# A Probabilistic-Constrained Leakage-Based Beamforming for Downlink Multiple Stream MU-MIMO System

Huiqin Du and Pei-Jung Chung
School of Engineering, The University of Edinburgh, UK
Email: H.Du@ed.ac.uk; P.Chung@ed.ac.uk

***Abstract*—Multi-user multiple-input and multiple-output (MU-MIMO) wireless system has the potential to provide a substantial gain by using transmit beamforming. The main challenge for transmit beamforming design is to suppress the interference from other users/data with imperfect channel state information at transmitter (CSIT). We propose a probabilistic-constrained beamforming design that combines the SLNR criteria with Alamouti coding for the multiple-stream-per-user MU-MIMO system. The proposed beamformer maximizes the average system performance and exterminates inter-data-interference while guaranteeing a low outage probability of serious leakage power performance. Simulation results show that the proposed beamformer achieves the highest signal-to-interference-and-noise ratio (SINR) reliability and provides the strongest robustness against channel imperfections, among the state-of-art transmit beamformers.**

## I. Introduction

MU-MIMO wireless system has gained considerable amount of interest since it can significantly increase data throughput and achieve higher diversity gain. Considering a multiple-stream-per-user MU-MIMO system, a base station (BS) communicates with multiple users in the same frequency and time slots, and each user receives multi-stream. It leads to the interference at the end users, including inter-stream interference and inter-user interference [1]. Thus, the suppression of interference is crucial to transmit beamforming design.

In an attempt to cancel interference, accurate channel information is required at transmitter side. However, it is usually not available due to errors induced by outdated/limited-feedback channel, leading to significant performance degradation. It motivates the development of robust transmit beamforming techniques which not only suppresses MU interference but also ensures robustness against the imperfections.

Recent advances in robust MU-MIMO transmit beamforming designs can be categorized into three classes: stochastic, deterministic and probabilistic approaches. The stochastic approach optimizes average system performance based on channel statistics, such as mean or covariance [1]–[3]. In contrast to stochastic approach that pays no attention to the extreme case, the deterministic approach optimizes the worst-case performance which leads to excessively conservative performance as the extreme operational condition is rare [4]–[7]. Recently, many researchers focus on probabilistic approach that considers the worst case scenario proportionally. Examples include receive adaptive beamforming [8]–[10], transceiver design of MISO systems [11], power minimization in multi-user MISO system [11], [12] and transmit beamformer design of single-stream MU-MIMO systems [13]. Compared with stochastic and worst-case approaches, the probabilistic approach maximizes average system performance while keeping a low probability of the server performance degradation. In this work, we adopt the probabilistic constraint strategy to beamformer design.

In order to eliminate the interference induced by multiple stream, we incorporates Alamouti code into robust transmit beamforming design. The resulting hybrid scheme gains orthogonal dimensions for each multiple stream vector. In this case, the beamforming design focuses on suppressing inter-user-interference. Two criteria work as inter-user-interference performance measurement of transmit beamformer, that is, signal-to-noise ratio (SINR) [5]–[7], [9], [14], [15] and leakage-based measurement [1], [4], [13], [16], [17]. As it couples optimization and feasibility simultaneously, the SINR solution easily arrives to infeasible region, especially with large number of users [18]. In contrast to SINR criteria, the leakage-based optimization problem only considers one user at each time, and admits an analytical closed form [17]. Hence, we pursue the signal-to-leakage-noise ratio (SLNR) as the measurement of beamforming design.

In this contribution, we propose a probabilistic-constrained beamforming scheme that combines Alamouti code with the leakage-based criterion for the multiple-stream-per-user MU-MIMO systems. The proposed beamforming maximizes the average signal power for the desired user, exterminates the inter-stream-interference, and ensures the robustness against the CSI errors by keeping a low outage probability of serious power leakage. We shall show the probabilistic-constrained optimization problem can be further replaced by a convex deterministic form by using the Markov's inequality and Lagrangian relaxation. The underlying problem can be efficiently solved by modern convex optimization algorithms and the final solution is obtained via randomization techniques. Simulation results show the proposed beamforming scheme provides the most reliable SINR performance and the strongest robustness in BER performance against channel imperfections, compared with several popular transmit beamforming techniques.

