Quantum Computing Simulated Annealing Algorithm Applying in Portfolio Optimization Problem

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Abstract. The quantum annealing algorithm is an optimization algorithm which utilizes the quantum tunnelling effect produced by quantum fluctuations to escape local optima, thus enhancing the chance of finding the global optimal solution. Portfolio optimization problems effectively describe activities such as ordinary stock investments and asset management as optimization problems in portfolio investments. The quantum annealing algorithm leverages the unique properties of quantum mechanics, which could significantly improve the efficiency and effectiveness of solutions. This paper proposes a modelling and solution approach for the portfolio investment problem based on a quantum computing framework. The method utilizes the quantum annealing algorithm to optimize the real stock price trends in financial markets, building on the foundation of modern portfolio theory.

Keywords: Quantum Annealing; Quadratic Unconstrained Binary Optimization; Portfolio Optimization; Modern Portfolio Theory

1 INTRODUCTION

The concept of quantum computing was born with the introduction of the famous quantum mechanical model of Turing machines by Paul Benioff[1] in 1980. Building on this, Richard Feynman[2] in 1985 proposed the idea of constructing computers based on quantum mechanical properties. Following these breakthroughs, Canadian company D-Wave has emerged as a leading force in commercializing quantum computing[3]. Their main focus is developing quantum annealing machines primarily designed for solving combinatorial optimization problems.

Quantum annealing algorithms (QA) leverage the unique properties of quantum mechanics, unlike traditional simulated annealing algorithms (SA). QA utilizes the quantum tunnelling effect, produced by quantum fluctuations, to escape local optima, thus enhancing the chance of finding the global optimal solution. Its application in portfolio optimization and other domains can significantly improve the efficiency and effectiveness of solutions[4].

Portfolio optimization aims to find the most suitable investment portfolio considering risk and return. This context has employed Modern Portfolio Theory[5] (MPT) for portfolio optimization. MPT suggests that by diversifying investments across multiple stocks and various assets in a portfolio, it is possible to achieve a certain level of profitability while mitigating price volatility risks in asset management. Based on the portfolio theory proposed by Harry Markowitz[6] in the

ICPDI 2023, September 01-03, Chongqing, People's Republic of China

DOI 10.4108/eai.1-9-2023.2338767

1950s, this concept emphasizes the importance of diversification in investment strategies. The overall price volatility risk of a portfolio is determined not only by the individual price volatility risks and their respective weights of the component stocks but also by the correlation coefficient that represents the movement relationship between any two stocks.

2 MODELLING AND IMPLEMENTATION

2.1 Introduction of the quantum annealing algorithm

The quantum annealing algorithm generally consists of two parts aimed at finding the global minimum of a given objective function. The first part is the quantum potential energy, which maps the objective function of the optimization to a potential field imposed on a quantum system. This mapping allows the quantum optimization problem to be represented as a quantum system. The second part is the quantum kinetic energy, which introduces a kinetic energy term with controllable amplitude. This term serves as a control over the penetration length of the quantum fluctuations in the system.

The quantum annealing algorithm involves the following steps [7].

STEP 1, the evaluation function $H_q = H_{pot} + H_{kin}$ for the quantum system is constructed based on the optimization problem at hand, representing the quantum Hamiltonian. The quantum potential H_{pot}, serving as the evaluation function in the simulated annealing algorithm, is denoted within this. H_{kin} represents the quantum kinetic energy.

STEP 2, the initialized state is x and its corresponding state energy is $H_{pot}(x)$. T_0 is the initial temperature of quantum annealing, while Γ represents the transverse field strength. The changing transverse field strength induces quantum transitions between different quantum states. The maximum number of iterations is MaxSteps. To start the process, each parameter needs to be initialized.

STEP 3, a random micro disturbance causes a state energy change $H_{pot}(x')$, resulting in a new state x'.

STEP 4, the calculated energy difference

$$\Delta H_{pot} = H_{pot}(x') - H_{pot}(x) \tag{1}$$

and

$$\Delta H_q = H_q(x') - H_q(x) \tag{2}$$

If

$$\Delta H_{pot} < 0 \ or \ \Delta H_q < 0 \tag{3}$$

the system accepts the new solution $x = x'$, and conversely, if

$$\exp(\frac{\Delta H_q}{T}) < random(0,1) \tag{4}$$

then $x = x'$, otherwise, repeat STEP 3.

STEP 5, performing the annealing operation results in a similar change in Γ as the temperature T in simulated annealing, with the transverse field strength changing accordingly.

$$\Gamma = \Gamma - \frac{\Gamma_0}{MaxSteps} \tag{5}$$

STEP 6, if $\Gamma = 0$, terminate the quantum annealing algorithm, otherwise, repeat STEP 3.

