Conditional Probability Density Function Based Signal Detection for OFDM-Based Transform Domain Communication Systems

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Abstract—**The orthogonal frequency division multiplexing-based transform domain communication system (OFDM-TDCS) is a promising candidate for signaling transmission in Cognitive Radio (CR) networks. An important issue of OFDM-TDCS system is the effective signal detection scheme design when channel coding is applied. In this paper, a class of new hard-demodulation (HD) and soft-demodulation (SD) algorithms are proposed in a unified signal detection framework. Although HD is based on the maximum likelihood (ML) estimation while SD is to calculate the log-likelihood ratio (LLR) of each coded bit, both of them are derived from an identical conditional probability density function. To further improve the implementation efficiency of SD detector, a code table is established to facilitate the needed searching operation. Finally, simulations under IEEE 802.22 Profile C channel validate the proposed signal detection detectors in terms of bit error rate (BER).**

I. INTRODUCTION

Cognitive radio (CR) [1]-[3] has become one promising technology for the improvement of spectrum utilization. An important aspect of cognitive radio is the agile physical layer transmission techniques, which should adaptively adjust the spectral waveform of transmitted signals to avoid collisions with primary users that have the priority to use spectrum.

Recently, a novel transceiver, called orthogonal frequency division multiplexing-based transform domain communication system (OFDM-TDCS), has been proposed as one signaling transmission system to solve the control channel design problem [4]. On account of the cyclic code shift keying (CCSK) modulation, OFDM-TDCS exhibits excellent bit error rate (BER) performance in the very low signal-to-noise ratio (SNR) region at the expensive of data rate [5]. Thus, not only is it suitable for the low-rate signaling transmission, but also the introduced interference on primary users can be mitigated significantly owning to the very low transmitting power. Meanwhile, this system is able to guarantee the transmission performance when a part of spectrum bands are occupied by primary users so that only the rest are available, and even though in the case of "spectrum heterogeneity" , in which the available spectrum bands at transmitter are not accordant with that at receiver [6][7]. In the initial phase of establishing CR networks, this "spectrum heterogeneity" phenomenon is inevitable but it can be solved via a signaling exchange mechanism supported by OFDM-TDCS [8].

To achieve the advantage introduced by OFDM-TDCS, it is necessary to detect signal in low SNR region. As cyclic code shift keying (CCSK) modulation [9] is the core of OFDM-TDCS, the signal detection algorithms of OFDM-TDCS are significantly different from that of existing OFDM systems, in which M-PSK or M-QAM modulations are adopted.

Basically, the signal detectors for the channel coded OFDM-TDCS can be classified into hard-demodulation (HD) and soft-demodulation (SD) [10]. The existing HD algorithm is based on the theory of CCSK demodulation to estimate the transmitted data. It can be simply implemented via searching the peak location of demodulated function. In this paper, a new HD algorithm is derived from the transmitted data's conditional probability density function (CPDF) according to maximum likelihood (ML) estimation. On the other hand, the existing SD algorithm is to calculate the log-likelihood ratio (LLR) values of all the coded bits in each transmitted data. It requires additional summation operations and then results in additional computational overhead. In this paper, a simple SD algorithm is proposed where the CPDF of the transmitted integer is involved and a code table is established to facilitate the needed searching operation. Furthermore, the proposed HD and SD detectors can be designed in a same detector framework because both of them are based on the common CPDF.

The rest of this paper is organized as follows. In Section II, the transmitter and receiver models of channel encoded OFDM-TDCS are described. In Section III, the new HD and SD algorithms are derived from a common CPDF and their implementations are provided in a unified framework. In Section IV, the simulation results are presented to validate the proposed algorithms. Finally, conclusions are drawn in the last Section.

II. SYSTEM MODEL

The channel encoded OFDM-TDCS transmitter model is described in Fig. 1. Each $\log_2 M_{\text{ary}}$ channel coded bits are converted into an integer S_i ($0 \le S_i \le M_{\text{ary}} - 1$), where the CCSK order M_{ary} is a power of 2 and not greater than N. Then, the i-th integer data S_i is modulated by a CCSK basis function. Finally, the frequency-domain OFDM signal vector $\mathbf{X}$ is generated and transmitted out.

