1. Fig.1 shows the effects of λ on the two period selling prices and the total profit.

It is shown that as the awareness of consumers increases, the profitability of investing also rises, which means the investment variable x will change from 0 to 1. Besides, we can see that firms are more likely to make higher profits by investing in green products with subsidies than they would be in the absence of subsidies. This may be explained as follows: Higher green awareness means that consumers are more likely to buy green products than non-green products, which also makes it more likely that companies will profit from producing environmentally friendly products. At the same time, businesses have more opportunities to pass on the increased costs of producing green products to consumers.

The results indicated that the high green awareness will give producers courage to invest in the greenness of products.

From the above analysis, we can draw the following interesting conclusions: Raising the green awareness of consumers will indeed help save energy and reduce emissions, and this may be one of the important directions of the Government's work in the future. But at the same time, it will also give businessmen more opportunities to shift costs. One of the parties, either government, manufacturer, or consumers, will pay for the non-economic aspects of environmentally friendly products, but when consumers find that their environmental awareness has been "exploited", will anyone be willing to be the "wrongdoer"? This may be a future research direction.

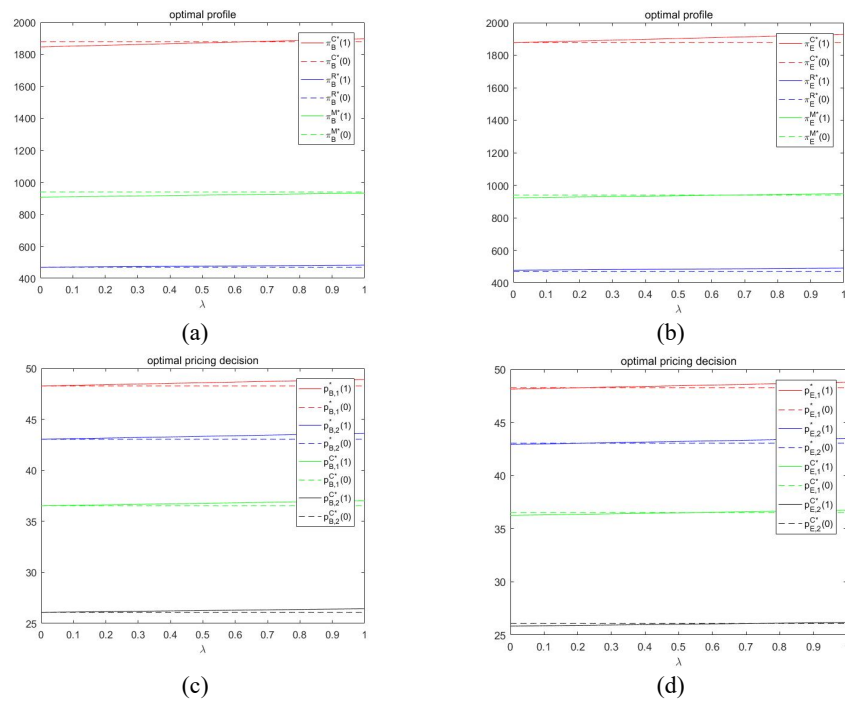
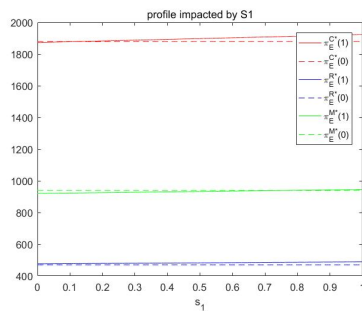


Figure 1. Effect of λ on optimal solutions

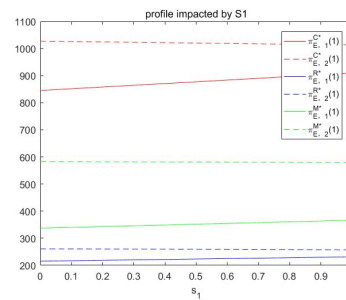
4.3 Effect of s_1 and s_2 on optimal solutions

In order to obtain more management implications, sensitivity analysis on s_1 and s_2 is conducted. Here we set $s_2 = 0.2$ when s_1 varying from 0 to 1, as same as s_2 . We similarly examine the effect of subsidy rates on whether firms invest in research and development of green products: when investing in green products makes firms more profitable, we consider that firms have sufficient incentives to engage in investment behavior, which means when $\pi^M(x = 1) > \pi^M(x = 0)$, $x = 1$, and the Investment behavior will happen.