In the case of quantum annealing, the system can tunnel through the potential energy barrier to reach the global optimum and achieve optimization of the target system. This is due to the quantum tunnelling effect, which is reflected in the quantum adiabatic theorem. According to this theorem, if the Hamiltonian H_q of the system varies slowly with T, the system will remain in the H_q ground state at the current moment. The quantum annealing algorithm takes advantage of this property by mapping the real problem's optimal solution to an instantaneous ground state[8]. The process begins with a worse initial solution, corresponding to the system's Hamiltonian H_q at the initial moment. As the Hamiltonian slowly changes, the system progresses through the adiabatic evolution process in quantum mechanics, eventually reaching the instantaneous ground state of the Hamiltonian H_q, which corresponds to the optimal solution of the actual problem.

2.2 Solutions to Existing Problems

Constructing an optimal investment portfolio that requires precise allocation of investment ratios becomes extremely challenging for individual investors with limited funds. This difficulty arises due to the volatility of individual stock prices and the need for fine-grained allocation of investment ratios. Consequently, it becomes arduous for individual investors with limited funds to replicate such an optimal investment portfolio in practice due to the significant amount of capital required. To address this challenge, two solutions are proposed. Firstly, the introduction of investment budget constraints involves explicitly setting upper limits on investment amounts, ensuring that individual investors do not exceed their economic capacity when constructing an optimal investment portfolio. Secondly, expressing the composition of the investment portfolio in terms of share quantities instead of monetary values allows for greater flexibility. By converting investment ratios into the number of shares purchased, investors can flexibly construct their investment portfolios and make reasonable adjustments based on their available financial resources.

2.3 Constructing the objective function

The vector P_{i0} represents the purchase price of the ith stock, the vector P_{ij} represents the sale price of the ith stock at moment j, and the vector w_i is the rate of return on the ith stock, which can be negative. These vectors are related as follows.

$$w_i = \frac{P_{ij} - P_{i0}}{P_{i0}} \tag{6}$$

The function that represents the total return of stocks is,

$$V_{return} = \sum_i w_i x_i \tag{7}$$

where x_i is a binary variable that takes on a value of 0 or 1, with a value of 1 indicating the stock is purchased, while the opposite indicates it is not purchased.

The function that represents the risk is,

$$V_{risk} = \sum_{i,j} \text{cov}(i,j) x_i x_j \tag{8}$$

where the variables x_1 and x_2 are binary variables corresponding to the ith and jth stocks respectively.

$\text{cov}(i,j)$ is the covariance between two stocks and is used to measure the relative volatility of stock prices. A positive covariance indicates that the two stocks have similar return trends. The formula that calculates the covariance of the ith and jth stocks is,

$$\text{cov}(A,B) = \frac{1}{n-1} \sum_{i,j} [(A_i - \overline{A})(B_j - \overline{B})] \tag{9}$$

where A_i, B_i are the stock Ath and Bth price at moment i, and n is the number of time periods observed. The returns of stocks for the issue is,

$$V = -L \times V_{return} + V_{risk} \tag{10}$$

where L is the smoothness index, adjusted to maximize returns or minimize risk.

To better simulate the real investment situations of investors, we need to impose restrictions on the invested callable principal[9]. In quantum annealing, it is commonly achieved by introducing a penalty function to enforce constraint implementation during the optimization process. Specifically for QUBO, there is a specific construction format when dealing with constraint formulation[10]. This construction uses mathematical methods to create a polynomial such that it equals zero when the constraints are satisfied and is greater than zero otherwise. This allows the constraint polynomial to exert an overall constraint impact on the objective function in quantum annealing.

$$H = (\sum_i P_{ij} x_i - K)^2 \tag{11}$$

Above all, the objective function for the issue is formula (12),

$$\begin{aligned} &\min \quad -Q = V + \lambda \times H + (\sum_i p_{ij} x_i - K) r \\ &s.t. \quad x_i \in \{0,1\} \end{aligned} \tag{12}$$

where λ is penalty coefficient, K is investment budget and r is risk-free interest rate.

2.4 Results

Python was used to simulate the implementation of the quantum annealing algorithm and construct two portfolio strategies based on investor risk preferences (risk-seeking and risk-averse). The investment budget K was set at 1000, the risk-free interest rate r at 2%, the smoothness index of risk-seeking portfolio L_s at 200, and the smoothness index of risk-averse portfolio L_a at 0.2. Subsequently, the investment returns and asset trends for each type of strategy were compared with the DOW index and stochastic measures as shown in Fig. 1.