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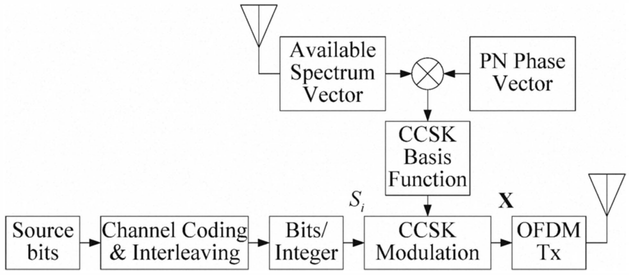

Fig. 1 Transmitter diagram of the encoded OFDM-TDCS

As it is shown in Fig. 2, the main idea of M-ary CCSK is modulating an integer data S_i by cyclically shifting an N-length time-domain CCSK basis function $b[n]$ by $D = S_i N / M_{\text{ary}}$ units [9]. In frequency domain, it can be explained that the corresponding frequency-domain CCSK basis function $B[k]$ is element-wisely multiplied by a complex exponent $\exp(-j2\pi S_i k / M_{\text{ary}})$ for $k = 0,1,..., N-1$ [10].

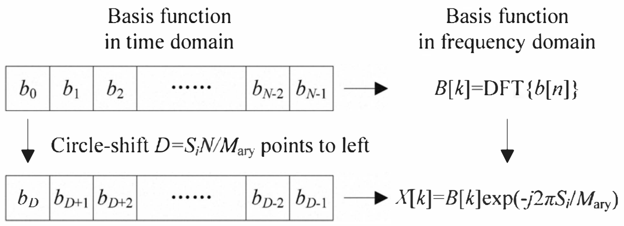

Fig. 2 The theory of CCSK modulation

In OFDM-TDCS systems, the frequency-domain CCSK basis function is composed of an available spectrum vector and a pseudo-noise (PN) phase vector. To mitigate the collisions with primary users, OFDM-TDCS systems only employ the free subcarriers to transmit signals. The states of all subcarriers can be determined by spectrum sensing, and then they are represented by an available spectrum vector, of which the element A_k =1 or 0 denotes that the k-th subcarrier is available or unavailable, respectively. To improve the anti-interference capability, a PN phase vector is adopted and its elements are derived from a confirmable PN sequence [4]. Thus, the frequency-domain CCSK basis function can be written as

$$B[k] = CA_k \exp\left(j2\pi m_k / M\right) / \sqrt{N} \tag{1}$$

where m_k is the k-th element of one PN sequence with period M , and C denotes the scaling factor which constrains the power of each OFDM symbol to be E_s , so that it is defined as

$$C = \sqrt{E_s N / N_T} \tag{2}$$

Here, N_T is the number of ones in the available spectrum vector. Consequently, the transmitted signal at the k-th subcarrier for an OFDM signal vector **X** can be expressed as

$$X[k] = \frac{1}{\sqrt{N}} CA_k \exp(j\frac{2\pi m_k}{M}) \exp(-j\frac{2\pi S_i k}{M_{\text{ary}}}) \tag{3}$$

In fading channel, the received OFDM signal at the k-th subcarrier can be expressed as

$$Y[k] = H[k]X[k] + W[k] \tag{4}$$

where $H[k]$ denotes the frequency-domain channel coefficient at the k-th subcarrier and $W[k]$ is the additional white Gaussian noise (AWGN) with variance N_0.

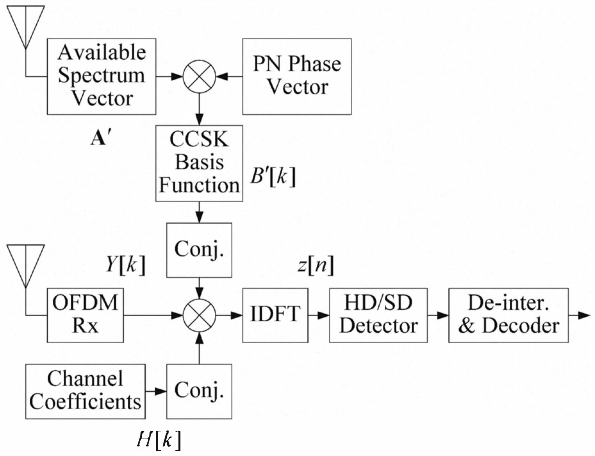

Fig. 3 Receiver diagram of the encoded OFDM-TDCS

The corresponding OFDM-TDCS receiver model is depicted in Fig. 3, where HD and SD are designed in a unified framework. On the one hand, another available spectrum vector is generated to receive signals at the free subcarriers. It is noticeable that the free subcarriers determined in receiver may be not identical with that in transmitter due to the spectral environment difference. Despite of that, the data information has been involved in all available subcarriers so that it can also be obtained from the common free subcarriers. On the other hand, a PN phase vector is generated to be used for CCSK demodulation, which is derived from a PN sequence as same as the one in transmitter. To keep identity of PN phase vector, the PN sequence adopted in transceiver can be stored in memory beforehand. Therefore, the CCSK basis function in receiver can be written as

