EAI Endorsed Transactions
on Digital Transformation on Industrial Processes Research Article EAI.EU

Solving Facility Location and Supply Chain Management Problems Using Modified Population-Based TP-AB Algorithm

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Abstract

INTRODUCTION: Locating optimal supply/ storage/ distribution facilities is critical in minimizing the establishment, transportation and distribution costs. Among the facility location/ supply chain management problems, "Mini-Sum" and "Mini-Max" problems are widely studied popular sub-domains of Operations Research.
OBJECTIVES: The objective is to proposes one model for estimating the facility centres based on any specific criterion. Also, the feasibility of grouping demand points into clusters based on the need is demonstrated.
METHODS: A modified TP-AB algorithm solves the facility location problems by considering European countries. The required population data from 1955 to 2025 and population and Gross Domestic Product per capita data for 2013, 2018 and 2023 are extracted from the literature. Capital cities of these countries are taken as the representative demand points in the supply chain network and facility centres are located using weighted and un-weighted distance. Since the data points are spread over the Earth's surface, Great Circle Distance is preferred over Euclidean Distance.
RESULTS: It is observed that the “population centre”, “economic centre" and the "access centre" do not merge showing variations in the population spread and economic strength among the European Union and European countries considered. Finally, two more real-time cases involving population change of 25 European Union countries since 1955 and establishing a central command centre to monitor the border cities of Germany are solved.
CONCLUSION: The results demonstrate the flexibility and adoptability of the modified TP-AB algorithm. The model could be effectively extended for the same country also considering different states/ districts/ cities.

Keywords: Facility Location, Access Centre, Population Centre, Economic Centre, TP-AB Algorithm

Received on 29 January 2025, accepted on 20 March 2025, published on 01 April 2025

doi: 10.4108/dtip.8561

1. Introduction

Facility Location Problems (FLP) and Supply Chain Management (SCM) are important sub-domains of Operations Research. They aim to minimize the costs involved which may be establishment, operating, transportation, distribution and storage costs (Celik Turkoglu and Erol Genevois, 2020). A metric or “distance function” is a function that defines the closeness between two non-negative real numbers. Distance functions (weighted or un-weighted) are frequently used as “cost functions” to be minimized in an optimization problem. Different distance metrics are being used by the scientific community for different applications. These include Euclidean Distance, Manhattan Distance, Chebyshev Distance, Minkowski Distance, Canberra Distance, Great Circle Distance and Ellipsoidal Distance. The supply/ storage points are usually called the "Facilities" and the final destinations are often

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referred to as the "Demand" points or customers. The "Optimal Facility" is the "Geometric Median (GM)" of the demand points. FLP are NP-Hard (Raeisi Dehkordi, 2019) and hence no exact mathematical solutions are available. As a result, several heuristics are being proposed to handle such problems. When the supply (facility) and demand points (customers) are located on a plane surface, “Euclidean Distance (ED)” will work nicely. Weiszfeld algorithm (1937) is frequently used to iteratively find the “GM” of a set of points in the Euclidean plane.

When the data points are located over the Earth's surface, ED will not yield the expected results. If the Earth is assumed as a perfect sphere, the “Great Circle Distance (GCD)” is considered by the researchers. GCD models are proposed by several authors for marine applications (Mwemezi and Huang, 2011; Baskar and Xavior, 2021, a). However; Earth is not a sphere but an oblate ellipsoid. In such a case, "Ellipsoidal Distance (ELD)" gives more accurate results. GCD and ELD are computed using the “Geodetic Coordinates” (latitudes and longitudes) using Haversine and Vincenty’s formulae respectively. Latitudes and longitudes are the imaginary lines that run in an east-west direction and north-south direction respectively along Earth’s surface. The equator is assumed to be at 0^{o} latitude. Latitude varies between zero and ninety degrees that are followed by the letter 'N' or 'S' to represent northern and southern locations. The longitudes are represented by both positive and negative values between 0^{o} and 180^{o} followed by the letter ‘E’ or ‘W’ to represent eastern and western locations. For converting the values to radians the relation being used is, a hundred and eighty degrees equal to 'π' radians.

This paper proposes a population-based modified TP-AB algorithm applied to FLP with any distance metric. The model is validated using the datasets available in the literature. This work considers the shortest distance without considering the actual driving distance.

Both GCD and ELD were studied by Katz and Cooper (1980). They confirmed that when the points are widely separated on the Earth's surface, the difference between GCD and ELD is significantly higher. One randomized algorithm proposed by Welzl (1991) computes the “smallest enclosing disks (balls and ellipsoids)” in linear time. Except for marine and aerial applications, land/ road distances (driving distances) are higher than GCD or ELD. Hence, the exact distance could not be computed exactly. This is due to the presence of curvatures, bridges, rivers, lakes and hills that exist in the transportation routes. This again is a function of rural or urban locations in different parts of the Earth. Hence, a correction factor termed the "Wiggle Factor" was proposed (Cooper, 1983; Domínguez-Caamaño et al., 2016). Computing the correction factor will be a problem in real-world applicability. To make the analyses simple, this is ignored in this work.

An algorithm was proposed by Shih (2015) which uses the driving distances on the spherical surface which could be used as a decision tool to explore facility locations. Evolutionary and population-based algorithms which are stochastic in nature are also popular in solving optimization problems that include FLP. They usually need several iterations to arrive at the optimal/near-optimal solution of any problem.

In recent years, many AI-driven FLP and real-world transportation constraint models have been proposed and analysed by researchers. Krishna Vaddy (2023) explored the application of artificial intelligence (AI) and machine learning (ML) to optimize supply chain routes. The study examined several AI-driven optimization algorithms and their roles in enhancing effective decision-making and reduction in costs. Their findings underscored the transformative impact of these cutting-edge technologies for developing more sustainable and efficient supply chain logistics. One “Reliable Un-Capacitated Facility Location (RFL)” was studied by Shen et al. (2024) and they proposed a data-driven approach considering random demands and facility disruptions. Their model selects facility locations that minimize both fixed and operational costs. It was claimed that their approach offers a theoretical guarantee in situations where the available data are limited.

Malladi and Muthuraman (2024) addressed the facility location problem under joint disruptions that affect multiple facilities simultaneously. A calibration algorithm was developed to model the dependencies among disruptions. It was claimed by the authors that their model could be adapted to other problems in logistics with joint disruptions. Zhang et al. (2024) proposed one "Hierarchical Facility Location Problem (HFLP)" model to identify the recharging locations of drones in addition to selection of service stations and demand points for urban delivery with drones. A mixed-integer programming model was used to solve this. A demand satisfaction constraint was also incorporated in the model.

The objective of this paper is to propose a flexible model to handle different types of facility location problems. “Mini-Sum” and “Mini-Max” cases are considered in this work. Real-time geographical, population and economic data pertaining to a few European countries are analysed. Finally, a “Command Centre” is located using forty-two border cities of Germany which shares its border with nine other countries. This paper uses one modified population-based TP-AB algorithm (Baskat et. al., 2024) for solving different FLPs.

