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A Two-Phase Hybrid GWO:TP-AB Algorithm for Benchmark and Engineering Optimization Problems

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Abstract

INTRODUCTION: Population-based algorithms are popular stochastic algorithms used for solving optimization problems. Grey Wolf Optimizer (GWO) proposed in 2014 is one of the most studied algorithms in the past decade. Population-based two-phase trigonometric AB (TP-AB) is a recently proposed algorithm for handling optimization problems.
OBJECTIVES: The objective of this work is to propose one new hybrid algorithm combining the strengths of two better performing algorithms in two different phases. The performance is analysed using popular benchmarks and the results are compared with a few popular algorithms available in the literature.
METHODS: One new two-phase hybrid algorithm is designed by taking GWO in its first phase and the second phase of the TP-AB algorithm in the second phase. In the second phase, the Levy Strategy is introduced which was not in the original TP-AB algorithm.
RESULTS: The performance of the new hybrid GWO:TP-AB algorithm is analysed using 23 classic mathematical functions, 10 numbers of the CEC2019 dataset and 18 real-world engineering problems In addition, to demonstrate its capability to handle higher dimension problems, 13 scalable problems are solved. These include unimodal and multimodal instances with dimensions 30, 100, 500 and 1000.
CONCLUSION: The results demonstrate the better performance of the GWO:TP-AB algorithm when compared to several optimization algorithms of recent times.

Keywords: Metaheuristic, Hybrid Algorithm, Grey Wolf Optimizer, Two-Phase AB (TP-AB) Algorithm, Real-World Applications, Industrial Optimization

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1. Introduction

Industrial revolutions increase the focus on industrial digital transformation (IDT) for its industrial value creation potential (Abiodun et al., 2023). In today's digital era, understanding the critical factors that drive industrial digital transformation is essential for any organization to evolve in the technological landscape (Tangwaragorn, 2024). Industrial optimization aims to balance the resources and their usage to achieve the set objectives.

Operations Research (OR) finds its place in almost all elements of manufacturing, marketing, sales and service activities to improve the profit and reduction in wastages. Designing optimal layouts, efficient scheduling and proper design of machines and machine elements are a few areas of focus in any industry.

Metaheuristic algorithms are popular nowadays in solving complex optimization problems in varying domains of Operations Research due to their versatility and flexibility (Tomar et al., 2024). We could find the applications of such optimization algorithms in solving constrained, unconstrained, single-objective, multi-objective, linear, and non-linear problems with continuous and discrete search spaces. They are more popular due to their ability to combine exploration and exploitation capabilities to reach optimal/near-optimal solutions.

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Exact methods could be effective for smaller problems only. When the problem dimensions increase the computing time increases exponentially. As a result, metaheuristics are widely used by researchers for larger problem sizes to obtain results with acceptable accuracy. The genetic algorithm proposed by Holland (1992) is an evolutionary heuristic algorithm which is population-based and stochastic. Since then, numerous heuristics have been proposed by researchers over the years which could be classified based on:

Nature inspired Evolutionary Algorithms (Example: Genetic Algorithms)

Collective behaviour of swarms inspired Swarm-Based Algorithms (Example: Particle Swarm Optimization of Kennedy and Eberhart, 1995)

Human Behaviour inspired Human-Based Algorithms (Example: Teaching-Learning Based Optimization of Rao et al., 2011)

Science-inspired Science-Based Algorithms (Example: Harmony Search Algorithm of Kim, 2016).

Maths inspired Maths-Based Algorithms (Example: Trigonometric Sine (AB) and T-Cos algorithms of Baskar; 2022, 2024)

Another way of classifying such algorithms is population-based or trajectory-based.

In population-based algorithms, a set (population) of approximate solutions is generated within the bounds and these solutions are iteratively moved towards the optimal/near-optimal solutions. Evolutionary and swarm-based algorithms fall under this population-based category. On the other hand, trajectory-based algorithms explore the search space by following a single solution path (trajectory) at a time and iteratively refine the single solution. Simulated Annealing (Kirkpatrick, 1983) is a good example of a trajectory-based algorithm.

Based on the number of objectives that need to be optimized, algorithms may also be classified as single-objective and multi/ many-objective optimization algorithms.

Another popular category is hybrid algorithms in which two or more strategies are extracted from better-performing algorithms available in the literature to improve the performance of the new hybrid algorithm.

This paper is one such hybrid algorithm that combines the Grey Wolf Optimizer (GWO) (Mirjalili et al., 2014) and TP-AB trigonometric algorithm (Baskar et al., 2024) whose performances were demonstrated using a set of well-known benchmark problem sets.

The rest of this paper is organized as follows: The newly proposed hybrid algorithm is detailed in Section 2 and Section 3 explains briefly the benchmarks used for analysing the performance of the proposed GWO: TP-AB hybrid algorithm. A comprehensive analysis is performed and some comparisons are performed in Section 4. Finally, in the conclusion section 5, we conclude with a summary, of the main advantages and future work involving the proposed hybrid algorithm.

2. Hybrid GWO: TP-AB Algorithm

The objective function of any optimization is usually represented in terms of a mathematical equation which is evaluated for the set of variables that result in minimization or maximization of the objective function. This process of optimization in practical cases is subjected to constraints that are equality or inequality or mixed in nature. The single-objective minimization function could be defined as presented in equation (1) and two types of constraints in equations (2) and (3).

Minimize $\boldsymbol{f(x_i)}$, $\boldsymbol{x_i \in X_i}$ ($\boldsymbol{i}$ = 1 to I, $\boldsymbol{X_i}$ = bounds in '$\boldsymbol{I}$' dimensions) (1)

$\boldsymbol{g_j(x_i) \leq 0}$ (inequality constraints, $\boldsymbol{j}$ = 1 to $\boldsymbol{J}$) (2)

$\boldsymbol{h_k(x_i) = 0}$ (equality constraints, $\boldsymbol{k}$ = 1 to $\boldsymbol{K}$). (3)

The objective function is negated for a maximization problem.