$$B'[k] = C'A'_k \exp\left(j2\pi m_k / M\right) / \sqrt{N} \tag{5}$$

where the scaling factor C' normalizes the power of received signal vector and it is

$$C' = \sqrt{E_s N / N_R} \tag{6}$$

Here, N_R is the number of ones in the available spectrum vector in receiver. In addition, although channel coefficients can be obtained via channel estimation [11], perfect channel estimation is assumed in this paper for simplicity.

III. SIGNAL DETECTION

In this section, we derive a class of new HD and SD detection from an identical conditional probability density function (CPDF) depending on a correlation function given as

$$z[n] \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftharpoons}} Z[k] = Y[k]H^*[k]B'^*[k] \tag{7}$$

In the existing HD scheme, it follows (3), (5) and (4) that the correlation function can be analyzed as

$$z[n] \approx \frac{\eta E_s}{N} \left(\sum_{k=0}^{N-1} \left| H[k] \right|^2 \right) \delta[n - \frac{S_i N}{M_{\text{ary}}}] + w[n] \tag{8}$$

where $\delta[\bullet]$ is Delta-Dirac function, η is correlation coefficient of available subcarriers in transceiver and it is defined as

$$\eta = \sum_{k=0}^{N-1} A_k A'_k \Bigg/ \left(\sqrt{\sum_{k=0}^{N-1} A_k} \sqrt{\sum_{k=0}^{N-1} A'_k} \right) \tag{9}$$

and $w[n]$ is the complex noise with variance

$$\sigma^2 \approx N_0 \eta^2 \sum_{k=0}^{N-1} \left| H[k] \right|^2 \tag{10}$$

It is concluded that the transmitted data S_i can be estimated directly via searching the peak location of the correlation function given in (8). In what follows, we will re-explain this conclusion from the perspective of the CPDF-based maximum likelihood (ML) estimation. Meanwhile, we will derive a new SD algorithm based on the same CPDF. Therefore, the new HD and SD detectors can be designed in a unified framework.

A. Hard-Demodulation Algorithm

1) Conditional Probability Density Function

It is assumed that the transmitted data S_i is s, where $s \in \{0,1,...,M_{\text{ary}}-1\}$. The CPDF formula of the received OFDM signal vector can be expressed as

$$\begin{aligned} p(\mathbf{Y}|s) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left\| \mathbf{Y} - \mathbf{H}\mathbf{X}_{(s)} \right\|^2}{2\sigma^2} \right\} \\ &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left\| \mathbf{Y} \right\|^2 + \left\| \mathbf{H}\mathbf{X}_{(s)} \right\|^2}{2\sigma^2} \right\} \times \\ &\quad \exp\left\{ \frac{\operatorname{Re}\left\{ \mathbf{Y}\mathbf{H}^{\mathrm{H}}\mathbf{X}_{(s)}^{\mathrm{H}} \right\}}{\sigma^2} \right\} \end{aligned} \tag{11}$$

where $\mathbf{Y} = \{Y[0], Y[1],..., Y[N-1]\}^T$ is the received OFDM signal vector, $\mathbf{X}_{(s)} = \{X_{(s)}[0], X_{(s)}[1],..., X_{(s)}[N-1]\}^T$ denotes the transmitted OFDM signal vector when the transmitted data is s, $\mathbf{H} = \operatorname{diag}\{H[0], H[1],..., H[N-1]\}$ denotes the frequency-domain channel coefficients matrix, and the equivalent noise variance σ^2 has been defined in (10). To simplify the CPDF expression, we define a coefficient

$$K_c = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left\| \mathbf{Y} \right\|^2 + \left\| \mathbf{H}\mathbf{X}_{(s)} \right\|^2}{2\sigma^2} \right\} \tag{12}$$

and a likelihood function

$$\alpha(s) = \frac{1}{\sigma^2} \operatorname{Re}\left\{ \mathbf{Y}\mathbf{H}^{\mathrm{H}}\mathbf{X}_{(s)}^{\mathrm{H}} \right\} \tag{13}$$