The paper is structured as follows: The introduction is followed by section 2 which gives a brief description of the population-based TP-AB algorithm. Discussions about different distance metrics are presented in section 3. Section 4 deliberates about the “Mini-Sum” and “Mini-Max” problems of supply chain management and section 5 analyses a few real-time cases pertaining to European countries. The paper culminates with the “Conclusions” section no. 6

2. Population-Based TP-AB Algorithm and Methodology Used

TP-AB algorithm is a two-phase population-based metaheuristic proposed recently by Baskar et al. (2024). Originally it was proposed for single objective optimization

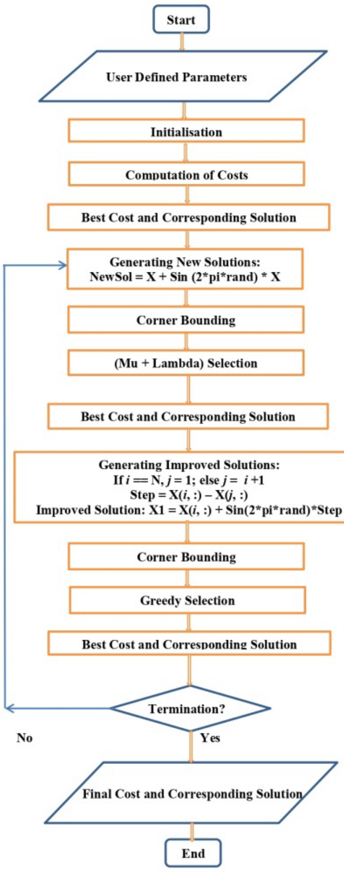


Figure 1, TP-AB Algorithm without Tuning (Baskar et. al., 2024)

with and without constraints. The algorithm without tuning could be explained in a few steps (Fig. 1).

The advantages of using this algorithm are its simplicity, absence of any user-defined parameter that influences the end results and ability to modify the codes to solve any optimization problem. The capability of the TP-AB algorithm in solving single objective, multi-objective, constrained and unconstrained, travelling salesman and flowshop scheduling problems has been well demonstrated by the authors.

The TP-AB algorithm is modified for each type of problem and validated before carrying out the analyses.

The strategy is explained in the following steps:

- Collect the real-time data and finalize the demand points
- Compute the fraction of weights for each country based on the population and economic criteria
- Find the “Geodetic Coordinates” (latitude and longitude) of each data point
- Select the initial approximate facility (usually the “Mass Centre" which is the average of latitudes and longitudes of the demand points)
- Compute the “Cost Function” (using weighted/ un-weighted GCD or ELD distance metric from the approximate facility to each demand point)
- Generate many approximate facilities (populations/ approximate solutions) from the initial approximate facility using the updating expressions of the TP-AB algorithm.
- Select the best facility.
- Generate more approximate solutions (populations/ approximate solutions) from this new approximate facility using the updating expressions.
- Select the best facility.
- Repeat the process till the termination criterion is met.

If tuning is to be applied, the “sine” terms in the updating expressions of Phase-I and Phase-II are to be changed to,

NewSol = X + r*Sin (2*pi*rand) * X (in Phase-I)

and**,** (1)

X1 = X(*i*, :) + r*Sin(2*pi*rand)*Step (in Phase-II)

where, (2)

r* = *a*-(*at*/*T*); *a*** = tuning parameter, ***t*** = current iteration number, ***T*** = the maximum number of iterations (“***a***” can be any positive value).

Keeping the updating expressions the same (with tuning by assuming ***a*** =1), codes are suitably modified for solving facility location problems (FLP).

3. Distance Metrics

In this work, we deal with only three distances (Fig. 2) whose mathematical expressions are presented below:

Euclidean Distance (ED) between points (*x1, y1*) and (*x2, y2*) in two dimensional spaces

$$=\sqrt{(x2-x1)^2+\ (y2-y1)^2}$$

Great Circle Distance (GCD) = (Earth Radius) * (Internal Spherical Angle in Radians)

[For GCD, Earth is considered a perfect square with a radius of 6371.009 km]

Internal Spherical Angle between two points

$$= 2\sin^{-1}\sqrt{(sin^2 \Delta L + cos\ L1 * cos\ L2 * sin^2\ \Delta M)}$$

Where:

ΔL – Average of latitudes of the two points = (L1+L2)/2

L1 – Latitude of first point, radians

L2 – Latitude of the second point, radians

ΔM – Average of longitudes of the two points = (M1+M2)/2

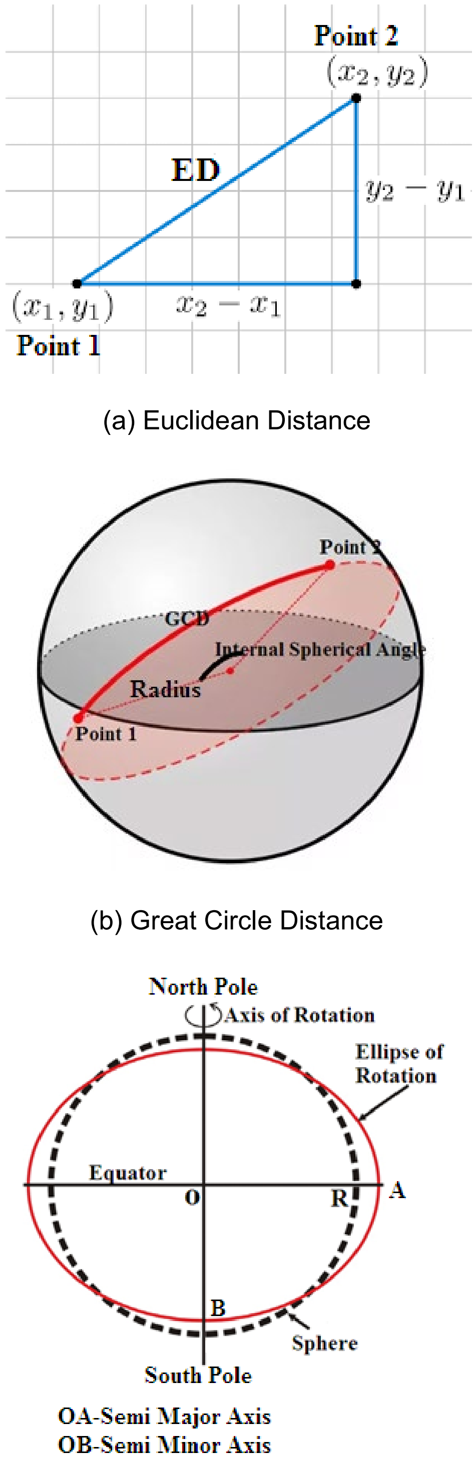
M1 - Longitude of the first point, radians

M2 – Longitude of the second point, radians.

Since the Earth is slightly flattened at the poles and bulges at the equator, it is considered an oblate spheroid. An oblate spheroid is an ellipsoid of revolution obtained by rotating an ellipse about its minor axis.

Vincenty’s formulae developed by Thaddeus Vincenty (1975) are widely used for computing the ellipsoidal distance (ELD). ELD yields better results than the GCD which is computed by using the Haversine formula.