This mathematical expression is evaluated for different input variables (randomly generated initially within the bounds) and the best ones are retained and used for generating the next set of approximate solutions during the next iteration. Different updating expressions are used for generating the new approximate solutions by different algorithms. The quality of newly generated approximate solutions is a function of these updating expressions.

Grey Wolf Optimizer (GWO) is a nature-inspired metaheuristic algorithm proposed by Mirjalili et al. in 2014. It is one of the most studied algorithms in the last decade, and several improvements including hybrid variants have been proposed. Şenel et al. (2019) proposed a hybrid algorithm fusing the exploration ability of the grey wolf optimizer (GWO) with the exploitation ability of the particle swarm optimization (PSO). Vo et al. (2024) combined GWO and Cuckoo Search Algorithm (CS) to solve one multi-objective spatial truss design problem.

TP-AB is a two-phase population-based trigonometric algorithm recently proposed by Baskar et al. (2024). This work proposes one two-phase hybrid version by combining GWO (with a linear reduction from 2 to 0) in its first phase and the 'second phase of TP-AB' (without tuning) in the second phase. The Levy Flight Strategy (LFT) is used in the second phase for more randomization.

LFT refers to a random walk in which the step sizes are drawn from a Lévy distribution. Compared to normal distribution, Levy distribution has a higher probability of generating larger steps. The mixture of larger and shorter steps permits to explore the search space and avoids local optima. Sharp peaks and asymmetry are important features of LFT's probability density distribution (PDF).

The shape parameter 'β' is taken between 0 and 2. In this work, the simulations are carried out by taking 1.5 as the shape parameter.

The Levy distribution could be approximated as given in equation (4).

$\boldsymbol{L(s) \sim s^{(-1-\beta)}}$ (Chawla and Duhan, 2018) where '$\boldsymbol{s}$' is the step size. (4)

The updating expression in the second phase of the TP-AB algorithm without tuning is given in equation (5).

$$X = X + Sine(2*pi*rand)*Step \quad (5)$$

Where "***Step***" is the difference between two adjacent approximate solutions.

In the proposed hybrid algorithm, the Levy function "***RL***" replaces the "***rand***" function keeping other things the same. That is, the updating expression used here is as given in equation (6).

$$X = X + Sine(2*pi*RL)*Step. \quad (6)$$

Figure 1 shows the flowchart of the proposed hybrid algorithm.

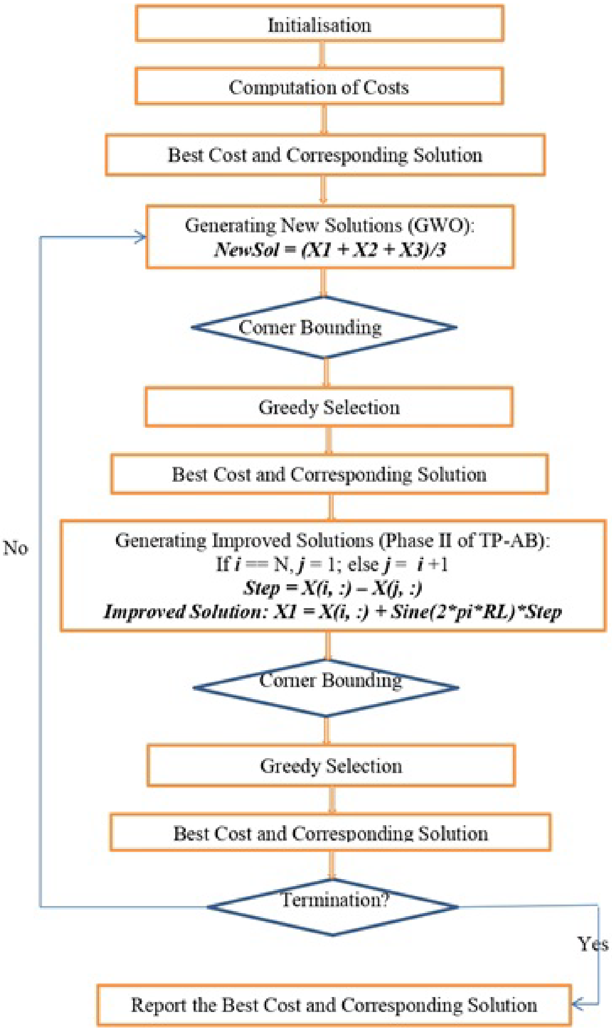

Figure 1. Flowchart of GWO:TP-AB Algorithm

"RL" in the flowchart refers to the Levy Function with a beta value of 1.5. Studies have shown that a beta value of 1.5 can provide a good balance between exploration and exploitation in various contexts (Cui et al., 2022). It allows for the generation of both small and large steps in the search space, increasing the chances of finding good solutions.

The choice of $\beta = 1.5$ in Lévy flight is a compromise between exploring a wide search space and exploiting promising regions, making it a popular and effective choice for various optimization problems.

The new algorithm is coded in MATLAB (online version) and run on a desktop i5 PC with 8 GB RAM.

3. Benchmarks Used

For assessing the performance of optimization algorithms, several benchmark datasets are available in the literature with their optimal values, number of variables, constraints and bounds. The first dataset of 23 classic functions (Mirjalili and Lewis, 2016) has three different sub-sets; the first seven problems are unimodal (F1-F7), the next six problems (F8-F13) are multimodal and the remaining problems (F14-F23) are fixed dimensional multimodal instances that have dissimilar search spaces. The second sub-set (F8-F13) has multiple local optima which increase exponentially with increasing dimensions.

The second dataset considered is 10 composite functions of CEC2019 (Table 1), popularly identified as the "100-digit challenge". Complete details could be seen in the technical report published by Price et al. (2018). The salient feature of this dataset is that the optimal value is "1" for all ten problems.