The remaining sections are organized as follows. In Section

CHINACOM 2010, August 25-27, Beijing, China

DOI 10.4108/chinacom.2010.62

II,we describe the system model and channel uncertainty briefly. The proposed design is formulated as stochastic optimization problem, and transfers the probabilistic constraint to a deterministic convex form in Section III. Numerical examples are presented and discussed in Section IV. Concluding remarks are given in Section V.

## II. System Model

Consider a multiuser downlink channel with $K$ users and a single base station. The base station is equipped with $N$ transmit antennas and each user has $M_k$ receive antennas. A multiple stream is transmitted from base station to each user with the length of multiple data equal to $L_k$. To prevent the inter-stream-interference caused by the non-orthogonal beamforming matrix, the multiple stream $\mathbf{s}_k \in \mathbb{C}^{L_k \times 1}$ is first exploited by Alamouti scheme [19]. Note that in this work, we consider the simplest case $L_k = 2$. The transmit coded block is given as follows,

$$\mathbf{s}_k = \begin{bmatrix} s_{k,1} \\ s_{k,2} \end{bmatrix} \Rightarrow \mathbf{S}_k = \begin{bmatrix} s_{k,1} & -s_{k,2}^* \\ s_{k,2} & s_{k,1}^* \end{bmatrix} , \quad (1)$$

where the superscript $*$ denotes complex conjugation without transposition, and the power of data vector $\mathbf{s}_k$ is assumed to be $\mathsf{E}\left[\mathbf{s}_i\mathbf{s}_i^H\right] = \mathbf{I}/2$. The transmit coded block is multiplied by beamforming $\mathbf{C}_k \in \mathbb{C}^{N\times 2}$ before being transmitted. The transmit signal matrix $\mathbf{X} \in \mathbb{C}^{N\times 2}$ can be presented as

$$\mathbf{X} = \sum_{k=1}^{K} \mathbf{C}_k\mathbf{S}_k , \quad (2)$$

where beamforming matrix $\mathbf{C}_k$ is normalized, so that $\mathrm{tr}\{\mathbf{C}_k^H\mathbf{C}_k\} \le 2$.

Assuming that the channel is slowly varied fading and user $i$ is the desired user, the received block for the desired user can be written as

$$\mathbf{Y}_i = \mathbf{H}_i\sum_{i=1}^{K}\mathbf{C}_i\mathbf{S}_i + \mathbf{N}_i = \mathbf{H}_i\mathbf{C}_i\mathbf{S}_i + \mathbf{H}_i\sum_{k=1,k\neq i}^{K}\mathbf{C}_k\mathbf{S}_k + \mathbf{N}_i , \quad (3)$$

where $\mathbf{N}_i$ is the noise matrix, each element of $\mathbf{N}_i$ is i.i.d complex normally distributed with zero mean and variance $\sigma_i^2$.

Denote $\mathbf{F}_i = \mathbf{H}_i\mathbf{C}_i \in \mathbb{C}^{M_i\times 2}$, a reconstructed new matrix $\bar{\mathbf{H}}_i \in \mathbb{C}^{2M_i\times 2}$ for the desired user can be expressed as

$$\bar{\mathbf{H}}_i = \begin{bmatrix} \mathbf{F}_i^{(1,1)} & \mathbf{F}_i^{(1,2)*} & \dots & \mathbf{F}_i^{(M_i,1)} & \mathbf{F}_i^{(M_i,2)*} \\ \mathbf{F}_i^{(1,2)} & -\mathbf{F}_i^{(1,1)*} & \dots & \mathbf{F}_i^{(M_i,2)} & -\mathbf{F}_i^{(M_i,1)*} \end{bmatrix}^T , \quad (4)$$

where $\mathbf{F}_i^{k,l}$ denotes the $(k,l)$-th element in matrix $\mathbf{F}_i$. The rearranged receive block (3) can be represented in terms of vector, that is