Fig.2 shows that the benefits to supply chain participants can indeed be improved to a large extent by increasing the rate of subsidy, in both the first and second stages. In particular, while increasing the subsidy rate would improve the performance of the supply chain in general, we find that increasing the subsidy rate at one stage reduces the revenue profile at another stage to some extent. This may be due to the fact that while a high subsidy reduces the price of the product, increase product attractiveness and rewards the environmental behavior, it also limits the scope for price reductions to some extent, thus making it difficult for merchants to gain a competitive advantage through concessions. Fig.3 demonstrates the minimum subsidy rate and the amount of subsidy required to make manufacturers willing to make green investments. Manufacturers will not engage in investment behavior when the subsidy rate or subsidy amount lies below a straight line, while the opposite is true above. In addition, studies have also shown that centralized decision-making allows the government to induce firms to invest in green R&D through lower subsidies compared to decentralized decision-making. However, in both decision scenarios, increasing the subsidy rate in the second stage (reduced subsidy rate for phase 1) would reduce overall subsidy expenditures, which means that the government can increase the efficacy of subsidies through this way.



(a)



(b)

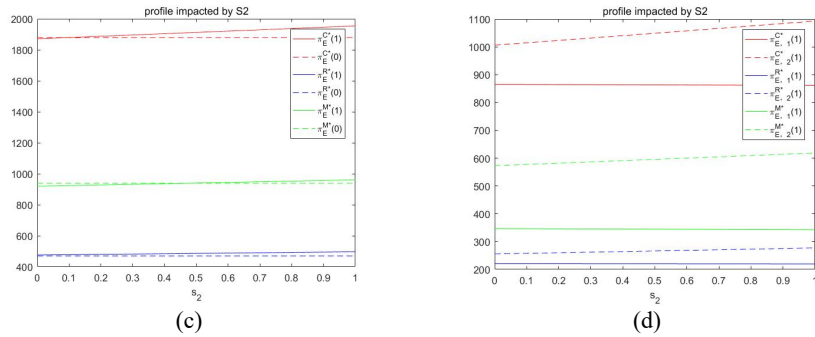


Figure 2. Effect of s_1 and s_2 on optimal solutions

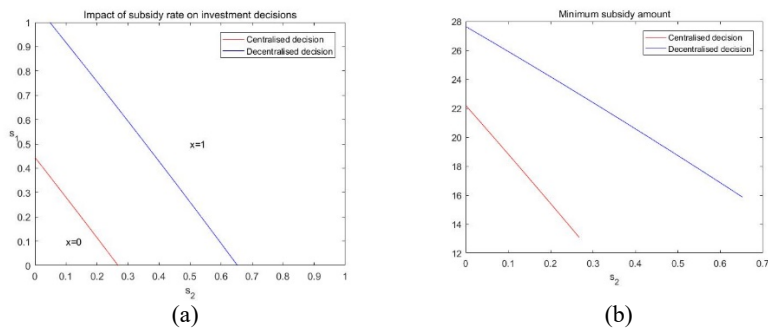


Figure 3. Minimum subsidized amount

4.4 Effect of b on pricing and profile

In this subsection we explore the impact of price sensitivity on supply chain participants. Here we set b varying from 0.5 to 1.5. Fig.4 shows the optimal pricing and maximum profitability Influenced by b .