Table 1. CEC2019, 100 Digit Challenge Test Functions [Price et al., 2018]

S. No.	Function	Dimension	Search Span	Optimal
1	Storn's Chebyshev Polynomial Fitting Problem	9	[−8192, 8192]	1
2	Inverse Hilbert Matrix Problem	16	[−16384, 16384]	1
3	Lennard-Jones Minimum Energy Cluster Problem	18	[−4, 4]	1
4	Rastrigin's Shifted and Rotated Function	10	[−100, 100]	1
5	Griewangk's Shifted and Rotated Function	10	[−100, 100]	1
6	Weierstrass Shifted and Rotated Function	10	[−100, 100]	1
7	Schwefel's Shifted and Rotated Function	10	[−100, 100]	1
8	Schaffer's Shifted and Rotated F6 Function	10	[−100, 100]	1
9	Happy Cat Shifted and Rotated Function	10	[−100, 100]	1
10	Ackley Shifted and Rotated Function	10	[−100, 100]	1

The third set of problems is 18 numbers of real-world constrained engineering problems. Finally, 13 scalable problems (Mirjalili and Lewis, 2016) with dimensions 30, 100, 500, 1000 and Cobb-Douglas Production Function are analysed. The number of variables, constraints and optimal costs are presented in the results and discussions section.

4. Results and Discussion

The performance is analysed using the above-described 23 classic benchmark functions, the tough CEC2019 test suite and, 18 constrained real-world engineering problems.

Arora et. al. (2019) proposed one hybrid algorithm by combining Grey Wolf Optimization and the Crow Search Algorithm (GWOCSA) and compared the performance with 10 other algorithms; Bat Algorithm (BA), Biogeography-based optimization (BBO), Crow Search Algorithm (CSA), Dragonfly Algorithm (DA), Genetic Algorithm (GA), Grey Wolf Optimizer (GWO), Particle Swarm Optimization (PSO), School Based Optimization (SBO), Enhanced Grey Wolf Optimizer (EGWO) and Accelerated Grey Wolf Optimization (AGWO). The authors used a population size (PS) of 30 for all benchmarks. The GWO:TP-AB is run for the same number of function evaluations (NFE) and the "mean" results are compared (Table 2 to Table 4). The PS is taken as 5 and 30 in separate simulations and the number of iterations varies to keep the NFE the same

Table 2. Classical 23 Functions: F1-F7 Results (NFE: 9000; Trials: 30)

F.No.	PS	GWO:TP-AB				Arora et. al.	
		Minimum	Mean	Maximum	STD	Best Mean	Algorithm
F1	5	2.6983e-47	***4.3867e-43***	1.1016e-41	2.0028e-42		
	30	2.9548e-24	1.7420e-22	2.2166e-21	4.2034e-22	1.01e-28	GWOCSA
F2	5	5.1319e-31	***6.5977e-29***	4.3354e-28	1.0716e-28		
	30	1.0425e-14	1.1618e-13	3.7946e-13	8.9822e-14	1.50e-17	GWOCSA
F3	5	4.2321e-03	1.8745	18.949	3.9349		
	30	0.1206	7.2749	52.776	11.122	*5.18e-04*	GWOCSA
F4	5	1.7251e-05	1.4758e-03	0.010886	2.5913e-03		
	30	1.0515e-03	0.022094	0.1184	0.025889	*2.07e-07*	GWOCSA
F5	5	25.133	***25.714***	28.717	0.7431		
	30	25.590	**26.147**	28.727	0.5528	27.0	GWOCSA
F6	5	8.1793e-05	0.048195	0.4988	0.1163	1.23	GWOCSA
	30	2.3421e-04	0.2890	0.9180	0.2286	*1.20e-03*	PSO
F7	5	1.4706e-03	6.3952e-03	0.014130	2.7275e-03		
	30	3.9778e-03	9.8687e-03	0.019780	4.0070e-03	*1.92e-03*	GWOCSA

Table 3. Classical 23 Functions: F8-F13 Results (NFE: 9000; Trials: 30)

F.No.	PS	GWO:TP-AB				Arora et. al.	
		Minimum	Mean	Maximum	Standard Deviation	Best Mean	Algorithm
F8	5	-9994.5	-8014.7	-5294.5	1214.8	-3.57e03	GWOCSA
	30	-8793.4	-6268.3	-5126.6	824.08	***-2.84e+71***	BA
F9	5	12.183	49.651	135.86	29.854		
	30	17.083	48.145	131.62	26.222	***1.19***	GWOCSA
F10	5	7.5495e-15	1.4181e-14	2.1760e-14	3.1959e-15		
	30	4.4453e-13	3.0334e-12	1.3938e-11	2.7647e-12	***1.37e-14***	GWOCSA
F11	5	0.0	0.012906	0.049090	0.013932		
	30	0.0	4.3439e-03	0.018402	6.0436e-03	***0.0***	GWOCSA AGWO
F12	5	4.1374e-06	***7.2490e-03***	0.2075	0.037868		
	30	2.4607e-05	0.026940	0.2142	0.045540	4.92e-02	GWOCSA
F13	5	1.4298e-04	0.1679	0.6053	0.1508	0.939	GWOCSA
	30	6.6747e-04	0.2722	0.5350	0.1383	**5.68e-02**	SBO

Table 4. Classical 23 Functions: F14-F23 Results (NFE: 9000; Trials: 30)