$$\mathbf{z}_i = \bar{\mathbf{H}}_i\mathbf{s}_i + \sum\nolimits_{k=1,k\neq i}^{K}\bar{\mathbf{H}}_k\mathbf{s}_k + \mathbf{n}_i , \quad (5)$$

where $\mathbf{z}_i = \left[\mathbf{Y}_i^{(1,1)}, \mathbf{Y}_i^{(1,2)*}, \dots, \mathbf{Y}_i^{(M_i,1)}, \mathbf{Y}_i^{(M_i,2)*}\right]^T$, and therefore the vector $\mathbf{n}_i$ is arranged correspondingly. According to (4), we have

$$||\bar{\mathbf{H}}_i||_F^2 = 2||\mathbf{F}_i||_F^2 = 2||\mathbf{H}_i\mathbf{C}_i||_F^2 , \quad (6)$$

Note that the transmit beamformer based on the SINR criterion couples optimization and feasibility simultaneously. The solution easily arrives at infeasible region, especially with large number of users [18]. Thus, we adopt the concept of *leakage* [16] using the signal-to-leakage-noise ratio (SLNR) criterion instead. According to (6) and $\mathsf{E}\left[\mathbf{s}_i\mathbf{s}_i^H\right] = \mathbf{I}/2$, the SLNR for the $i$-th user is given by

$$\begin{aligned} \mathrm{SLNR}_i &= \frac{\mathsf{E}\left[\mathbf{s}_i^H\bar{\mathbf{H}}_i^H\bar{\mathbf{H}}_i\mathbf{s}_i\right]}{M_i\sigma_i^2 + \mathsf{E}\left[\mathbf{s}_i^H\bar{\mathbf{H}}_i^H\bar{\mathbf{H}}_k\mathbf{s}_i\right]} \\ &= \frac{\mathrm{tr}\left\{\mathbf{C}_i^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{C}_i\right\}}{M_i\sigma_i^2 + \mathrm{tr}\left\{\mathbf{C}_i^H\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i\mathbf{C}_i\right\}} , \end{aligned} \quad (7)$$

where $\tilde{\mathbf{H}}_i \in \mathbb{C}^{\sum_{k\neq i} M_k\times N}$ denotes an extended channel matrix that excludes $\mathbf{H}_i$, i.e. $\tilde{\mathbf{H}}_i = \left[\mathbf{H}_1^T, \dots, \mathbf{H}_{i-1}^T, \mathbf{H}_{i+1}, \dots, \mathbf{H}_K^T\right]^T$. The superscript $T$ denotes transposition without conjugation, and the superscript $H$ is complex conjugate transposition. Thanks to Alamouti scheme, the cross term of $\mathsf{E}\left[s_k^H s_i\right]$ disappears, which means the beamforming design only needs to suppress co-channel interference. Moreover, compared with the hybrid scheme proposed in [14], our scheme only assume that $N \le \sum_{k\neq i} M_k$. It is more reasonable since the number of transmit antenna could be less than receive antenna, especially in broadcast channel.

However, in real scenario, only imperfect channel information can be accessed at transmitter. For the desired user (the $i$-th user), the presumed channel $\mathbf{H}_{i_p} \in \mathbb{C}^{M_k\times N}$ can be expressed as

$$\mathbf{H}_i = \mathbf{H}_{i_p} + \mathbf{E}_i , \quad (8)$$

where the error matrix $\mathbf{E}_i \in \mathbb{C}^{M_k\times N}$ consists of i.i.d. complex normally distributed entries with variance $\sigma_e^2$. The subscript $p$ is used to denote the *presumed* channel information. The corresponding interference channel $\tilde{\mathbf{H}}_{i_p} \in \mathbb{C}^{(\sum_{k=1,k\neq i}^{K} M_k)\times N}$ can be written as