The flattening factor and radius are taken as 1/298.257223563 and 6378137 m (according to WGS84) respectively. Since the formulae are complicated, they are not reproduced here.



(a) Euclidean Distance

(b) Great Circle Distance

(c) Ellipsoidal Model

Figure 2, Three Distance Metrics

4. Facility Location and Supply Chain Management Problems Considered

Facility Location Problems (FLP) linked with Supply Chain Management (SCM) is a class of optimization problems. They fall under the Operations Research (OR) domain. It determines the best location(s) for a facility/supply point to be selected based on geographical demands, the number of facilities required, facility costs involved, the transportation distances between each facility and the demand points connected with it.

Supply chain management is one of the critical aspects of these kinds of problems.

In this work two types of FLP are considered:

- Mini-Sum (*k*-Median) Problems
- Mini-Max (Minimum Covering Dome) Problems.

4.1. Mini-Sum (*k*-Median) Problems

In this FLP problem, the objective is to minimize the sum of the distances between a facility (supply point) and existing demand points. The distance may be weighted or un-weighted. It is common to consider Euclidean Distance (ED) as the distance metric for most of the FLP problems. However, when FLP problems are optimized for points located over the Earth's surface, ED may not yield the expected results. In such cases, Great Circle Distance (GCD) or Ellipsoidal Distance (ELD) is preferable.

The number of facilities, 'k' may be one or more.

That is, $k \leq n$ where n = number of demand points.

A typical "mini-sum" problem for a single facility can be mathematically expressed as,

$$\text{Minimize } f(x) = \sum_{i=1}^{n} (w_i * d(O, P_i));\ \text{for } i = 1 \text{ to } n \quad (3)$$

Where, w_i = Weight associated with 'i'th demand point

$d(O, P_i)$ = Distance between the facility 'O' and data point 'P_i'.

The facility is the 'Geometric Median' of the 'n' data points.

4.2. Mini-Max (Minimum Covering Dome) Problems

The objective here is to minimize the maximum distance (radius) between the facility and the existing data (demand) points. That is, the maximum distance (radius) between the facility and the farthest demand point has to be minimized. When the data points are spread over the Earth's surface, the problem is called the "Minimum Covering Dome" problem instead of the "Minimum Covering Circle".

Here also, the number of facilities, 'k' may be one or more, $1 \leq k \leq n$.

The distance may be weighted or un-weighted similar to a "Mini-Sum" problem.

The mathematical function for a single facility "mini-max" FLP is:

$$\text{Minimize } f(x) = \text{maximum } [d(O, P_i)];\ \ \text{for } i = 1 \text{ to } n \quad (4)$$

Where 'O' is the new facility and $d(O, P_i)$ is the distance between the 'i^{th}' data point 'P_i' and facility 'O'.

When the demand (data) points and the facility (supply point) are located on the curved exterior surface of the earth (dome), ED is not preferable and we should go for either GCD or ELD.

When all the data and facility points are located on a plane then, this becomes a "minimum covering circle" problem.

5. Results and Discussion

The codes are written in MATLAB and run on an i5 Desktop PC with 4 GB RAM.

The developed codes for GCD and ELD are verified by checking the distance between two airports; Invercargill Airport, New Zealand [46°24′44″S, 168°18′46″E] and, Svalbard Airport, Longyear, Norway [78°14′45″N, 15°27′56″E].

The GCD reported in a web portal (https://www.airmilescalculator.com/distance/ivc-to-lyr/) is 16292.601 km and ELD 16273.342 km.

When the developed codes are used we get GCD = 16292.606068 km and ELD = 16273.346571 km. That is, the results are almost matched with the reported values with minimum deviation.

5.1. Validation of Modified TP-AB Codes

To validate the modified codes of the TP-AB algorithm, data available in the literature are considered. The population size (PS), number of iterations (IT) and number of trials (TR) are mentioned in the respective Table itself.

Two datasets of Cazabal-Valencia et al. (2016); set-A and set-B are compared with the results obtained (Table 1) using the TP-AB algorithm.

Table 1. Lucia Datasets, 10 Locations Each [PS: 5, IT: 1000, TR: 30] No Weight, Mini-Sum

	Dataset A		Dataset B	
Distance Metric	Facility (Latitude, Longitude), rad.	Total Distance, km	Facility (Latitude, Longitude), rad.	Total Distance, km
GCD (Lucia)	0.88135, 0.058565	85679.42703	1.1010, 1.0780	70,654.08890
ELD (Lucia)	0.88135, 0.058571	85610.24031	1.0880, 1.0696	70,636.37066
GCD (FP-AB)	0.881348, 0.058576	85679.54813	1.1010, 1.0780	70654.19805
ELD (FP-AB)	0.881351, 0.058575	85603.32635	1.1164, 1.0876	70588.50724
GCD (TP-AB)	0.881347,0.058563	85679.5425256	1.1010, 1.0780	70654.1980461
ELD (TP-AB)	0.881347,0.058563	85603.3175504	1.1163, 1.0876	70588.5072376

Both the Lucia datasets A and B consist of ten locations each used for the “Mini-Sum” analysis. The Great Circle Distance, GCD (TP-AB) and Ellipsoidal Distance, ELD (TP-AB) obtained using the TP-AB algorithm are very close to the results of Lucia, GCD (Lucia) and ELD (Lucia). The results are also comparable with the results of the "Four-Point Direction Search Algorithm (FP-AB)” proposed by Baskar and Anthony Xavior (2021). The ellipsoidal distance is slightly less than the great circle distance.

Table 2. Random Dataset of 10 Locations

S.No.	Location	Latitude, deg.	Longitude, deg.
1.	Yukon, Canada	60.170638	-130.827364
2.	Kamchatka Krai, Russia	62.424437	169.684973
3.	Durazno, Uruguay	-33.195543	-55.429532
4.	London, UK	51.5085300	-0.1257400
5.	Ihosy, Madagascar	-22.488918	45.657826
6.	Hamrin Mountain, Iraq	35.050944	43.636343
7.	Nenets Autonomous Okrug, Russia	68.031820	61.372730
8.	Thellai, India	12.776006	79.028060
9.	Omakau, New Zealand	-45.062944	169.629765
10.	Hulunbuir, Inner Mongolia, China	49.753488	124.590197
Mass Centre	Al Udayd Saudi Arabia	23.8968458	50.7217258

Table 3. Random Dataset of 10 Locations [PS: 5, IT: 1000, TR: 30] No Weight, Mini-Sum

Distance Metric	Facility Latitude, rad.	Facility Longitude, rad.	Total Distance, km
1 Cluster			
GCD (FP-AB)	1.1745	1.0884	67835.95306
ELD (FP-AB)	1.1764	1.0861	67755.91956
GCD (TP-AB)	1.17454959	1.08836721	67835.9530550
ELD (TP-AB)	1.17642257	1.08612890	67755.9195570
2 Clusters; Initial Centre1 = [-1 -1]; Initial Centre2 = [1 1] GCD (TP-AB)			
Cluster-I	-0.78649674	0.627069964	19237.036088 +
Cluster-II	1.18737926	1.071156209	27875.88757011
Cluster Size	3	7	=47112.9236584