F.No.	PS	GWO:TP-AB				Arora et. al.	
		Minimum	Mean	Maximum	Standard Deviation	Best Mean	Algorithm
F14	5	0.9980	3.6044	18.304	4.2355		
	30	0.9980	***0.9980***	0.9980	1.8281e-13	***9.98e-01***	GWOCSA
F15	5	3.0753e-04	5.1487e-03	0.020363	8.5396e-03		
	30	3.7232e-04	1.9196e-03	0.020363	5.0142e-03	***3.38e-04***	GWOCSA
F16	5	-1.0316	***-1.0316***	-1.0316	6.3477e-16		
	30	-1.0316	***-1.0316***	-1.0316	1.2596e-13	***-1.03***	All
F17	5	0.3979	***0.3979***	0.3979	4.5608e-14		
	30	0.3979	***0.3979***	0.3979	9.3996e-15	***3.98e-01***	All except GA
F18	5	3.0000	8.4000	84.000	20.550		
	30	3.0000	***3.0000***	3.0000	1.0615e-14	***3.0000***	CSA, DA, GWO, PSO, EGWO, AGWO, GWOCSA
F19	5	-3.8628	***-3.8628***	-3.8628	2.2809e-15		
	30	-3.8628	***-3.8628***	-3.8628	5.0154e-12	***-3.86***	All except GA
F20	5	-3.3220	-3.2719	-3.1376	0.062771		
	30	-3.3220	-3.2824	-3.2031	0.057005	***-3.31***	GWOCSA
F21	5	-10.153	-7.9446	-2.6305	3.2742	-6.80	GWOCSA
	30	-10.153	***-9.2054***	-2.6305	2.1916	-9.14	GWO
F22	5	-10.403	-9.2624	-1.8376	2.6578	-8.76	GWOCSA
	30	-10.403	***-10.403***	-10.403	3.3586e-05	***-10.4***	GWO
F23	5	-10.536	-9.0955	-2.4217	2.9890	-8.82	GWOCSA
	30	-10.536	***-10.536***	-10.536	3.9977e-05	-9.72	GWO

When all 23 functions are considered, GWOCSA accounts for 13/23 best "mean" results, GWO:TP-AB is closely behind at 12/23 for best results, and GWO reports the best results in 7/23 cases.

That is, GWO:TP-AB performs reasonably well among the considered 12 algorithms especially better than the original GWO and at par with the hybrid GWOCSA for this dataset.

Figure 2 shows the convergent curve for the unconstrained “sphere” function for 20 trials (dimension: 30) with a broader search range of [-100, 100].

The convergence is steep during earlier iterations and remains almost flat after 40 iterations. The curve shows the consistency of convergence also for this function.

Lei et. al. (2023) proposed one Enhancing Grey Wolf Optimizer with Levy Flight (LFGWO) and analysed the performance with eight other algorithms; AHA, AO, DA, DMOA, GBO, HGS, HHO, and MVO. It was concluded that LFGWO reported the best "mean" results in 9 of the 10 problems of the CEC2019 dataset. However, LFGWO reported values below “1” in 7 of the 10 cases whereas; the optimal values are “1” in all cases. Hence, the results of LFGWO are discarded and the second-best "mean" results

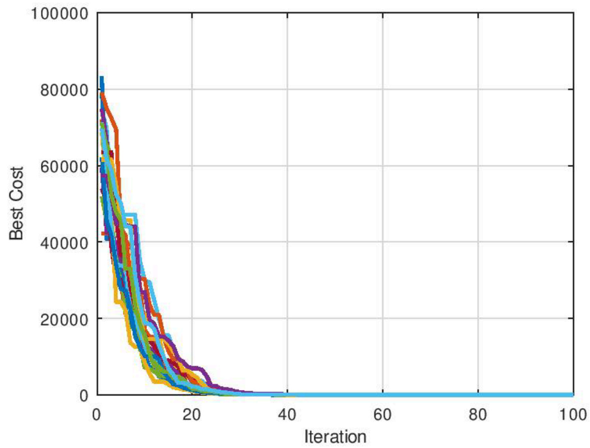

Figure 2. Convergent Curve for the Un-constrained Sphere Function

are considered. GWO:TP-AB is run again for the same NFE and results are compared (Table 5).

For the tough CEC2019 dataset, GWO:TP-AB reports better "mean" values in 5/10 cases and HGS (Hunger Games Search) in 4/10 cases.

One hybrid algorithm of Grey Wolf Optimizer and Harris Hawks Optimization (HGWO) was proposed by Tu et. al. (2023) and the performance was analysed in a few datasets including four real-world engineering problems.

The results obtained after 15000 NFE were compared with seven popular optimization algorithms; GWO, WOA, GJO, MVO, SSA, SOA, and AOA and three hybrid algorithms; AGWO_CS, PSOGWO and AGWO. GWO:TP-AB is simulated for the same NFE (Table 6, Table 7) and results are compared. The results show that in all four cases, GWO:TP-AB is ahead of all other algorithms when the population size is 5 and better in 2 of the 4 cases if the population size is increased to 30 keeping NPE constant. The penalty approach is applied with a penalty parameter of 10^9.

Finally, the GWO:TP-AB algorithm is tested against a few more constrained real-world engineering other than those listed in Table 6. The results are presented in Table 8, the summary of which in Table 9. In this simulation also, NFE is taken as the same 15000 and the best costs reported in 20 trials are taken. The first 11 problems are taken from the paper of Bayzidi et. al., 2021, Himmelblau's Function (Himmelblau, 1972), discrete spring (Deb and Goyal, 1997) and, continuous stepped cantilever beam (MathWorks Help Center) are the other three problems analysed. The penalty approach is applied with a penalty parameter of 10^9 here also.