Studies have shown that as consumer sensitivity to current prices increases, the optimal price of the product and the profitability of the supply chain decreases. In addition, while the optimal price of a product in the first stage is always higher than in the second stage, it is also more likely to be influenced by consumers' price sensitivity. When competition reaches the second stage, it seems that firms don't need to make as many concessions on price, but, compared to phase one, are more likely to lose significant profits due to increased price sensitivity among consumers.

This phenomenon may be due to the fact that when consumers care more about the price of the product, enterprises will have to reduce the initial price of the product for this reason. It also further limits the room for enterprises to reduce the price in the second stage which is planned to stimulate consumption and grab more profits by reducing the price.

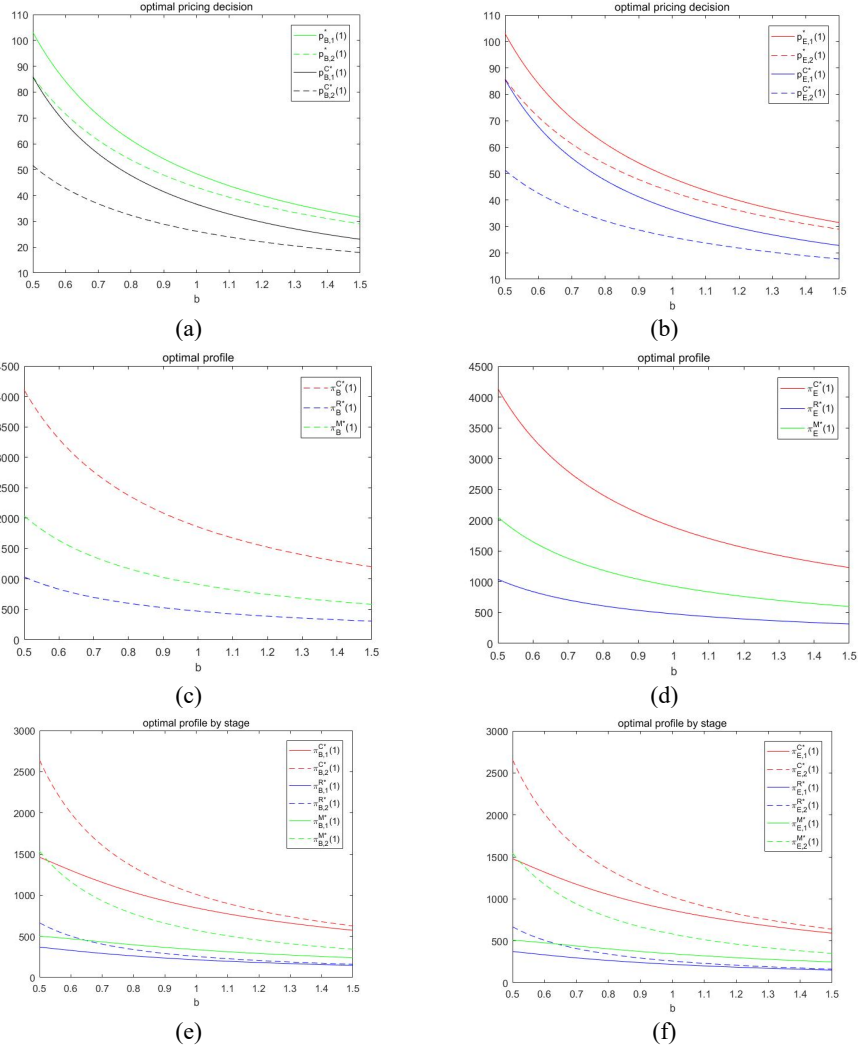


Figure 4. Effect of b on optimal solutions

4.5 Effect of k on optimal solutions

In this subsection we explore how consumer sensitivity to price differentials affects the performance of supply chain participants. For this purpose, we set k varying from 0 to 0.2.