Table 4. Radius of Minimum Covering Dome [PS: 5, IT: 1000, TR: 30] No Weight, Random Dataset of 10 Locations, Mini-Max

Facility Latitude, rad	Facility Longitude, rad.	Minimum Great Circle Radius, km	Total GCD, km
GCD (FP-AB)			
0.13269	0.97653	12455.35575	78838.91365
GCD (TP-AB)			
0.132691827	0.97653532	12455.35164420	78838.90453645

A random dataset of Baskar and Anthony Xavior (2021) is considered (Table 2) for two types of analyses as a "Mini-Sum" problem. Initially, all data points are considered as a single cluster and the sum of both GCD and ELD are minimized. Then by considering only GCD these 10 points are grouped into two clusters using the TP-AB algorithm. In the single cluster case, TP-AB reports the same values as that of the "Four-Point Direction Search Algorithm (FP-AB)" (Table 3). When two clusters are considered for the same random dataset, the total GCD comes down to 47112.9236584 km from 67835.9530550 km. Three points are in one cluster and seven in the other cluster (Table 3).

Similarly, for the "Mini-Max" case, the radius of the minimum covering dome is minimized. That is the radius of the spherical dome to circumscribe all the ten points is estimated. TP-AB reports slightly better results than the "Four Point Direction Search Algorithm (FP-AB)" (Table 4). As the covering radius is minimized, the sum of GCDs increases from 67835.9530550 km (one cluster, Table 3) to 78838.91365 km (Table 4).

Finally, 661 districts of India (2011 census) are considered. The facility is located by minimizing the sum of GCDs (Table 5) using the TP-AB algorithm. The headquarters of each district of India and their latitude and longitudes are collected.

Table 5. Table 5. 661 Districts of India [PS: 5, IT: 10000, TR: 30] No Weight, Mini-Sum

Facility Latitude, rad.	Facility Longitude, rad.	Total Distance, km	Location
GCD (FP-AB)			
0.41766	1.41085	537412.9 7730	Ubra, Katni, Madhya Pradesh, India
GCD (TP-AB)			
0.4176570 6	1.4108573 9	537412.9 7655024	Ubra, Katni, Madhya Pradesh, India

Since the number of data points is significantly higher, the number of iterations is increased to 10000 and population size is kept the same. A total of thirty trials are conducted and the minimum value is recorded. The sum of GCDs reported by TP-AB is marginally less than that of the "Four-Point Direction Search Algorithm (FP-AB)". The located facility lies at "Ubra" of Madhya Pradesh state.

Above results demonstrate the perfect working of the modified TP-AB Algorithm for the FLP datasets available in the literature.

5.2. Analysis of EU, EFTA and EU Candidate Countries

In this analysis, 36 European countries are considered. This includes 27 EU countries, 3 European Free Trade Association (EFTA) countries (Iceland, Norway and Switzerland) and 6 EU candidate countries (Albania, Bosnia and Herzegovina, Montenegro, North Macedonia, Serbia and Türkiye).

It is assumed that we have to build a supply chain network to connect all countries represented by their capital cities. Number of clusters may be one or more depending on the need. Facility centres are to be identified to optimize the distance (weighted or un-weighted) function. If the supply chain has to be built after considering the population spread then, the facility will be located using the fraction population share of each country as its weight multiplied by the distance from the facility. If the facility is based on economic strength then, the fraction of GDP per capita is the weight. If the weights are the same for each country then there will not be any shift among the facilities and they will converge to a single point.

The capital city of each country is taken as its representative demand point and simulations are carried out. Since we will be analysing the relative position of facilities, this will satisfy our objectives.

The latitudes and longitudes of the capital cities are taken from a single source (www.findlatitudeandlongitude.com) to have uniformity (Table 6).

The population and GDP per capita data are extracted from the EU website (https://ec.europa.eu/eurostat/statistics-explained/index.php?title=GDP_per_capita,_consumption_per_capita_and_price_level_indices) for the years 2013, 2018 and 2023 (Table 6).

In 2023, Luxembourg recorded the highest level of GDP per capita followed by Ireland, both are EU countries. The third and fourth positions are occupied by two EFTA countries, Norway and Switzerland respectively.

Albania and Bosnia and Herzegovina (EU candidate countries) are placed in the last two positions.

When the population is considered for the year 2023, Türkiye has the highest population among all. The next populous countries are Germany and France followed by Italy in descending order. Iceland is the least populated country and Malta comes next as per 2023 data.

In the analysis, only GCD is considered as the distance metric for locating all facilities. The facility centres are given a different name based on the nature of the weights attached to the GCD.

If only the distance is considered in "mini-sum" FLP without any attached weight, we call the facility the "Access Centre (AC)". The centres with population fractions as the weights are named "Population Centres (PC)" and if the fraction GDP per capita is multiplied by the distance they are termed, "Economic Centres (EC)".

In the case of "mini-max" (minimum covering dome) FLP without weights, the facility centre is designated as the "MCC".

The integer suffixed refers to the year concerned for "PCxxxx" and "ECxxxx" centres and the number of countries for "ACxx" and "MCCxx".

The abbreviations used in this paper are:
AC – Access Centre
MCC – Minimum Covering Dome Centre
AC36 – Access Centre for 36 Countries
AC27 – Access Centre for 27 Countries
MCC36 – Minimum Covering Dome Centre for 36 Countries
MCC27 – Minimum Covering Dome Centre for 27 Countries

PC2013 – Population Centre for the Year 2013
PC2018 – Population Centre for the Year 2018
PC2023 – Population Centre for the Year 2023
EC2013 – Economic Centre for the Year 2013
EC2018 – Economic Centre for the Year 2018
EC2023 – Economic Centre for the Year 2023.

The objective of this work is to locate a central facility for specific supply chain requirements for all 36 countries. In the “Mini-Sum” case, the total GCD (weighted and un-weighted) to connect all countries’ capitals is minimized and in the second case of “Mini-Max”, the minimum dome radius GCD (un-weighted) to cover all these capitals is estimated and the corresponding facilities are located.

5.3. Discussions about AC, PC and EC

The Access Centre, “AC”, is a function of distance and independent of population, GDP, or any other weight parameter. Its location depends on the relative distances between the capital cities of countries.

Population centre, “PC” is a function of both distance and population share. Since the population share can never be the same for two countries, “PC” and “AC” will never coincide.

On the other hand, irrespective of the population, the GDP per capita can be the same or very close. Hence, if the GDP per capita is almost the same for all the member countries in a cluster then the economic centre, “EC” and “AC” can be very close to each other and may coincide.

Similarly, if the GDP share instead of GDP per capita is the weight and the GDP is proportional to the population, “PC” and “EC” (GDP) will approach each other and may also coincide.

Initially, the “Access Centre” of 36 countries (AC36) is located by considering only the un-weighted GCD. The “Mass Centre” which is the average of all points is assumed as the initial approximate facility. This initial point is then iteratively moved towards the optimal/near-optimal centre (Table 7). The optimal facility is located in Vienna, Austria with coordinates [48.20835398, 16.37250402] deg. Total GCD is 35468.38873905 km which is the sum of the un-weighted GCD of all capital cities from the facility, “AC36".