Table 5. CEC2019: Results (NFE: 10000; Trials: 30)

F.No.	PS	GWO:TP-AB Minimum	Mean	Maximum	Standard Deviation	Lei et. al. 2nd Best Mean	Algorithm
F1	5	8.0475e+06	***1.0401e+09***	5.9214e+09	1.3994e+09		
	100	4.6057e+07	***1.2903e+09***	3.2562e+09	9.0679e+08	9.95e+09	HHO
F2	5	18.3429	***18.3429***	18.3429	1.0565e-13		
	100	18.3434	***18.3445***	18.3459	6.6920e-04	64.4	AHA
F3	5	13.7024	13.7024	13.7024	4.2034e-07		
	100	13.7024	13.7024	13.7024	1.1931e-07	***12.7***	AHA
F4	5	17.9143	***50.8669***	107.9310	25.0574		
	100	22.2618	***49.1965***	65.6646	11.0490	646	HGS
F5	5	2.0129	2.2382	2.6805	0.1690		
	100	2.0754	2.5548	2.7451	0.1531	***1.43(I)*** ***1.63 (II)***	GBO HGS
F6	5	10.1574	12.0530	13.2727	0.8014		
	100	9.7858	12.0989	13.2506	0.8323	***7.41***	HGS
F7	5	70.8095	***475.9432***	1.1057e+03	282.8184		
	100	206.2003	638.0031	996.8844	196.0447	608	HGS
F8	5	2.1125	5.3242	6.8696	1.0878		
	100	2.3297	4.7987	6.5713	1.0689	***4.59***	HGS
F9	5	3.3918	***3.6207***	4.0527	0.1822		
	100	3.4218	***3.7690***	4.0961	0.1744	64.2	GBO
F10	5	1.0000	20.7783	21.6454	3.7372		
	100	3.7712	20.9394	21.7370	3.2436	***20.2***	HGS

Table 6. Constrained Engineering Problems [Tu et. al., 2023]

Problem	No. of Variables	No. of Constraints	Optimal Value	GWO:TP-AB	APD	HGWO (Tu et. al.)
Pressure Vessel	4	4	5885.332773	***5885.3327736***	1.01948E-08	5885.334276
Speed Reducer	7	11	2994.424466	***2994.4244658***	0	2994.613691
Three-Bar Truss	2	3	263.8958434	***263.89584338***	0	263.8958539
Welded Beam	4	7	1.724852308597	***1.724852317362***	5.08159E-07	1.725201706

Table 7. Four Real-World Engineering Problems: Results (NFE: 15000; Trials: 20)

		GWO:TP-AB			
Problem	PS	Minimum	Mean	Maximum	Standard Deviation
Pressure Vessel	5	***5885.3327736164592351997271180153***	6330.3735844438333515427075326443	7319.0007020502443992882035672665	566.80301810640924031758913770318
	30	5885.8082213758616489940322935581	6063.7418269105755825876258313656	6901.1766607911768005578778684139	298.20401896057711610410478897393
Speed Reducer	5	***2994.4244657567364811256993561983***	2994.4245234660684218397364020348	2994.4256119880406004085671156645	0.0002562154783499746607718094981 7205
	30	***2994.4428549646836472675204277039***	2994.5214090403569571208208799362	2994.6129967797955941932741552591	0.054145148091894391984002510298524
Three Bar Truss	5	***263.89584337652343037916580215096***	263.89697982965100209185038693249	263.90784471260030841222032904625	0.002627642957340218729522796081 7101
	30	***263.89585118651882567064603790641***	263.89696991620769495057174935937	263.90683749351478581957053393126	0.0024504290524198489055474059483 686
Welded Beam	5	***1.7248523173623659943132224725559***	1.7259957511342129965470348906820	1.7399454875679394305620917293709	0.0033564521984124533540705392908 876
	30	1.7252744030046709422521189480904	1.7290330812261551773900691841845	1.7357306458796928350096777649014	0.0027866841355704730261166890414 870

Table 8. GWO:TP-AB; Real-World Engineering Problems: Results (PS: 5; NFE: 15000; Trials: 20)

Problem	Minimum	Mean	Maximum	Standard Deviation
Gear Train	2.7008571488865133510618295276270e-12	3.8856002658424227023371842362580e-10	0.000000001361649139063991413866632958431	4.9339316589255334254863205465302e-10
X = [49.47794032673360931084971525706 4, 19.176286014264217527625078218989, 16.478744814526081086114572826773, 43.347494958533580700077436631545]				
Himmelblau	-30665.538671745929605094715952873	-30665.090257237839978188276290894	-30662.869794851903861854225397110	0.8300349063934077120663346249784 8
X = [78.0, 33.000000000131585409235412953421, 29.995256025882124362169633968733, 44.99999999989768184605054557323, 36.775812905371800809462001780048]				
Spring (Cont.)	0.01266528886308712111008478728990 7	0.01289306938628879094232981827872 2	0.01369830907051986304545643235997	0.00028557353393006412733967636086163
X = [0.051740250298048857946486833725 43, 0.35795038007560764370396100275684, 11.217070935744811066570036928169]				
Spring (Disc.)	2.6585591659695992916567774955183	2.6820824521916977190016950771678	2.800157076098202413305671143462	0.03432821363519733548486101426533 4
X = [0.25859072291166024237796250417887, 0.85302727237647013591725908554508, 1.2230410099638071574901232452248]				
Corrugated Bulk Head	6.8429580100808378162469125527423	6.8435522519508662497855766559951	6.8500078169689917473306195461191	0.0016631368313005936380305493926812
X = [57.692307692307692307692307692308, 34.147620348674351475892763119191, 57.692307692307692307692307692308, 1.05]				
I-Beam Deflection	0.01307411890522333468289684077490 2	0.01307411901222258442112345022678 7	0.01307411985370254305094395164132 9	0.0000000002543858803023858375654 7308798888
X = [80.0, 50.0, 0.9, 2.3217922606924643584521384928717]				
Piston Lever	8.4126983231064453860881258151494	79.989713713725706156765227206051	167.47275226290031469034147448838	81.187070423920388861915853340179
X = [0.050000000000003909372825461332468, 2.0415135899181180434425186831504, 4.0830271798362192114950630639214, 120.0]				
Car Side Impact	22.842969461066683578565061907284	22.943048739212301256884529720992	23.838793553863858676322706742212	0.2316743907627490661749192213392 2
X = [0.50000000007658884637606888645678, 1.1163457460717085556467509377399, 0.5, 1.3022301986541147122977690742118, 0.50000000000161870516990347823 59, 1.5, 0.50000008710249288501614728374989, 0.64531670307963129307182725824532, 0.76652592030216937235564955699374, -19.565078840901410472952193231322, -0.000771331081181118047600436648636446]				
Cantilever Beam	1.3399585868683236977716433102614	1.3400844935600217944227097177645	1.3404967537678189604122280798038	0.0001358521801842391379510494031 6349
X = [6.0184156597786753195578057784587, 5.3081637374453833899679011665285, 4.4881755042395390375986607978 12, 3.5034094977655114711012629413744, 2.1555309031478775416701409994857]				
Tubular Column	26.486361472447814691122403019108	26.486361491141117596725962357596	26.486361845942873571857489878312	0.00000008351167623202757931930603 8498072