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i , \quad (9)$$

where the error matrix $\tilde{\mathbf{E}}_i$ is composed of $(K-1)$ error matrix $\mathbf{E}_i$, that is, $\tilde{\mathbf{E}}_i = [\mathbf{E}_1^T, \dots, \mathbf{E}_{i-1}^T, \mathbf{E}_{i+1}^T, \dots, \mathbf{E}_K^T]^T$. Since the CSIT for each user is independent, the constructed matrix $\tilde{\mathbf{E}}_i$ has the same distribution of each component $\mathbf{E}_i$, that is, i.i.d complex normally distributed entries with variance $\sigma_e^2$. Note that we assume that $\tilde{\mathbf{H}}_i$ and $\tilde{\mathbf{H}}_{i_p}$ have the same rank, that is, $\mathrm{rank}(\tilde{\mathbf{H}}_i) = \mathrm{rank}(\tilde{\mathbf{H}}_{i_p}) = N$.

## III. Beamformer Design based on Probabilistic-constraint Optimization

To tackle performance degradation caused by imperfect channel estimates, we consider a probabilistic-constrained approach. In contrast to the minimax approach [4] and the stochastic approach [1]–[3], [17], the server degradation performance is considered proportionally by probabilistic constraint, which is favorable to an achievable SLNR performance on the desired user, and prevents a pessimistic result.

Based on the error model (8) and (9), the SLNR becomes a function of the random errors $\mathbf{E}_i$ and $\tilde{\mathbf{E}}_i$, that is

$$\begin{aligned} & \mathrm{SLNR}_i(\mathbf{E}_i, \tilde{\mathbf{E}}_i) \\ = & \frac{\mathrm{tr}\left\{\mathbf{C}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{C}_i\right\}}{M_i\sigma_i^2+\mathrm{tr}\left\{\mathbf{C}_i^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)\mathbf{C}_i\right\}} \ . \end{aligned} \quad (10)$$

In this work, instead of maximizing the SLNR directly, we maximize the average power allocation on the desired user while eliminating inter-user-interference and keeping a low outage probability of the serious leakage power performance. The proposed beamforming design can be formulated as the following probabilistic constrained optimization problem

$$\text{maximize} \quad \mathsf{E}\left[\mathrm{tr}\left\{\mathbf{C}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{C}_i\right\}\right] \ , \quad (11)$$

subject to

$$Pr\left\{M_i\sigma_i^2+\mathrm{tr}\left\{\mathbf{C}_i^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)\mathbf{C}_i\right\} \geq \gamma_{th_i}\right\} \leq p_i \quad (12)$$

$$\mathrm{tr}\{\mathbf{C}_i\mathbf{C}_i^H\} \leq 2 \ , \quad (13)$$

where $Pr\{A\}$ denotes the probability of the event $A$, $\gamma_{th_i}$ denotes a pre-specified leakage power level, and $p_i$ is an outage probability. Eq. (13) dictates the transmit power constraint that guarantees each symbol has unit transmit power.

To simplify the expression, we define a new parameter $\mathbf{W}_i$ as follow

$$\mathbf{W}_i \triangleq \mathbf{C}_i\mathbf{C}_i^H \ , \quad \mathbf{W}_i \geq 0 \ , \quad \text{and} \quad \mathrm{rank}(\mathbf{W}_i) = 2 \ , \quad (14)$$

where $\mathbf{W}_i \geq 0$ denotes $\mathbf{W}_i$ is semi-positive definite.

### A. Objective Function

Given the presumed channel $\tilde{\mathbf{H}}_{ip}$ at transmitter, the objective function is obtained by taking the expectation of the power allocated on the desired user with respect to error matrix $\mathbf{E}_i$,

$$\begin{aligned} & \mathsf{E}\left[\mathrm{tr}\left\{\mathbf{C}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{C}_i\right\}\right] \\ = & \ \mathrm{tr}\left\{(\mathbf{H}_{i_p}^H\mathbf{H}_{i_p}+M_i\sigma_e^2\mathbf{I})\mathbf{W}_i\right\} \ . \end{aligned} \quad (15)$$

In this work, the number of transmit antennas could be smaller than the number of all receive antennas combined. With $N$ transmit antennas, only $N-1$ degree of freedom is provided, which is less than the subspace of all users. That means, we can not guarantee the subspaces of all users are orthogonal to each other. Therefore, eigen-decomposition approach can not be easily implemented into objective function.