In the subsequent analysis, the population fraction and GDP per-capita fraction of each nation are assumed as the weight and corresponding facilities are located for the years 2013, 2018 and 2023 (Table 7). These facilities are termed PC2013, PC2018 and PC2023 for the population centres (PC) computed for the years 2013, 2018 and 2023 respectively. Similarly for the GDP per capita data, the economic centres EC2013, EC2018 and EC2023 are arrived at. The results show that the total GCD and corresponding facilities are slightly different from the initial facility (AC36) without any weight.

These 36 countries can be split into many clusters if required based on the supply chain requirements. For example, in the same Table 7, these 36 countries are grouped in two clusters (without considering any weight) so that the sum of the total GCD of Cluster-I and Cluster-II is minimized. The two initial approximate centres are assumed at [0.765, 0.321] radians and [0.969, 0.228] radians. In this two-cluster case, total GCD = 26874.41199877 km as against 35468.38873905 km of a single Cluster total GCD without weight. The number of countries in the cluster is not equal but, 20 and 16 respectively. In this way, the given set of demand points can be grouped under any number of clusters based on weighted distances. In another simulation, the minimum covering dome problem (Mini-Max) is solved for these 36 countries (Table 8).

The minimum dome radius (mini-max) case, the GCD reported comes down to 2437.29002810 km as against the radius of 2888.54501078 km reported for the “mini-sum” case. However, the total GCD increases to 39165.15925476 km from 35468.38873905 km of the “mini-sum” case. The facility for the “mini-max” case is located in Poland.

The facility centres (un-weighted) are computed for single and two clusters (mini-sum). The single cluster centre (C), cluster centre 1 (CC1) and cluster centre 2 (CC2) for two clusters are mapped. In the same map (Fig. 3), the minimum covering dome (mini-max) centre (MCC) is also graphically represented. “CC1” and “CC2” are located on either side of “C” whereas; “MCC” is located between “CC2” and “C”. All facilities lie almost in a straight line.

Now, facilities with population share and GDP per-capita share as the weights are computed. PC2013, PC2018, PC2023 (population centres) and, EC2013, EC2018, EC2023 (economic centres) for the years 2013, 2018 and 2023 respectively are plotted along with un-weighted cluster centre, “C” (Fig. 4). The observation is that the centres do not coincide but are separated marginally. The economic centres are located above “C” whereas; the population centres are below ‘C’. The economic centre for the year 2023, EC2023 is at the top right of the cluster centre (C) whereas the population centre for the same year PC2023 is at the bottom left of “C”. That is, the population and economic centres for the year 2023 are opposite to each other. Now, the simulation is carried out for the EU countries only.

After 31 January 2020, with the withdrawal of the United Kingdom from the EU, there are 27 EU countries the results of which are presented in Table 9.

The Access Centre for 27 countries, “AC27” is located in the South Bohemian Region, Southwest, Czechia ([49.08701177, 15.34068070] deg.) with a total GCD of 25546.96185925 km. The maximum cluster radius (given in brackets, column 4 of Table 9) is 2260.62137447 km.

The population and economic centres for the year 2023, PC2023 and EC2023 are also estimated which differ slightly from “AC27”.

If the 27 countries are split into two clusters (no weight), unlike the 36 countries case, we have almost the same number of countries in each cluster. 13 countries form a cluster and in the other cluster, there are 14 countries. The two initial approximate centres are assumed at [0.793, 0.285] rad. and [0.941, 0.233] rad. respectively for each cluster. Total un-weighted GCD also comes down from 25546.96185925 km to 20591.78692921 km.

Table 6. Data of 36 countries; 27 EU, 3 EFTA and 6 EU Candidate Countries

Country	Capital	Latitude, deg.	Longitude, deg.	GDP per Capita			Population		
				2013	2018	2023	2013	2018	2023
Luxembourg	Luxembourg (city)	49.611277	6.129799	277	258	237	5,43,066	6,07,913	6,54,768
Ireland	Dublin	53.34938	-6.260559	134	192	213	45,88,832	48,34,507	50,56,935
Netherlands	Amsterdam	52.37308	4.892453	136	130	133	1,68,67,352	1,72,86,042	1,76,18,299
Denmark	Copenhagen	55.686724	12.570072	129	127	125	56,25,385	57,66,686	59,10,913
Austria	Vienna	48.208354	16.372504	131	126	120	84,79,539	88,40,513	89,58,960
Belgium	Brussels	50.846557	4.351697	121	116	118	1,11,03,257	1,14,48,595	1,16,86,140
Germany	Berlin	52.510885	13.398937	126	125	116	8,16,80,591	8,28,96,696	8,32,94,633
Sweden	Stockholm	59.325117	18.071094	127	118	114	96,48,932	1,01,62,298	1,06,12,086
Malta	Valletta	35.898982	14.513676	91	107	107	4,37,525	4,91,586	5,35,064
Finland	Helsinki	60.167488	24.942747	114	109	105	54,38,872	55,15,461	55,45,475
France	Paris	48.85889	2.320041	109	103	99	6,33,35,180	6,42,77,808	6,47,56,584
Italy	Rome	41.89332	12.482932	100	97	98	6,03,12,599	5,98,77,425	5,88,70,762
Cyprus	Nicosia	35.17465	33.363878	84	91	97	11,66,968	12,18,831	12,60,138
Slovenia	Ljubljana	46.050027	14.506929	82	86	92	20,72,374	21,05,924	21,19,675
Spain	Madrid	40.416705	-3.703582	90	91	91	4,66,03,459	4,67,92,043	4,75,19,628
Czechia	Prague	50.059629	14.446459	85	93	90	1,05,14,552	1,05,34,593	1,04,95,295
Lithuania	Vilnius	54.687046	25.282911	73	81	87	30,28,189	28,76,128	27,18,352
Portugal	Lisbon	38.707751	-9.136592	77	77	81	1,04,64,535	1,02,89,835	1,02,47,605
Estonia	Tallinn	59.437216	24.745369	76	82	80	13,17,982	13,22,148	13,22,765
Romania	Bucharest	44.436141	26.10272	54	66	78	2,00,66,546	1,96,06,783	1,98,92,812
Poland	Warsaw	52.231958	21.006725	67	72	77	3,86,07,353	3,85,21,457	4,10,26,067
Hungary	Budapest	47.497879	19.040238	68	72	77	98,94,639	97,76,358	1,01,56,239
Croatia	Zagreb	45.842641	15.962231	61	64	76	43,08,854	41,60,485	40,08,617
Slovakia	Bratislava	48.151699	17.109306	77	70	74	54,14,739	54,46,745	57,95,199
Latvia	Riga	56.949398	24.105185	60	66	70	20,31,486	19,35,630	18,30,211
Greece	Athens	37.975565	23.734832	71	66	69	1,09,14,505	1,06,33,271	1,03,41,277
Bulgaria	Sofia	42.697703	23.321736	46	53	64	74,31,167	71,17,431	66,87,717
Norway	Oslo	59.91333	10.73897	185	156	171	50,80,668	53,12,320	54,74,360
Switzerland	Bern	46.948474	7.452175	170	157	154	80,88,367	85,14,434	87,96,669
Iceland	Reykjavik	64.145981	- 21.942237	124	129	135	3,24,024	3,52,946	3,75,318
Turkey	Ankara	39.920776	32.85405	62	63	72	7,66,67,864	8,44,15,969	8,72,70,501
Montenegro	Podgorica	42.441524	19.262108	41	48	51	6,33,946	6,31,455	6,26,485
Serbia	Belgrade	44.817813	20.456897	42	41	49	75,66,676	74,33,818	71,49,077
North Macedonia	Skopje	41.996182	21.431921	38	41	41	21,02,215	21,13,491	20,85,679
Albania	Tirana	41.328148	19.818444	29	30	36	28,87,014	28,77,013	28,32,439
Bosnia and Herzegovina	Sarajevo	43.851977	18.386687	30	32	36	36,17,559	34,00,129	32,10,847