X = [5.4521807362239060879005592141766, 0.29162642929940890690332366830262]				
Stepped Cantilever (Cont.)	63182.591890177442110143601894379	64635.926705121681152377277612686	67554.012411140152835287153720856	1148.5021549520163262059213593602
X = [3.0207850831358924459379977633944, 60.379446489702502276486484333873, 2.8362681880493596509040798991919, 56.715153988680000907152134459466, 2.5404275476987918658267062710365, 50.805714405453464621587045257911, 2.2156790986227705708699886599788, 44.293090732297372369430377148092, 1.7565262685745965942629709388711, 34.935821838849271614435565425083]				
Stepped Cantilever (Disc.) P=10^3	63940.586042174501926638185977936	64567.88959520826028892770409584	66332.014895901331328786909580231	717.98820111639406604808755218983
X = [0.51644851041369410626913349915412, 1.0, 0.99964915861882264191962121913093, 0.67043182206102891473875615702127, 0.36468171849495778502614484750666, 0.5682596817710487968611232645344, 2.2045556994630848279825841018464, 44.091113756159664660572161665186, 1.7497572849831612984417006373405, 34.995137571156725186938274418935]				
P=10^9	64334.688137216333416290581226349	65285.229253080658963881433010101	68830.705043758425745181739330292	1285.0265043029960452258819714189
X = [0.41141878796092445913501478571561, 0.94554554531470202949350323251565, 1.0, 0.51079837050743448489953379976214, 0.39966761790870086734628330304986, 0.59517948631771078193963830926805, 2.2250047407244641917145600018557, 44.500089954629025612575787818059, 1.7497724998207522251192358453409, 34.995389553054110365337692201138]				
Pressure Vessel	6059.7143400267977995099499821663	6601.4943750160846320795826613903	7544.4925179250849396339617669582	506.42467853889866091776639223099
X = [12.968038549028149120090347423684, 6.6985812743516381928543523827102, 42.098445554985254091207025339827, 176.63659634892306371511949691921]				
RCC Beam	359.20799999999996998667484149337	360.13980000493563693453324958682	362.63400000000001455191522836685	1.4625369378701769118578113193507
X = [0.26595640638212081352520499422099, 0.48467334436690662213820246506657, 8.5]				

The convergent curve for the constrained "car side impact" problem is presented in Figure 3 for 20 trials. The lower bound is [0.50, 0.50, 0.50, 0.50, 0.50, 0.50, 0.50, 0, 0, -30, -30] and the upper bound is [1.50, 1.50, 1.50, 1.50, 1.50, 1.50, 1.50, 1, 1, +30, +30].

It is one of the toughest problems with 11 variables and 10 constraints. Unlike the sphere function, the variations in cost are slightly higher during earlier stages of iteration. However, the curves remain flat after 60 iterations. Another observation is that the curves are not steep but moderately curved while converging.

18 problems considered (Table 7 and Table 9) have a mixture of continuous or discrete or both discrete and continuous variables. APD in Table 9 refers to Average Per cent Deviation from the optimal value. Most of the variables are continuous and, the following benchmarks have discrete or discrete and continuous variables:

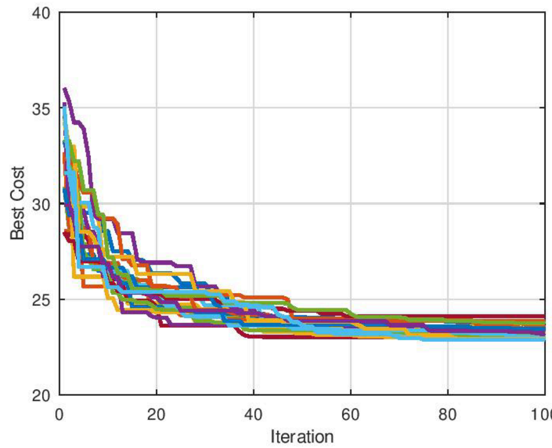

Figure 3. Convergent Curve for the Constrained Car Side Impact Function

Table 9, Constrained Engineering Problems (PS: 5; NFE: 15000; Trials: 20)

S.No.	Problem	Dimensions	Constraints	Optimal Value	GWO:TP-AB	APD
1.	Cantilever Beam	5	1	1.3399576	1.339958586868	7.36492E-05
2.	I-shaped Beam	4	2	0.0130741	**0.013074118905**	0
3.	Tubular Column	2	6	26.486361473	**26.486361472448**	-2.08409E-09
4.	Piston Lever	4	4	8.41269832311	**8.412698323106**	0
5.	Corrugated Bulkhead	4	6	6.8429580100808	**6.842958010081**	0
6.	Pressure Vessel (different version)	4	4	6059.714335048436	6059.714340026798	8.2155E-08
7.	Tension/Compression Spring (Continuous)	3	4	0.01266051	0.012665288863	0.037746212
8.	Gear Train	4	0	2.70085714e-12	**2.70085714e-12**	0
9.	Reinforced Concrete Beam	3	2	359.2080	**359.2080**	0
10.	Car Side Impact	11	10	22.84296954	**22.842969461067**	-3.45546E-07
11.	Cantilever Stepped Beam (Discrete)	10	11	63893.43079587 (violating the sixth constraint) ---	63893.43130309 (violating the sixth constraint) 64334.68813722 (satisfying all constraints)	7.93853E-07
12.	Himmelblau's Function	5	6	−30665.5398	-30665.538671745930	-3.67922E-06
13.	Tension/Compression Spring (Discrete)	3	8	2.65855916	2.658559165970	2.24558E-07
14.	Cantilever Stepped Beam (Continuous)	10	11	63408.9	**63182.591890177442**	-0.356902753