### B. Probabilistic Constraint

To maintain the leakage power under an acceptable level, the probabilistic constraint is introduced to guarantee a low probability that the power leakage becomes higher than a pre-specified threshold. The instantaneous leakage power has to satisfy the following probabilistic constraint,

$$Pr\left\{M_i\sigma_i^2+\mathrm{tr}\left\{\mathbf{C}_i(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}})^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}})\mathbf{C}_i^H\right\} \geq \gamma_{th_i}\right\} \leq p_i \ . \quad (16)$$

Since the probabilistic constraint is not convex, the deterministic form will be derived as follows.

**Proposition**: Under the assumption of Gaussian-distributed error, the probabilistic constraint (16) can be replaced by the following deterministic convex constraint

$$\mathrm{tr}\left\{\left(\tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p}+\sigma_e^2 n_i\mathbf{I}\right)\mathbf{W}_i\right\} \leq p_i\bar{\gamma}_{th_i} \ . \quad (17)$$

where $\bar{\gamma}_{th_i} = \gamma_{th_i} - M_i\sigma_i^2 > 0$, and the matrix $\mathbf{I}$ is an identity matrix.

*Proof* : Define

$$\mathbf{T} = (\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i) \ ,$$

the probabilistic constraint (16) can be rewritten in terms of $\mathbf{T}$ and $\mathbf{W}_i$

$$Pr\left\{\mathrm{tr}\left\{\mathbf{T}\mathbf{W}_i\right\} \geq \bar{\gamma}_{th_i}\right\} \leq p_i \ , \quad (18)$$

where $\bar{\gamma}_{th_i} = \gamma_{th_i} - M_i\sigma_i^2$.

Applying the Markov's inequality [20], an upper bound for the probability in (18) could be obtained as follows

$$Pr\left\{\mathrm{tr}\left\{\mathbf{T}\mathbf{W}_i\right\} \geq \bar{\gamma}_{th_i}\right\} \leq \frac{\mathsf{E}\left[\mathrm{tr}\{\mathbf{T}\mathbf{W}_i\}\right]}{\bar{\gamma}_{th_i}}. \quad (19)$$

In order to keep the power leakage below an acceptable level, we set the upper bound (19) being less than $p_i$,

$$\frac{\mathsf{E}\left[\mathrm{tr}\{\mathbf{T}\mathbf{W}_i\}\right]}{\bar{\gamma}_{th_i}} \leq p_i \ . \quad (20)$$

Moreover, under the assumption of the elements of the error matrix $\tilde{\mathbf{E}}_i$ are complex Gaussian-distributed, we have

$$\mathbf{T} \sim \mathcal{CW}_N(n_i, (\sigma_e^2\mathbf{I})^{-1}\tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p}, \sigma_e^2\mathbf{I}),$$

where $\mathcal{CW}_N(n_i, (\sigma_e^2\mathbf{I})^{-1}\tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p}, \sigma_e^2\mathbf{I})$ denotes that the matrix $\mathbf{T}^{N\times N}$ is complex Wishart distributed with degree of freedom $n_i = 2\sum_{k=1,k\neq i}^{K} M_k$, non-centrality parameter $(\sigma_e^2\mathbf{I})^{-1}\tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p}$ and covariance matrix $\sigma_e^2\mathbf{I}$. Based on the result in [21], the mean of complex Wishart-distributed matrix $\mathbf{T}_i$ can be expressed as

$$\mathsf{E}\left[\mathbf{T}_i\right] = n_i\sigma_e^2\mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p} \ . \quad (21)$$

Since the expectation and trace are both linear operators, we can express $\mathsf{E}\left[\mathrm{tr}\{\mathbf{T}\mathbf{W}_i\}\right]$ in the following form

$$\mathsf{E}\left[\mathrm{tr}\{\mathbf{T}\mathbf{W}_i\}\right] = \mathrm{tr}\{\mathsf{E}\left[\mathbf{T}\mathbf{W}_i\right]\} = \mathrm{tr}\{\mathsf{E}\left[\mathbf{T}\right]\mathbf{W}_i\} \ . \quad (22)$$

Substituting (21) and (22) into (20), it immediately leads to the deterministic inequality (18). Note that to guarantee the validity of the probabilistic constraint (17), $\bar{\gamma}_{th_i}$ should be positive, namely $\bar{\gamma}_{th_i} > 0$, because of nonnegative-definite Wishart random variables.