Table 7. Centres of 36 European Countries [PS: 5, IT: 10000, TR: 30]; GCD, Mini-Sum

Performance Metric	Optimal Centre (Latitude, Longitude), rad.	Optimal Centre (Latitude, Longitude), deg.	Total GCD, km	Location
One Cluster	Mass Centre = [0.8409, 0.2531] radians			
AC36 (No Weight)	[0.84139450, 0.28575410]	[48.20835398, 16.37250402]	35468.38873905 (R=2888.54501078)	Vienna, 1010, Austria
2013 Population Share: PC2013	[0.84046327, 0.28635057]	[48.15499839, 16.40667891]	35471.57748454	Vienna, 1100, Austria
2018 Population Share: PC2018	[0.84043570, 0.28448834]	[48.15341868, 16.29998093]	35470.16472041	Vienna, 1230, Austria
2023 Population Share: PC2023	[0.84060564, 0.28253323]	[48.16315548, 16.18796186]	35476.01813718	Bezirk Mödling, 1130, Austria
2013 GDP per Capita Share: EC2013	[0.84283437, 0.28358352]	[48.29085243, 16.24813881]	35487.01699986	Tulln, 3400, Austria
2018 GDP per Capita Share: EC2018	[0.84282710, 0.28037728]	[48.29043590, 16.06443503]	35497.58626814	Tulln, 3441, Austria

2023 GDP per Capita Share: EC2023	[0.84176871, 0.28708719]	[48.22979436, 16.44888449]	35479.85289971	Vienna, 1220, Austria
Two Clusters (No Weight); Initial Centre1 = [0.765, 0.321] radians; Initial Centre2 = [0.969, 0.228] radians				
Cluster-I (CC1)	[0.76536138, 0.32090823]	[43.85197700, 18.38668702]	14395.29133113 (R1=2358.0390498)	City of Sarajevo, Bosnia and Herzegovina
Cluster-II (CC2)	[0.96543406, 0.22249041]	[55.31529680, 12.74776134]	12479.12066764 (R2=2141.5485186)	Vellinge kommun, Skåne County, Sweden
Cluster Size	Cluster-I: 20	Cluster-II: 16	26874.41199877	---

Table 8. MCC of 36 European Countries [PS: 5, IT: 1000, TR: 30] No Weight; Mini-Max

Min. Dome Radius (GCD), km	Optimal Centre (Latitude, Longitude), rad.	Optimal Centre (Latitude, Longitude), deg.	Total GCD, km	Location
2437.29002810	[0.91982547, 0.25774417]	[52.70211706, 14.76765313]	39165.15925 476	Dojazd pożarowy, West Pomeranian Voivodeship, Poland

Table 9. Centres of 27 EU Countries [PS: 5, It: 10000, 30 Trials] GCD; Mini-Sum

Performance Metric	**Optimal Centre (Latitude, Longitude), rad**	**Optimal Centre (Latitude, Longitude), deg**	**Total GCD, km (Dome Radius, km)**	**Location**
One Cluster; Mass Centre = [0.8462, 0.2545] radians				
AC27 **(**No Weight)	[0.85672998, 0.26774539]	[49.08701177, 15.34068070]	25546.96185925 (R=2260.62137447)	South Bohemian Region, Southwest, Czechia
2023 Population Share: PC2023	[0.85784894, 0.26893386]	[49.15112356, 15.40877488]	25548.31532492	Vysočina Region, Southeast, Czechia
2023 GDP per Capita Share: EC2023	[0.85611502, 0.26735011]	[49.05177717, 15.31803284]	25547.24682246	Okres Jindřichův Hradec, South Bohemian Region, Southwest, Czechia
Two Clusters (No Weight); Initial Centre1 = [0.793, 0.285] radians; Initial Centre2 = [0.941, 0.233] radians				
Cluster-I (CC1)	[0.79326416, 0.28527889]	[45.45068852, 16.34527641]	10924.71885457 (R1=2221.3465540)	Nova Drenčina, Croatia
Cluster-II (CC2)	[0.94162115, 0.23356701]	[53.95091762, 13.38240372]	9667.06807464 (R2=1292.1744948)	Mecklenburg-Vorpommern, Germany
Cluster Size	Cluster-I: 13	Cluster-II: 14	20591.78692921	--

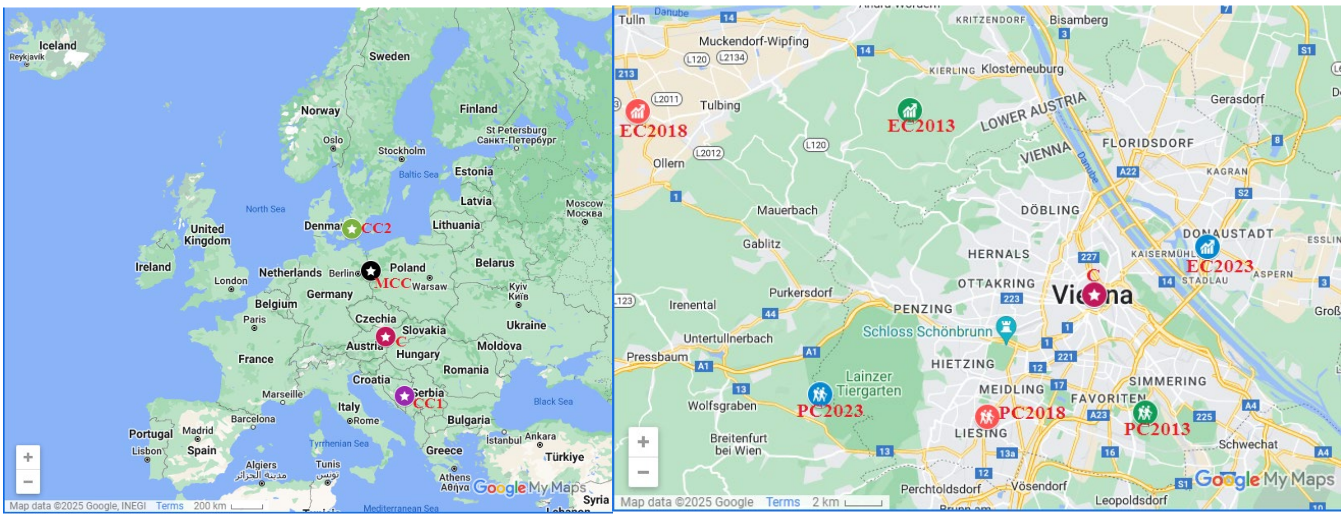


Figure 3, Facilities' Centres (36 Countries) **Figure 4**, Performance Centres (36 Countries)