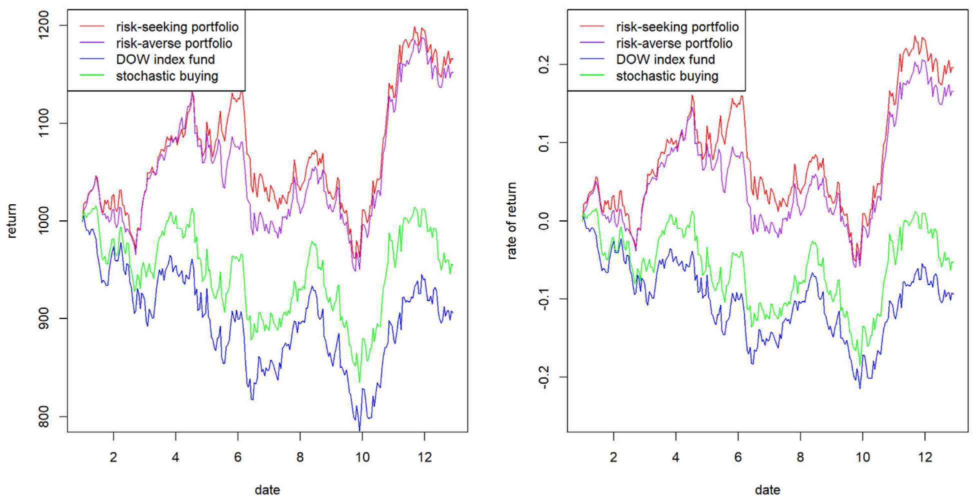

Fig. 1. Time series of portfolio return(left) and rate of return(right)

The portfolio strategies for risk-seeking and risk-averse investors are represented by the red curve and the purple curve, respectively. The blue curve represents an investment strategy for buying the DOW index fund. The green curve illustrates a stochastic buying approach without any specific investment strategy.

3 CONCLUSIONS

In summary, we have developed a model framework to solve the portfolio optimization problem in quantum computing. The optimized data significantly outperformed the original data, and by adjusting the given parameters, it could effectively reflect individual investment preferences. However, determining how to adjust the parameters or provide quantitative parameter strategies remains an unresolved problem. Additionally, it is unclear under what conditions quantum annealing algorithms are consistently superior to simulated annealing algorithms[11]. From this problem, it appears that quantum annealing algorithms excel in both speed and stability compared to simulated annealing algorithms when the data size is sufficiently large. In summary, our model framework improves portfolio optimization in quantum computing by delivering superior results, addressing issues of stability, and exploring the conditions for the superiority of quantum annealing algorithms.

The basic framework of quantum annealing algorithm theory and quantum computing has been developed. As quantum computing technology advances, increasingly sophisticated quantum annealing devices become available, offering new possibilities for solving various forms of optimization problems. However, their genuine potential remains an open question. This paper attempts to provide some insight into the use of quantum computing techniques to solve complex problems through modelling studies of portfolio optimization problems.

Our objective is to explore the feasibility of using web crawling techniques to retrieve news information or trading data related to a specific stock. By adjusting the stock value based on the nature of the data (positive or negative), we aim to provide predictive investment portfolio strategies for investors. The optimization model, although capable of finding the optimal solution for the given data, is not yet sufficient for predicting investment portfolio strategies due to the numerous factors that influence stock prices[12].

REFERENCES

[1] Benioff P. The computer as a physical system: A microscopic quantum mechanical Ha miltonian model of computers as represented by Turing machines[J]. Journal of statistical phy sics, 1980, 22: 563-591.

[2] Feynman R P. Quantum mechanical computers[J]. Optics news, 1985, 11(2): 11-20.

[3] Yarkoni S, Raponi E, Bäck T, et al. Quantum annealing for industry applications: Intr oduction and review[J]. Reports on Progress in Physics, 2022.

[4] Glover F, Kochenberger G, Du Y. Quantum Bridge Analytics I: a tutorial on formulat ing and using QUBO models[J]. 4or, 2019, 17: 335-371.

[5] Elton E J, Gruber M J. Modern portfolio theory, 1950 to date[J]. Journal of banking & finance, 1997, 21(11-12): 1743-1759.

[6] Markowitz H M. The early history of portfolio theory: 1600–1960[J]. Financial analys ts journal, 1999, 55(4): 5-16.

[7] Rajak A, Suzuki S, Dutta A, et al. Quantum annealing: an overview[J]. Philosophical Transactions of the Royal Society A, 2023, 381(2241): 20210417.

[8] Glover F, Kochenberger G, Du Y. A tutorial on formulating and using QUBO models [J]. arXiv preprint arXiv:1811.11538, 2018.

[9] Ayodele M. Penalty weights in qubo formulations: Permutation problems[C]//European Conference on Evolutionary Computation in Combinatorial Optimization (Part of EvoStar). Ch am: Springer International Publishing, 2022: 159-174.

[10] Zaman M, Tanahashi K, Tanaka S. PyQUBO: Python library for mapping combinatori al optimization problems to QUBO form[J]. IEEE Transactions on Computers, 2021, 71(4): 83 8-850.

[11] WANG BaoNan, SHUI HengHua, WANG SuMin, et al. Theories and applications of quantum annealing: A literature survey[J].SCIENTIA SINICA Physica, Mechanica & Astronom ica, 2021,51(08):5-17.

[12] Schenker J. Quantum: Computing Nouveau[M]. Beijing: posts & telecom press, 2021 : 131-134.