Since both $\mathbf{W}_i$ and $(\tilde{\mathbf{H}}_{i_p}^H\tilde{\mathbf{H}}_{i_p})$ are semi-positive definite, the constraint (17) is convex. □

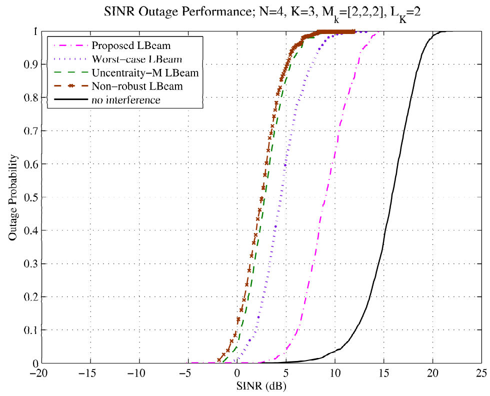


Fig. 1. SINR outage probability performance over multiple stream MU-MIMO system at $\text{SNR} = 10$ dB

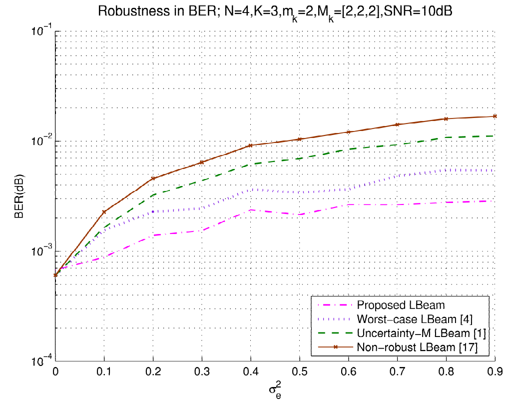


Fig. 2. Robustness in BER at $\text{SNR} = 10$dB

In order to convert the optimization problem into convex form, we introduce Lagrangian relaxation [22] to drop the rank constraint in (14), and only the positive semi-definite matrix constraint is left. In this case, we expect to find a lower bound solution $\mathbf{W}_i$ with a lower cost than (14).

Dropping the rank-one constraint in (14), reformulating the objective function (16) and probabilistic constraint (17), the proposed beamforming design can be reformulated as

$$\underset{\mathbf{W}_i}{\text{maximize}} \quad \text{tr}\left\{(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + M_i \sigma_e^2 \mathbf{I})\mathbf{W}_i\right\} \ , \tag{23}$$

$$\text{subject to} \quad \text{tr}\left\{\left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + \sigma_e^2 n_i \mathbf{I}\right)\mathbf{W}_i\right\} \leq p_i \overline{\gamma}_{th_i} \tag{24}$$

$$\text{tr}\{\mathbf{W}_i\} \leq 2 \ , \tag{25}$$

$$\mathbf{W}_i \geq 0 \ , \quad i = 1 \ , \ldots , K \ , \tag{26}$$

which can be efficiently solved by standard tools of mathematical programming [23]. Note that the rank of the solution $\mathbf{W}_i$ is usually higher than the rank of $\mathbf{C}_i$ and, therefore, the optimal weight vector cannot be directly recovered from $\mathbf{W}_i$. As suggested in [24], a common approach is to use *randomization techniques*. First, we generate a set of matrices which are distributed as $\mathcal{CN}(\mathbf{0}, \mathbf{W}_i)$, and then the *best* solution is selected among such randomly generated candidates. Due to the randomization, the constraint (24) may be violated by some of the weight matrix candidates. The feasible weight vector can be found by simply scaling the vector. Finally, the best candidate that satisfies the constraint (24) and maximizes the objective function (23) is selected as the solution.