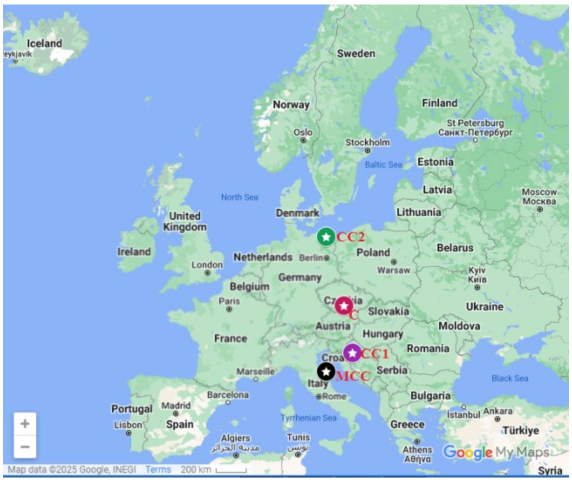
In this way, the given set of demand points can be grouped under any number of clusters based on weighted distances.

Table 10. MCC of 27 European Union Countries [PS: 5, IT: 1000, TR: 30] No Weight; Mini-Max

Min. Radius, km	Optimal Centre (Lat.,Lon.) rad.	Optimal Centre (Lat.,Lon.) deg.	Total Distance, km	Location
1962.47746791	[0.76697003, 0.23381867]	[43.94414595, 13.39682274]	30131.21936640	Ocean

Similarly, the minimum covering dome radius (mini-max case) with no weight is now estimated. The optimum great circle radius reported is 1962.47746791 km (Table 10). This is less than the radius of 2260.62137447 km obtained in the "mini-sum" case. Here, the estimated facility is located inside the ocean. Total GCD increases to 30131.21936640 km from 25546.96185925 km.

(a) 27 EU Countries



(b) Facility Centres C, MCC, CC1 and CC2

Figure 5, Facility Centres (27 EU Countries)

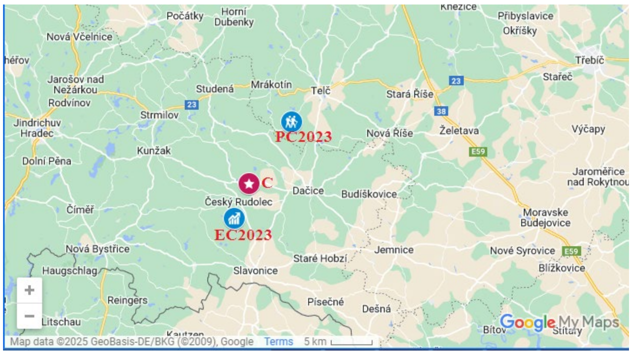


Figure 6, Performance Centres (27 Countries), PC2023, EC2023 and C

The facility centres are plotted and presented in Fig. 5 and Fig. 6. Fig. 5 shows the locations of cluster centre 1 (CC1) and cluster centre 2 (CC2) of two clusters with reference to the centre of a single cluster (C). "C" lies between "CC1" and "CC2". However, the minimum covering dome centre (MCC) is offset and lies inside the ocean near Italy.

Fig. 6 shows the relative locations of the population centre for the year 2023 (PC2023) and the economic centre for the same year (EC2023) with respect to "C". PC2023 is above "C" and EC2023 is below "C"' which is in contrast with the relative positions located for 36 countries.

Table 11. Distance between Performance Centres

From	To	36 Countries	27 Countries
		GCD, km	**GCD, km**
AC	PC2013	6.4512925	--
AC	PC2018	8.1379699	--
AC	PC2023	14.5750979	8.6821390

AC	EC2013	12.9979371	--
AC	EC2018	24.5687716	--
AC	EC2023	6.1405497	4.2511339
PC2013	EC2013	19.1348993	--
PC2018	EC2018	23.1652489	--
PC2023	EC2023	20.7105910	12.8714420
Cluster Size	Cluster-I	20	13
	Cluster-II	16	14
CC1	CC2	1336.8294997	968.67982519
AC36	AC27	123.657774	
MCC36	MCC27	979.0544386	
AC	MCC	512.4031691	590.8338029

Table 11 shows the distance between different facility centres for 36 and 27 countries cases. The two cluster centres (CC1 and CC2) are separated by a GCD of 1336.8294997 km for 36 countries case and, 968.67982519 km in the case of 27 EU countries.

The access centres, "AC36" for 36 countries and "AC27" for 27 countries are 123.657774 km apart. The minimum covering dome centres, "MCC36" and "MCC27" are separated by a larger distance of 979.0544386 km.

The access centre (AC) and minimum covering dome centre (MCC) are separated by a GCD of 512.4031691 km when all 37 countries are considered and, 590.8338029 km when only 27 EU countries are included.

The Population and Economic Centres for different years considered here are very close to the Access Centres (AC).

5.4. Analysis using Real-Time Data: Two Cases

To verify the applicability to real-time problems, two cases are analysed and discussed in this section.

Case I: Population Movement of 25 European Countries over the Past Seven Decades (Mini-Sum Problem)

Another real-time application is discussed here. The objective is to analyse the population movements of 25 European countries from the year 1955 to 2025. The countries considered are: Belgium, Czech Republic, Denmark, Germany, Estonia, Greece, Spain, France, Ireland, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Sweden and the United Kingdom.

The population data are collected from a single source (https://www.macrotrends.net/global-metrics/countries/). The population centres (PCs) are estimated by taking the population fraction share of each country as the weight. The centres (Geometric Medians that minimize the sum of weighted GCDs), the sum of GCDs from the PCs to the capital city of each country and the covering radius are presented in Table 12. PC1955 represents the population centre for the year 1955 and so on.

Table 12. Population Centres of 25 European Countries; PS: 5, IT: 30000; Trials: 100 [Mini-Sum Problem]

Centre	[Latitude, Longitude], rad.	Total GCD, km [minimized]	Covering Radius, km
No Weight, AC	[0.87473, 0.24539]	24053.13705	2278.78775
PC1955	[0.87522, 0.24188]	24054.91683	2291.84033
PC1960	[0.87559, 0.24619]	24054.22147	2279.72813
PC1965	[0.87522, 0.24072]	24056.31719	2295.49090
PC1970	[0.87567, 0.24792]	24056.26828	2274.64027
PC1975	[0.87419, 0.24502]	24053.51255	2277.75638
PC1980	[0.87470, 0.24060]	24056.85591	2293.81569
PC1985	[0.87305, 0.24252]	24058.03073	2281.08616
PC1990	[0.87283, 0.23795]	24067.10208	2294.80607
PC1995	[0.87452, 0.24577]	24053.18323	2276.71555
PC2000	[0.87737, 0.24465]	24058.58526	2291.81998
PC2005	[0.87379, 0.24573]	24053.99859	2273.89613
PC2010	[0.87337, 0.23706]	24067.41619	2299.81335
PC2015	[0.87538, 0.24197]	24054.84135	2292.21009
PC2020	[0.87179, 0.24175]	24064.68430	2278.51698
PC2025	[0.87553, 0.23938]	24058.34363	2300.99831

All centres are located in the Czech Republic (Czechia) and do not coincide. That is, every five years the centre keeps on changing depending on the population share of each country. Only randomly selected five PCs for the years 1955, 1970, 1990, 2005 and 2025 are plotted to demonstrate the movements along with the "Access Centre, C" (Fig. 7).