Here, K_c can be regarded as a constant since $\left\| \mathbf{Y} \right\|^2$ and $\left\| \mathbf{H}\mathbf{X}_{(s)} \right\|^2$ are invariable for a specific received OFDM symbol. As likelihood function $\alpha(s)$ depends on the symbol s, it is complicated to calculate the whole likelihood function values corresponding to each possible symbol s. Fortunately, the likelihood function can be represented by the correlation function given in (7), so that (13) can be simplified as

$$\begin{aligned} \alpha(s) &= \frac{1}{\sigma^2} \operatorname{Re}\left\{ \sum_{k=0}^{N-1} Y[k] H^*[k] X_{(s)}^*[k] \right\} \\ &= \frac{1}{\sigma^2} \operatorname{Re}\left\{ \sum_{k=0}^{N-1} Y[k] H^*[k] B'^*[k] \exp(j2\pi sk/M_{\text{ary}}) \right\} \\ &= \frac{N}{\sigma^2} \operatorname{Re}\left\{ z[sN/M_{\text{ary}}] \right\} \end{aligned} \tag{14}$$

where the correlation function $z[n]$ has been defined in (7). Then, the CPDF can be rewritten as

$$p(\mathbf{Y}|s) = K_c \exp\{\alpha(s)\} \tag{15}$$

2) Maximum Likelihood Estimation

According to the CPDF given in (15), the ML estimation of transmitted data can be estimated as

$$\begin{aligned} \hat{S}_i &= \arg \max_{0 \le s \le M_{\text{ary}}-1} \left\{ \ln p(\mathbf{Y}|s) \right\} = \arg \max_{0 \le s \le M_{\text{ary}}-1} \left\{ \alpha(s) \right\} \\ &= \arg \max_{0 \le s \le M_{\text{ary}}-1} \left\{ \operatorname{Re}\left\{ z[sN/M_{\text{ary}}] \right\} \right\} \end{aligned} \tag{16}$$

It means that the transmitted data can be estimated directly via searching the peak location of the correlation function at some special locations. Thus, the estimated data $\hat{S}_i$ is converted into $\log_2 M_{\text{ary}}$ coded bits $\hat{c}_m$. Finally, the whole code block is de-interleaved and decoded via a channel decoder, such as Viterbi decoder or *a posteriori probability* (APP) decoder.

B. Soft-Demodulation Algorithm

The existing SD algorithm is to calculate the LLR values of all the coded bits in each transmitted integer. However, it requires additional summation operations and then results in additional computational overhead. In this subsection, the LLR values are calculated based on the CPDF which has been utilized in HD algorithm. Not only a simple SD formula is derived but also the new HD and SD detectors can be implemented in a unified framework. Furthermore, a code table is established to improve the implementation efficiency.

1) Computation of Log-Likelihood Ratio

For the given received OFDM signal vector $\mathbf{Y}$, the log-likelihood ratio [12] of the m-th bit in data S_i is defined as

$$LLR(c_m) = \ln \frac{P(c_m = 0|\mathbf{Y})}{P(c_m = 1|\mathbf{Y})} \tag{17}$$

According to the Bayesian rule, the posteriori probability can be expanded as

$$P(c_m = 0|\mathbf{Y}) = \sum_{\forall s: c_m = 0} p(s|\mathbf{Y}) = \sum_{\forall s: c_m = 0} \frac{p(\mathbf{Y}|s) p(s)}{p(\mathbf{Y})} \tag{18}$$

where $p(\mathbf{Y}|s)$ is the CPDF given in (15), $p(s)$ is the symbol priori probability which is assumed to be equiprobable, and $p(\mathbf{Y})$ is the probability that the received OFDM signal vector is equal to $\mathbf{Y}$. The notation "$\forall s: c_m = 0$" denotes that all the possible symbols whose m-th bit is zero. It follows (15) and (18) that (17) can be rewritten as

$$LLR(c_m) = \ln \sum_{\forall s: c_m = 0} \exp\{\alpha(s)\} - \ln \sum_{\forall s: c_m = 1} \exp\{\alpha(s)\} \tag{19}$$

where the likelihood function $\alpha(s)$ has been defined in (14). Here, we utilize the approximation $\ln \sum e^{x_i} \approx \max_i \{x_i\}$ [13]. Then, LLR formula can be further simplified as

$$\begin{aligned} LLR(c_m) &\approx \max_{\forall s: c_m = 0} \{\alpha(s)\} - \max_{\forall s: c_m = 1} \{\alpha(s)\} \\ &= \frac{N}{\sigma^2} \left(\max_{\forall s: c_m = 0} \operatorname{Re}\left\{ z[\frac{sN}{M_{\text{ary}}}] \right\} - \max_{\forall s: c_m = 1} \operatorname{Re}\left\{ z[\frac{sN}{M_{\text{ary}}}] \right\} \right) \end{aligned} \tag{20}$$

where σ^2 is the equivalent noise variance, and $z[n]$ is the correlation function. According to (20), the LLR value of each bit in data S_i can be obtained.