## IV. Simulation

In our simulation, we consider a multi-user MIMO system with one base station (BS) equipped with 4 antennas and 3 users each equipped with 2 antennas. The data stream of each user with length $L_k = 2$ is generated using QPSK modulation, and the results are averaged over 1000 channel realizations. The proposed SLNR-based beamformer (abbr. as Proposed LBeam) (22)-(25) is compared with the worst-case SLNR-based beamformer (abbr. as Worst-case LBeam) [4], uncertainty-modified SLNR-based beamformer (abbr. as Uncertainty-M LBeam) [1], non-robust SLNR-based beamformer (abbr. as Non-robust LBeam) [17], and no-interference beamformer as a benchmark. Without loss of generality, we assume the following:

- The channel realization is generated by zero-mean and unit-variance i.i.d complex Gaussian-distributed. The variance of AWGN noise per receive antenna is assumed to be the same for all user, $\sigma_1^2 = \ldots = \sigma_i^2 = \sigma^2$. We set the variance of uncertainty as $\sigma_e^2 = 0.9$.
- Parameters in the probabilistic constraint (24): We set the normalized threshold $\overline{\gamma}_{th_i} = 0.9$ and $p = 5\%$.

To understand the behavior of the proposed algorithm, Fig. 1 gives the outage performance of SINR reliability (14) at $\text{SNR} = 10$dB. The proposed beamforming technique provides the lowest SINR outage probability, around $10\%$ at $SINR = 6$dB. It means for $90\%$ of the channel realizations, the achieved SINR is higher than 6dB. As shown in the figure, using the proposed beamformer results in an 5dB improvement in $10\%$ outage value compared to the worst-case SLNR-based beamformer, and an 7 dB improvement compared to other SLNR-based beamformers. Note that although it is a suboptimal solution with respect to SINR criteria, the proposed scheme still outperforms than all other beamformers. It is because that the leakage power from the desired user is suppressed at low threshold, which consequently tends to reduce the interference from all other users.

Fig. 2 illustrates the robustness of beamformers with respect to the BER performance. Given a certain SNR, the BER performance degrades with increased error variance. It shows that the proposed beamformer provides the strongest robustness to channel uncertainty, with absolute increase $2.2 \times 10^{-3}$, when the variance of error is varied from 0 to 0.99. In the same scenario, the worst-case SLNR-based beamformer performs better than using the uncertainty-modified SLNR-based and non-robust SLNR-based beamformers. The most serious performance degradation occurs when using the uncertainty-modified SLNR-based and non-robust SLNR-based beamformers, having absolute increase of $1.04 \times 10^{-2}$ and $1.52 \times 10^{-2}$ respectively.

## V. Conclusion

We proposed a probabilistic-constrained transmit beamforming design that combines the SLNR criteria with Alamouti code for the multiple-stream-per-user MU-MIMO communications. In such a hybrid scheme, the Alamouti scheme eliminates the inter-data-interference, while the probabilistic-constrained approach suppresses the inter-user-interference. The proposed beamformer maximizes the average desired signal power, while keeping a low outage probability of the leakage power larger than an acceptable level. The probabilistic constraint was transformed into a convex one. By introducing Lagrange relaxation, the underlying problem can be efficiently solved by modern software packages. Simulation results showed that the proposed beamformer provides the best SINR reliability and the highest robustness against imperfect channel information.

## References


[1] M. Sadek, A. Tarighat, and A.H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1711–1721, May 2007.

[2] M. Botros and T.N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE Journal on Selected Topics in Signal Processing*, vol. 1, no. 4, pp. 714–724, December 2007.

[3] X. Zhang, D.P. Palomar, and B. Ottersten, "Statistically robust design of linear MIMO transceivers," *IEEE Transactions on Signal Processing*, vol. 56, no. 8, pp. 3678–3689, August 2008.