This shows that the PCs do not remain static but are dynamic. If the distance between centres is estimated; GCD between PC1970 and PC1990: 44.554 km, GCD between "C" and PC1955: 14.653 km and GCD between "C" and PC2025: 25.042 km. They are close and marginally separated from each other.

In a similar way by taking appropriate weights, the movements of other centres like the "Economic Centre", and "Literacy Centre' could be effectively analysed to arrive at a conclusion. This could be expanded to country level, state level, district level etc.

(a) Movement of Population Centres over the Year (Closer View)

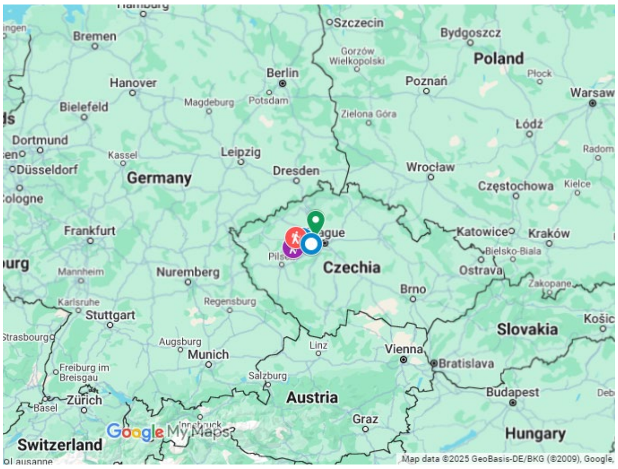


Figure. 7, Movement of Population Centres over the Years (Distant View)

Case II: Command Centre in Germany to Monitor International Border (Mini-Max Problem)

Germany shares its 3767 km land border with nine other countries which is one of the highest among the European countries:
North: Denmark
East: Poland and the Czech Republic
South: Austria and Switzerland
West: France, Luxembourg, Belgium, and the Netherlands.

The problem is to establish a central command centre to monitor the international border. The minimum radius is to be estimated for this which makes this problem a "Mini-Max" one.

For this analysis, 42 cities located on the border/ near the border (Table 13) are selected and their geodetic coordinates are estimated.

Table 13. Forty-Two Border/ Near Border Cities of Germany

City	[latitude, longitude] deg.	City	[latitude, longitude] deg.
Niebüll	54.786909, 8.825585	Hauzenberg	48.655197, 13.627678
Flensburg	54.783302, 9.433326	Passau	48.574823, 13.460974
Kappeln	54.664177, 9.93176	Trostberg	48.03211, 12.565436
Kiel	54.322709, 10.135555	Rosenheim	47.853927, 12.127262
Heiligenhafen	54.371745, 10.980924	Kempten	47.726706, 10.316884
Lübeck	53.866444, 10.684738	Friedrichshafen	47.650028, 9.480086
Wismar	53.890983, 11.464793	Stockach	47.853094, 9.013452
Rostock	54.088671, 12.140021	Müllheim	47.809674, 7.599352
Stralsund	54.309631, 13.082085	Offenburg	48.471656, 7.944378
Anklam	53.856053, 13.688091	Saarbrücken	49.234362, 6.996379
Pasewalk	53.505368, 13.988905	Bitburg	49.973276, 6.524947
Schwedt/Oder	53.058637, 14.284086	Aachen	50.776351, 6.083862
Eberswalde	52.835081, 13.799654	Krefeld	51.333121, 6.562334
Frankfurt (Oder),	52.341227, 14.549452	Lingen	52.522466, 7.316584
Cottbus	51.756745, 14.335731	Emden	53.367054, 7.20583
Görlitz	51.156319, 14.991018	Wilhelmshaven	53.527879, 8.106301
Chemnitz	50.832353, 12.918914	Varel	53.395777, 8.137215
Aue	50.586993, 12.699269	Bremerhaven	53.550539, 8.585195
Hof	50.321902, 11.917881	Cuxhaven	53.86878, 8.698286
Weiden in der Oberpfalz	49.675275, 12.163164	Sankt Peter-Ording	54.317266, 8.625494
Cham	49.217819, 12.666383	Husum	54.485414, 9.053794

The official Geometric Centre (Centroid) of Germany is estimated to be at Niederdorla [51.16344, 10.4476] deg. If the "Mass Centre" of these 42 cities is estimated it lies at [51.692, 10.634] deg. That is, these two points are separated by a GCD of 60.206 km.

The problem is considered as a "Mini-Max" one. The population size is taken as 5 and 1000 iterations are carried out. The minimum great circle radius estimated from 30 trials is 409.28150 km to cover all 42 cities. The central command centre is located at [51.165145, 9.920920] deg. The location moves closer to the official centroid of Germany and is away from it by just a GCD of 36.725 km. Another interpretation is that a circle of radius approximately 409.2815 km can circumscribe the all-important border cities of Germany. If the international border locations are considered, this value will be higher. If the area of 357596 square kilometres is converted to a regular circle, its radius will be about 337.38 km.

The accuracy of analysis increases with more number of cities considered and their closeness to the border.

6. Conclusion, Limitations and Future Work

This paper proposes one simple model to develop a supply chain framework and establish facility locations based on any criterion. Analyses are carried out by considering 36 European countries and 27 European Union countries with geodetic, population and GDP per capita data for the years 2013, 2018 and 2023. Facilities are located by considering only the great circle distance and also by taking the fraction population share as well as the fraction GDP per-capita share as the weights. These "performance centres" do not overlap but are separated by a small distance. This shows the varying levels of economic activity and population spread. Also, the feasibility of grouping the countries into clusters based on the need is demonstrated. The model could be effectively extended for the same country also considering different states/ districts/ cities. The modified population-based TP-AB algorithm is used for these analyses. Two more real-time problems are analysed using the population data of 25 European countries and 42 border cities of Germany. The advantage of using this model is that it helps in making management decisions while locating any facility/ distribution centre/ warehouse/ command centre etc. Micro-level analyses using more data points to village levels improve the solution quality. The tracking of the movement of a specific activity is possible as shown by the population activity of 25 European countries in the past seventy years. The limitation of this work is that the road distance is more than GCD/ ELD in most of the cases. The correction factor (Wiggle Factor) is ignored in this work. Future work includes analysing with different weights like the agricultural strength and education levels and using the "Wiggle Factor" also in the investigation.

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