Pressure Vessel: The first two are discrete variables
Gear Train: All four are discrete variables
Reinforced Cement Concrete Beam: The first two are discrete variables
Tension/ Compression Spring (Discrete): The first two are discrete variables
Cantilever Stepped Beam (Discrete): The first six are discrete variables.

In 11 out of 18 instances, GWO:TP-AB reports the optimal values (marked in bold) whereas in the remaining 7 cases the results are very close to the optimal values.

In the case of the stepped cantilever beam (discrete) problem, we get a higher cost of 64334.68813722. The variables' set is,
X = [3.0, 60.0, 3.1000000000000000888178419700125, 55.0, 2.6000000000000000888178419700125, 51.0, 2.2250047407244641917145600018557, 44.500089954629025612575787818059, 1.7497724998207522251192358453409, 34.995389553054110365337692201138] and the constraints' set is,
G = [-3.2339e-01, -3.8246e+02, -6.9151e+02, -1.2034e+03, -1.1111e+02, -2.3665e-07, -3.4544e-05, -2.1842e-06, -3.8462e-01, -2.2581e+00, 0].

Here, though the reported cost is high, all constraints are satisfied.

However, when the penalty parameter is reduced to 10^3 from 10^9, the new cost obtained is 63893.43130309. Corresponding variables and constraint sets are:
X = [3.0, 60.0, 3.1000000000000000888178419700125, 55.0, 2.6000000000000000888178419700125, 50.0, 2.2045556994630848279825841018464, 44.091113756159664660572161665186, 1.7497572849831612984417006373405, 34.995137571156725186938274418935];
G = [-5.0285e-05, -2.8800e-06, -1.5385e+02, -1.2034e+03, -1.1111e+02, 4.7155e-02, -4.6455e-06, -1.0574e-07, -7.6923e-01, -2.2581e+00, 0].

Here though the cost is less, the sixth constraint is violated.

SNS algorithm (Bayzidi et. al., 2021) reports a cost of 63893.4307958715 (NFE: 20000) which is close to the value obtained above for 15000 NFE. However, when the results given in the SNS paper are analysed, the obtained constraints set is, G = [-1.4881e-06, -2.3780e-06, -1.5385e+02, -1.2034e+03, -1.1111e+02, 4.7155e-02, 0, -4.5361e-09, -7.6923e-01, -2.2581e+00, 0]. That is, the sixth constraint is violated here also.

Above discussed analyses demonstrate the effectiveness of the newly proposed hybrid algorithm in solving real-world engineering problems.

4.1. Performance on a few High Dimensional Benchmarks

Abualigah et al. (2021) proposed one Arithmetic Optimization Algorithm (AOA) and its performance were compared with several benchmarks. The benchmarks include scalable sets of unimodal and multimodal test functions (F1–F13) with four dimension spaces (30, 100, 500, and 1000). The results were compared with 11 other popular algorithms which include GWO, GA, PSO and Differential Evolution (DE). GWO was ranked second behind AOA among 12 algorithms

For the same NFE, the proposed algorithm is run and results are presented in Table 10.

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Table 10. Results of Functions: F1 to F13 [Abualigah et al., 2021]

Function	Dimensions	Min	Mean	STD
F1	30	1.8001e-165	7.0612e-156	3.8134e-155
	100	1.5797e-89	7.5366e-85	1.7290e-84
	500	4.5379e-48	2.4857e-45	6.3428e-45
	1000	1.1060e-38	1.3048e-36	1.8493e-36
F2	30	8.5273e-107	2.9422e-100	7.9454e-100
	100	4.9259e-62	1.9907e-59	2.9870e-59
	500	2.6689e-37	1.8016e-36	1.9349e-36
	1000	36.9111	1.7440e+305	Inf
F3	30	2.9824e-17	1.6685e-07	9.1082e-07
	100	187.7449	2.1837e+03	1.9332e+03
	500	5.2263e+05	6.4115e+05	7.8782e+04
	1000	2.1779e+06	2.9122e+06	4.7946e+05
F4	30	3.9841e-18	7.7940e-14	2.6726e-13
	100	0.2699	3.8297	3.5131
	500	60.1462	66.8094	3.7809
	1000	72.3021	76.3728	2.8889
F5	30	23.9472	24.2096	0.1600
	100	93.7587	95.9218	1.9525
	500	497.0563	497.8931	0.1848
	1000	996.9183	997.6922	0.1875
F6	30	2.5473e-14	1.0584e-10	2.2125e-10
	100	4.9810	6.6586	0.8581
	500	92.2365	95.1988	1.6096
	1000	211.8232	215.4090	1.8141
F7	30	8.4695e-04	0.0019	8.6477e-04
	100	0.0026	0.0074	0.0023
	500	0.0131	0.0280	0.0090
	1000	0.0240	0.0449	0.0114
F8	30	-1.0122e+04	-8.2140e+03	1.2301e+03
	100	-2.9545e+04	-1.8776e+04	5.3723e+03
	500	-8.5594e+04	-5.1633e+04	1.9095e+04
	1000	-1.3466e+05	-7.6548e+04	3.2235e+04
F9	30	10.9446	46.6085	26.8965
	100	21.3406	186.8156	124.6664
	500	3.4043	116.9499	156.7475
	1000	11.0667	63.9113	43.7354
F10	30	3.9968e-15	7.1942e-15	1.0840e-15