2) Efficient Implementation

The most complicated computation in (20) is to calculate the binary-to-decimal mapping operations and search for the optimal symbol s^* that maximizes the likelihood function among all the potential symbols. The total number of the binary-to-decimal mapping operations for each transmitted data amounts to $M_{\text{ary}} \times \log_2 M_{\text{ary}}$ times. Moreover, these mapping operations should be implemented repeatedly for every transmitted data. Thus, it results in lower implementation efficiency.

Table 1 An example of code table (3 bits involved)

$s=(c_1c_2c_3)_2$	c_1		c_2		c_3	
	0	1	0	1	0	1
Potential symbols	0	4	0	2	0	1
	1	5	1	3	2	3
	2	6	4	6	4	5
	3	7	5	7	6	7

To improve implementation efficiency of mapping operations, we establish a special code table to facilitate the fast mapping and searching. In this code table, all the symbols are classified into two groups according to whether one symbol's m-th bit c_m is 0 or 1. One advantage of this table is that the binary-to-decimal mapping operations can be replaced by looking up the code table. Also, it benefits to search for the optimal symbol among all the potential symbols for c_m is 0 or 1 respectively on account of the classified groups. Moreover, this code table can be utilized for all transmitted data symbols without any changes. Therefore, the implementation efficiency is improved significantly via looking up this code table.

An example of the code table is given in Table 1, where M_{ary} is given to be 8 and one symbol involves 3 coded bits. Take the first bit c_1 as an example, when $LLR(c_1)$ is calculated, the group for $c_1 = 0$ include potential symbols $s = 0,1,2,3$, while the opposite group for $c_1 = 1$ is composed of potential symbols $s = 4,5,6,7$. Similarly, the symbols can be classified into two groups for an arbitrary bit. It is worth of mentioning that some additional memory units are required to store this table. Fortunately, it is acceptable that the total number of required memory units is linear with parameter M_{ary} approximately. When M_{ary} equals to 1028 or 2048, the total number of memory units is 10240 or 22528, respectively.

3) Unified detector framework

Since both HD and SD algorithms are derived from the identical conditional probability density function, the HD and SD detectors can be designed in a unified detector framework as shown in Fig. 4. For HD detector, the transmitted data can be estimated directly via searching the peak location of the correlation function, and then the estimated data is converted into bits information. This implementation is similar to the existing HD detector. For SD detector, the LLR values of all the coded bits in each transmitted data should be calculated according to (20) and associated with a code table contributing to the -efficient implementation.

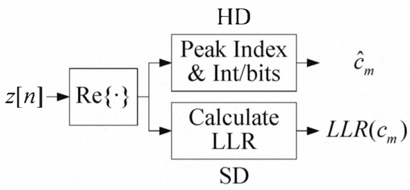

Fig. 4 The unified framework of HD and SD detectors

IV. Simulation Results

The simulation parameters are given as follows. The simulation is performed under the IEEE 802.22 Profile C channel [14]. The system parameters of channel encoded OFDM-TDCS are designed in Table 2. As every OFDM frame includes L_d OFDM symbols, the code block size is $L_d \times \log_2 M_{\text{ary}}$ and a matrix interleaver with L_d rows and $\log_2 M_{\text{ary}}$ columns is applied.