[4] V. Sharma, S. Lambotharan, and A. Jakobsson, "Robust transmit multiuser beamforming using worst case performance optimization," in *IEEE Conference on Vehicular Technology Conference, 2008. (VTC Spring 2008.)*, May 2008, pp. 1132–1136.

[5] G. Zheng, K.K. Wong, and T.-S. Ng, "Robust linear MIMO in the downlink: A worst-case optimization with," *EURASIP Journal on Advances in Signal Processing*, pp. 1–15, 2008.

[6] N. Vucic and H. Boche, "Robust qos-constrained optimization of downlink multiuser MISO systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 714–725, Feburary 2008.

[7] I. Wajid, Y.C. Eldar, and A. Gershman, "Robust downlink beamforming using covariance channel state information," in *2009 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2009)*, June 2009.

[8] Y. Rong, S.A. Vorobyov, and A.B. Gershman, "Robust linear receivers for multiaccess space-time block-coded MIMO systems: A probabilistically constrained approach," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1560–1570, August 2006.

[9] W. Cheng, E.K.S Au, and V.K.N Lau R.D. Murch, "Closed-form outage probability and ber of mimo zero-forcing receiver in the presence of imperfect csi," in *IEEE 7th Workshop on Signal Processing Advances in Wireless Communications, 2006. (SPAWC'06.)*, June 2006, pp. 1–5.

[10] B.K Chalise and A. Czylwik, "Robust uplink beamforming based upon minimum outage probability criterion," in *IEEE 47th Global Telecommunications Conference, 2004. (GLOBECOM '04)*, December 2004, pp. 3974 – 3978.

[11] M. Botros Shenouda and T. N. Davidson, "Outage-based designs for multi-user transceivers," in *2009 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2009)*, 2009, pp. 2389–2392.

[12] B.K. Chalise, S. Shahbazpanahi, A. Czylwik, and A.B. Gershman, "Robust downlink beamforming based on outage probability specifications," *IEEE Transactions on Wireless Communication*, vol. 6, no. 10, pp. 3498–3505, October 2007.

[13] H.-Q Du and P.-J. Chung, "Robust leaking-based transmit beamforming with probabilistic constraint for downlink multi-user MIMO system," in *The 2009 IEEE Workshop on Statistical Signal Processing (SSP2009)*, September 2009.

[14] Q.H. Spencer, A.L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 461–471, February 2004.

[15] S.-J Kim, A. Magnani, A. Mutapcic, S.P. Boyd, and Z.-Q Luo, "Robust beamforming via worst-case SINR maximization," *IEEE Transactions on Information Theory*, vol. 56, no. 4, pp. 1539–1547, April 2008.

[16] A. Tarighat, M. Sadek, and A.H. Sayed, "A multiuser beamforming scheme for MIMO channels based on maximizing signal-to-leakage ratios," in *IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005. Proceedings. (ICASSP '05)*, 2005, pp. 1129–1132.

[17] M. Sadek, A. Tarighat, and A.H. Sayed, "Active antenna selection in multi-user MIMO communications," *IEEE Transactions on Signal Processing*, vol. 55, no. 4, pp. 1498–1510, April 2007.

[18] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, January 2004.

[19] S.M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, March 1998.

[20] J. Michael Steele, *The Cauchy-Schwarz Master Class: An Introduction to the Art of Mathematical Inequalities*, Cambridge University Press, 2004.

[21] D. Maiwald, *Breitbandverfahren zur Signalentdeckung und –ortung mit Sensorgruppen in Seismik– und Sonaranwendungen*, Dr.–Ing. Dissertation, Faculty of Electrical Engineering, Ruhr–Universität Bochum, Shaker Verlag, Aachen, 1995.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[23] M. Grant, S. Boyd, and Y. Ye, "CVX User's Guide for cvx version 1.1," (build 565), November 2007.

[24] N.D. Sidiropoulos, T.N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Transactions on Signal Processing*, vol. 54, no. 6, pp. 2239– 2251, June 2006.