	100	7.5495e-15	1.4300e-14	2.8529e-15
	500	2.8866e-14	3.6445e-14	4.2483e-15
	1000	3.9524e-14	5.4564e-14	6.9621e-15
F11	30	0	0.0076	0.0092
	100	0	0.0065	0.0138
	500	1.1102e-16	6.4139e-04	0.0035
	1000	2.2204e-16	0.0048	0.0143
F12	30	7.9174e-16	0.0138	0.0358
	100	0.0440	0.0901	0.0403
	500	0.6355	0.7016	0.0326
	1000	0.7715	0.8252	0.0243
F13	30	3.5036e-13	0.0433	0.0701
	100	1.5172	3.3119	0.7783
	500	43.1530	44.4169	0.6382
	1000	93.4243	95.5878	1.0833

The results show that the proposed hybrid GWO:TP-AB algorithm performs better than GWO in 35 out of 52 cases when the "mean" values are considered. It outperforms GWO in 34 cases if "standard deviation" is the metric.

It performs well in 10 problems of dimension 30, 9 each of dimensions 100 and 1000 and, 7 problems of dimension 500 for the "mean" results.

The SD is better in 11 cases of dimension 500, 9 of dimension 1000, 8 of dimension 30 and 6 problems of dimension 100.

4.2. Cobb-Douglas Production Function

The Cobb-Douglas production model (Felipe and Adams, 2005) is a maximization problem. It has two input variables; units of capital (***K***) and, the units of labor (***L***) invested in the economy. "***A***" refers to the total factor of productivity,

The general model can be expressed as presented in equation (7).

$$Y(K,L) = A * L^{\alpha} * K^{(1-\alpha)} \quad (7)$$

If a manufacturing activity is more labor-intensive, "α" takes a higher value. In most of the cases, it takes a value of 0.6. This is one of the widely studied forms of a production problem which includes economics of production also.

To demonstrate the ability of the proposed algorithm, the following objective function (equation 8) is solved.

Maximize Production, $Y = 300 * L^{0.6} * K^{0.4}$. (8)

In this case, the capital cost "***K***" is assumed as 130 units and the labor cost "***L***" as 85 units. If the maximum cost is fixed as 100000 then, the inequality constraint can be expressed as in equation (9).

$$Total\ Cost = 130*K + 85*L \leq 100000. \quad (9)$$

The search range is taken as [0, 500] for both '***K***' and '***L***'.

Table 11. Cobb-Douglas Problem: Results for Different Population Sizes and Iterations

Population Size: 5				
IT 100	IT 200	IT 500	IT 750	IT 1000
-139541.1059	-139541.1181	-139541.1734	-139541.1837	-139541.1866
Number of Function Evaluations: 10000				
PS: 5	PS: 10	PS: 20	PS: 25	PS: 50
-139541.1884	-139541.1876	-139541.1679	-139541.1700	-139541.1435
Standard Deviation				
7.1064	3.10963	1.8551	2.2806	1.6191

Table 11 shows the results for this problem when different population sizes (5, 10, 20, 25 and 50) and number of iterations (100, 200, 500, 750 and 1000). For 1000 iterations (NFE = 10000), the obtained cost is 139541.1866 which is very close to the known value of 139541.6776.

As the number of iterations increases, the results improve. When the population size is 5, the result is better when compared to the population size of 50. However, the standard deviation (SD) is high when the population size is 5.

Figure 4 shows the convergence curve for 100 iterations and 20 trials. The steep curves below 10 iterations show the improvements are high during initial stages and remain almost flat after 40 iterations.

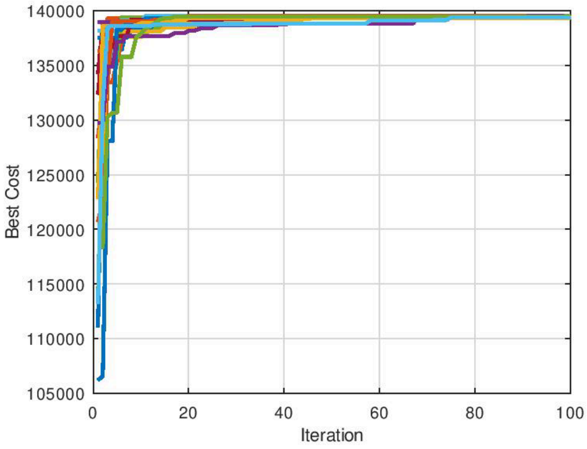

Figure 4, Convergent Curves for the Cobb-Douglas Problem

5. Conclusion, Limitations and Future Work

This paper combines the capabilities of one of the most popular optimization algorithms in the literature, GWO and another recent population-based TP-AB algorithm. It is a two-phase hybrid algorithm in which GWO improves the population in phase I and, the second phase of the TP-AB algorithm in phase II. In the second phase, the Levy strategy is applied to randomize the created solutions.

The new hybrid algorithm GWO:TP-AB is tested against 23 classic test functions, a tough CEC2019 dataset and 18 real-world engineering problems. Real-world engineering problems include continuous, discrete, discrete and continuous variables. In all cases, the performance is better than many other popular algorithms of recent times. To demonstrate the ability of the algorithm to handle larger dimension problems, 13 unimodal and multimodal problems are tested for 30, 100, 500 and 1000 dimensions. The “mean” and “standard deviation” results indicate its better performance over several algorithms including GWO.

The preliminary analyses show that the proposed hybrid GWO:TP-AB algorithm is a fair competitor for similar algorithms available in the literature. The main limitation of this algorithm is that the problems and constraints should be represented as mathematical expressions accurately. Future work includes the feasibility of solving multi-objective and other industrial optimization problems in the digital domain using the proposed hybrid algorithm.

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