Table 2 System parameters

Parameter titles	Values
Number of subcarriers	2048
Cyclic prefix	352
Sampling time	0.125 us
bandwidth	8 MHz
OFDM symbol duration	0.3 ms
LFSR order	11
CCSK order	128-2048
OFDM frame length	60

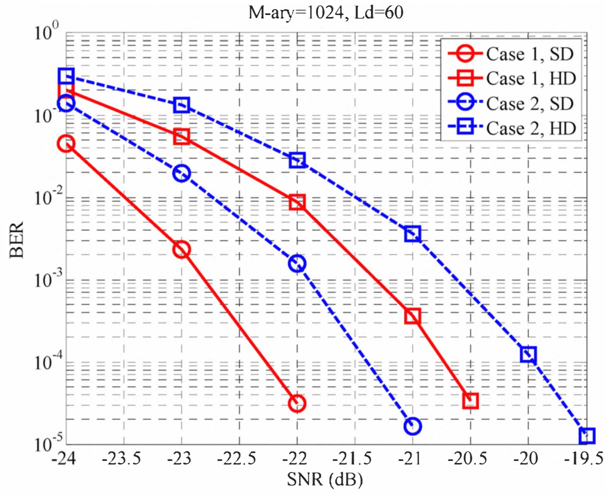

Fig. 5 BER performance in different spectrum cases

In Fig. 5, BER performance is presented in two spectrum scenarios. For scenario one, it is assumed that all the spectrum resources with 8 MHz bandwidth are available for both transmitter and receiver. For the other one, it is provided that the spectrum bands in the range of 0-7 MHz are available for transmitter, while the spectrum bands limited in 0-6 MHz can be used by receiver due to the emergence of primary users. The correlation coefficient of available subcarriers defined in (9) is 1.00 or 0.92 in these two scenarios respectively. It can be seen that the performance gap between these two scenarios is less than 1 dB both for HD and SD schemes, which means that both HD and SD schemes are effective in "spectrum heterogeneity" scenario.

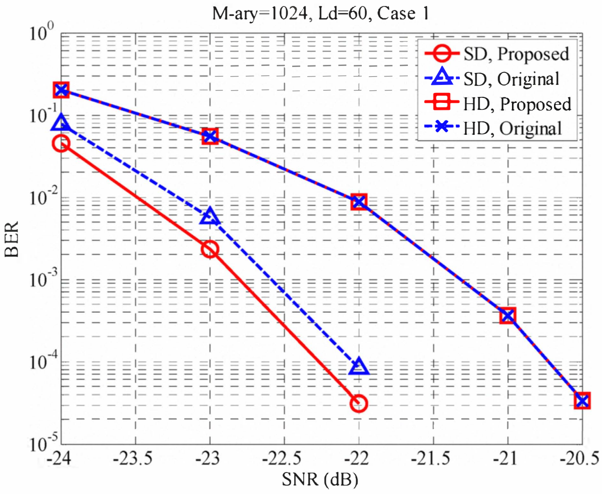

Fig. 6 Compared with original algorithms

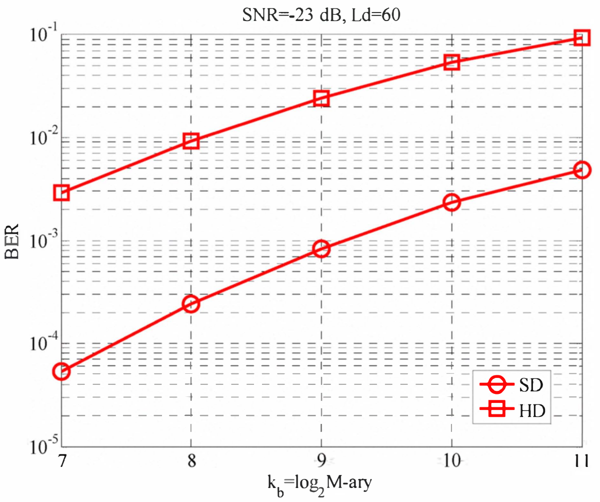

Fig. 7 BER performance with different CCSK orders

The performance comparison of proposed and original algorithms is shown in Fig. 6. For the same implementation scheme, the BER performance of proposed HD algorithm is identical with original one. However, the performance of proposed SD algorithm is slightly superior to original SD algorithm which modifies LLR values by a scaling function and then it results in performance degradation. Furthermore, it is much worth of mentioning that the implemental complexity of the proposed SD algorithm is significantly lower than that of original one due to the simple LLR formula and an efficient code table.

The BER performance of HD and SD with different CCSK orders is shown in Fig. 7. When the symbol power E_s is fixed, the lower CCSK order leads to less BER and lower complexity, but more sacrifice of data rate.

V. CONCLUSIONS

For channel encoded OFDM-TDCS, a new HD and SD algorithms are proposed under a unified framework based on an identical conditional probability density function. To improve the implementation efficiency of SD detector, a code table is further established to facilitate the fast mapping operations and symbol searching. Simulation results validate the proposed schemes.

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