Fig.5 shows that as k rises, the prices will rise in the first phase and fall in the second phase while the profit perform contrarily. Though not obvious, total profits do go up as consumers pay more attention to the price difference. In addition, we can see that retailers and firms with centralized decision-making are less affected by this change,

while manufacturers with decentralized decision-making have to make significant changes.

This phenomenon suggests that merchants can indeed utilize promotional activities to improve profitability. At the same time, merchants should also set reasonable initial prices for price-sensitive products to allow room for price reductions in future promotions.

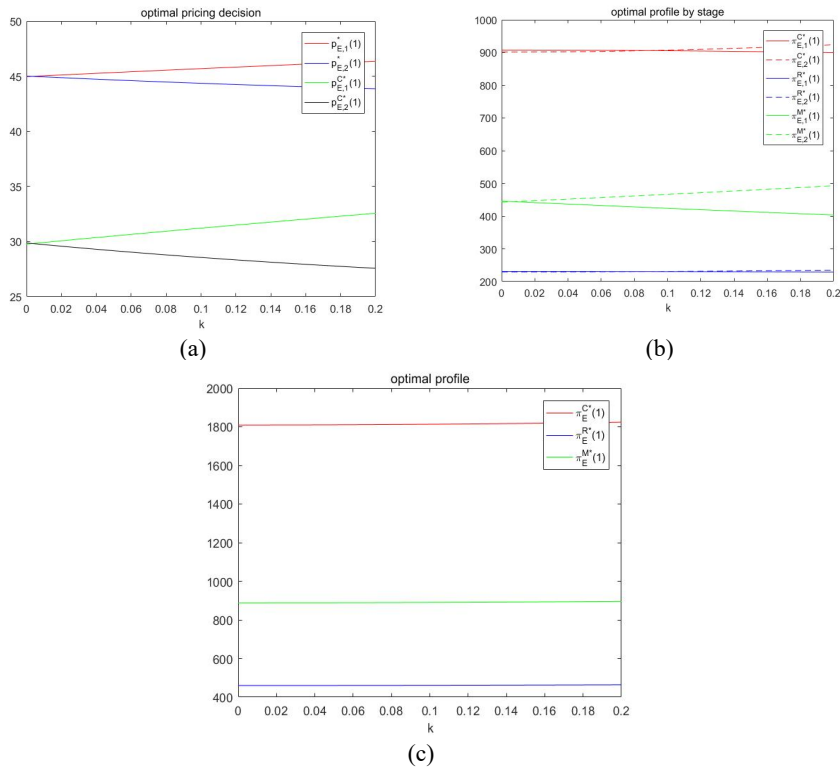


Figure 5. Effect of k on optimal solutions

5 Conclusions

5.1 Conclusions and managerial implications

This paper examines a two-stage pricing strategy and subsidizing issues for a green supply chain under two different decision situations and mainly investigates the investment behavior of manufacturer and the pricing behavior of retailer. Through the preceding analyses, the results are presented as following: Not only does centralized decision-making allow for greater efficiency at lower prices, but it also reduces government efforts to induce manufacturers to produce environmentally friendly goods. Firms also have more incentive to introduce discounts when price reductions can better attract consumers, which allows firms to reserve space for price reductions

by raising the initial price of the product and gain profit by this way. However, when consumers are more interested in the current price of a product, the room for price cuts is therefore limited, which highly constrain the capacity of obtaining benefits for the companies who produce green products.

In the actual production application, enterprises should reasonably determine whether consumers are more concerned about the current price of the product or the strength of the price reduction, in order to choose a reasonable pricing strategy. In addition, for the government, choice a different rate of subsidy at different stage can indeed improve the effect of subsidies to a certain extent.

5.2 Future directions

This paper seeks to add the impacts of subsidy rate changes and price changes on supply chains to traditional green supply chain research. However, a large number of shortcomings remain. Firstly, the time span of the study is limited to two phases and there is a lack of exploration of extending the model to multiple phases. Second, the article only discusses the impact of changes in subsidy rates, and future research could be expanded to include the impact of changes in subsidy strategies.

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